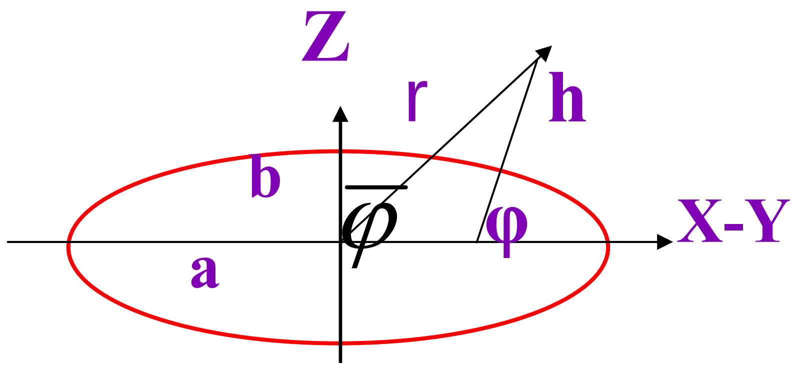


Meridian plane

$\bar{\varphi}$ = geocentric latitude

φ = geodetic latitude

r = radial distance, h = ellipsoidal height

a = semi-major axis, b = semi-minor axis

z = axis of rotation, 1900. flattening = (a-b)/a.

Coordinate-systems

Example:

Frederiksværk

$\varphi=56^0$, $\lambda=12^0$, h= 50 m

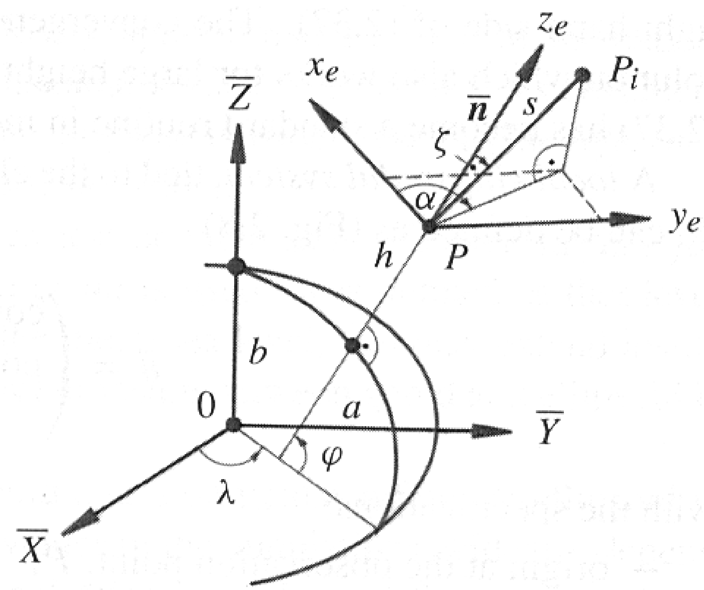
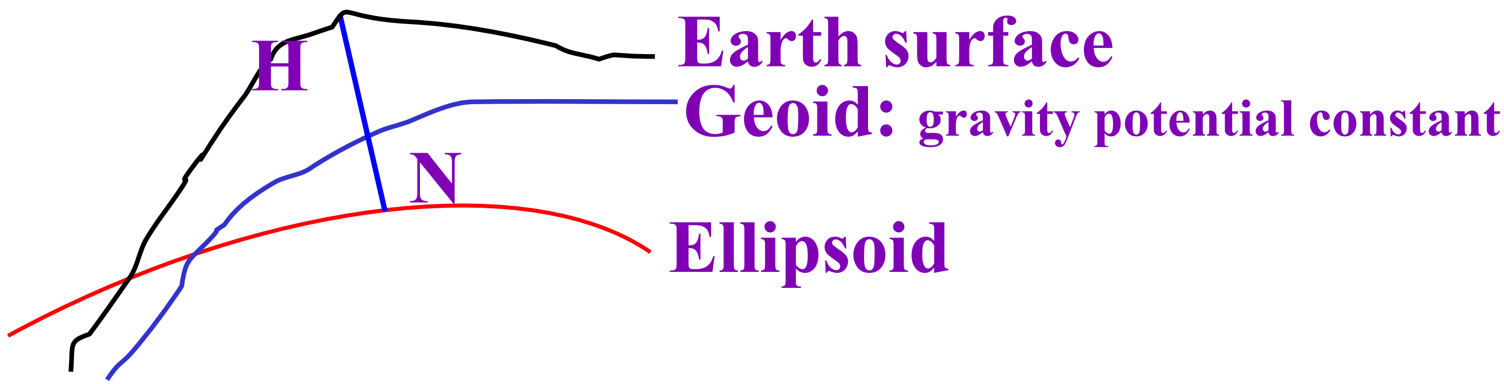


Figure 2.8. Global and local ellipsoidalsystems

Geoid and mean sea level



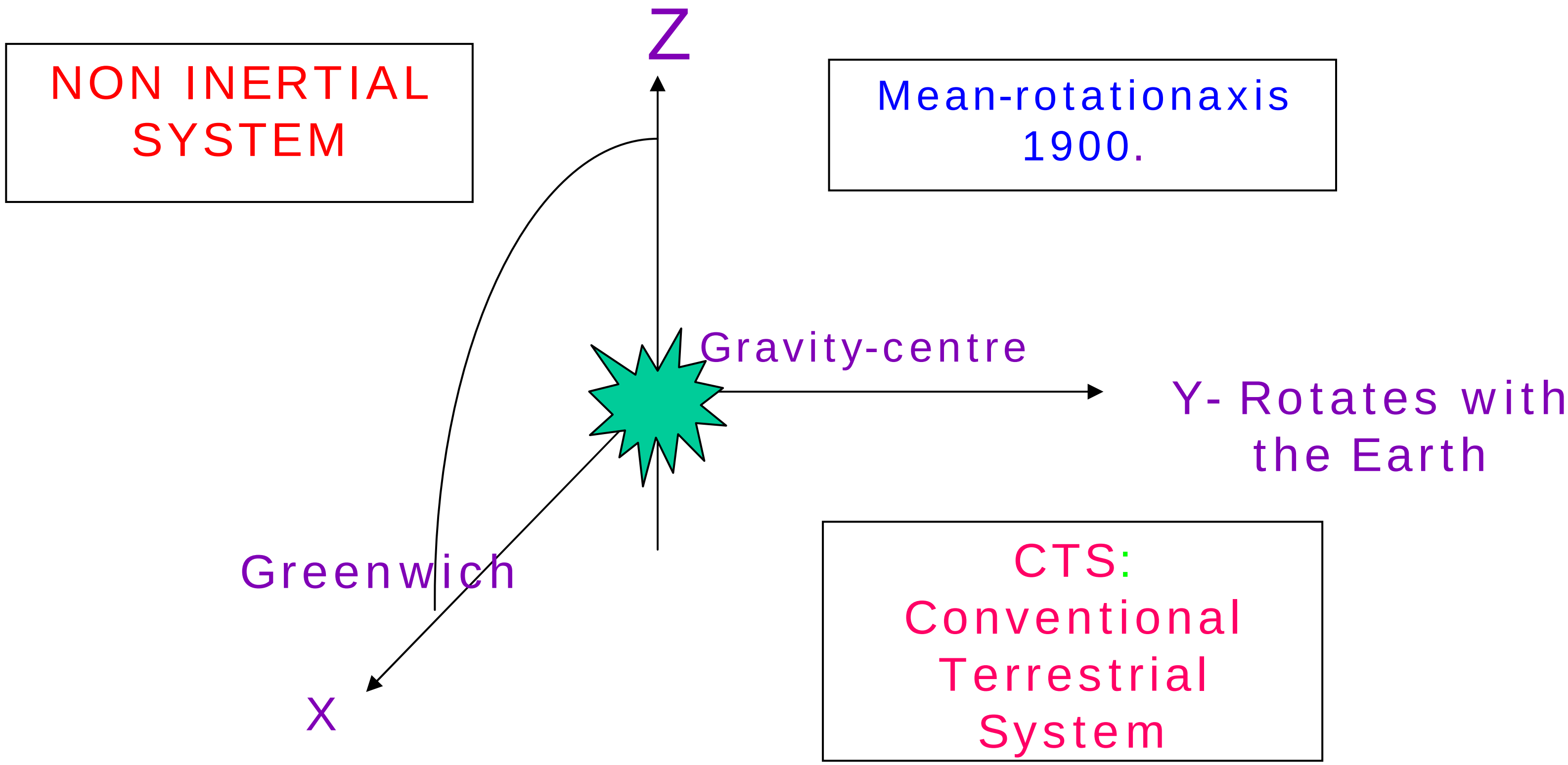
h=H+N=Orthometric height + geoid height

along plumb-line

$=H^N+\zeta$=Normal height + height anomaly,

along plumb-line of gravity normal field

Coordinate-systems and time.



POLAR MOTION

- Approximatively circular
- Period 430 days (Chandler period)
- Main reason: Axis of Inertia does not co-inside with axis of rotation.
- Rigid Earth: 305 days: Euler-period.

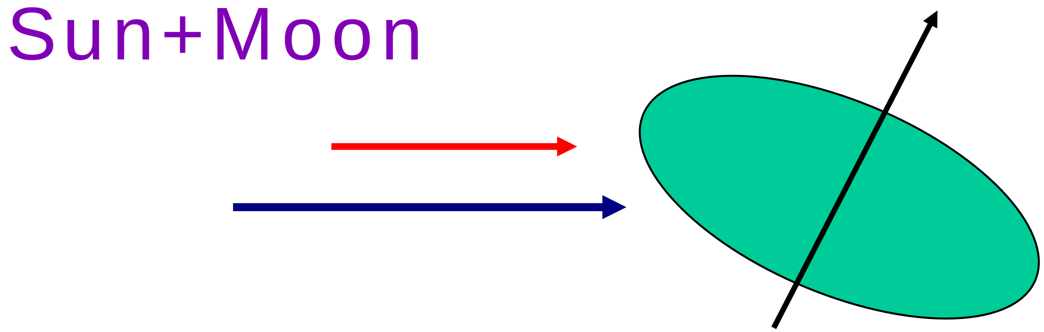
POLBEVÆGELSEN

- http://aiuws.unibe.ch/code/erp_pp.gif

- .

Ch. 3, Transformation CIS - CTS

- Precession
- Nutation
- Rotation+
- Polar movement



$$\vec{r}_{CTS} = SNP\, r_{CIS}$$

Gravity potential, Kaula Chap. 1.

- **Attraction (force):** $$F = k\frac{mM}{r^2}$$
- **Direction from gravity center of m to M.**
- **With m = 1 (unitless), then acceleration**

$$a = \frac{kM}{r^2} = \frac{\mu}{r^2}$$

Gradient of scalar potential, V,

$$\vec{a} = \nabla V = \begin{Bmatrix} \dfrac{\partial V}{\partial x} \\ \dfrac{\partial V}{\partial y} \\ \dfrac{\partial V}{\partial z} \end{Bmatrix}, \quad V(x, y, z) = \frac{k \bullet M}{r} \text{ point - mass}$$

Volume distribution, ρ(x,y,z)

$$V(x',y',z') =$$

$$\iiint_{Earth} k \frac{\rho(x,y,z)}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}\, dx\, dy\, dz =$$

$$k \iiint_{Earth} \frac{\rho}{r} d\Omega$$

- **V fulfills Laplace equation**

$$\Delta V = \frac{\partial^2}{\partial x^2}V + \frac{\partial^2}{\partial y^2}V + \frac{\partial^2}{\partial z^2}V = 0, \text{outside masses}$$

Spherical coordinates

- **Geocentric latitude** $\overline{\varphi}$
- **Longitude, λ, r = distance to origin.**

$$x = r\cos\overline{\varphi}\cos\lambda$$

$$y = r\cos\overline{\varphi}\sin\lambda$$

$$z = r\sin\overline{\varphi}$$

$$\text{Volume-measure}: d\Omega = dxdydz = \cos\overline{\varphi}r^2 d\overline{\varphi}d\lambda dr$$

Laplace in spherical coordinates

$$\Delta V = \frac{1}{r^2}\left[\frac{\partial}{\partial r} r^2 \frac{\partial V}{\partial r}\right] + \frac{1}{\cos\overline{\varphi}} \frac{\partial}{\partial \overline{\varphi}} (\cos\overline{\varphi} \frac{\partial V}{\partial \overline{\varphi}}) +$$

$$\frac{1}{\cos\overline{\varphi}^2} \frac{\partial^2 V}{\partial \lambda^2}$$

$$\text{Solution:} V = R(r)\Phi(\overline{\varphi})\Lambda(\lambda),$$

$$R(r) = r^{-n-1} \text{ or } r^n,\ \Lambda(\lambda) = \cos m\lambda \text{ or } \sin m\lambda$$

$$\Phi(\overline{\varphi}) = P_{nm}(\sin\overline{\varphi}), \text{n} = \text{degree}, \text{m} = \text{order}.$$

Spherical harmonics

- **Define:**

$$V_{nm}(r,\overline{\varphi},\lambda)=\frac{a^n}{r^{n+1}}P_{n|m|}(\sin\overline{\varphi})\begin{cases}\cos m\lambda, & m\geq 0\\ \sin|m|\lambda, & m<0\end{cases}$$

then

$$V(r,\overline{\varphi},\lambda)=kM\sum_{n=0}^{\infty}\sum_{m=-n}^{n}C_{nm}V_{nm}(r,\overline{\varphi},\lambda)$$

Orthogonal basis functions

- **Generalizes Fourier-series from the plane**

$$\int_{-90}^{90}\int_{0}^{180} V_{nm}(r,\overline{\varphi},\lambda)V_{ij}(r,\overline{\varphi},\lambda)\cos\overline{\varphi}\,d\overline{\varphi}\,d\lambda =$$

$$\begin{cases} kMC_{nm} \text{ for n = i, m = j} \\ 0 \qquad \text{for } n \neq i \text{ or } m \neq j \end{cases}$$

Functions may be (fully)normalized, then

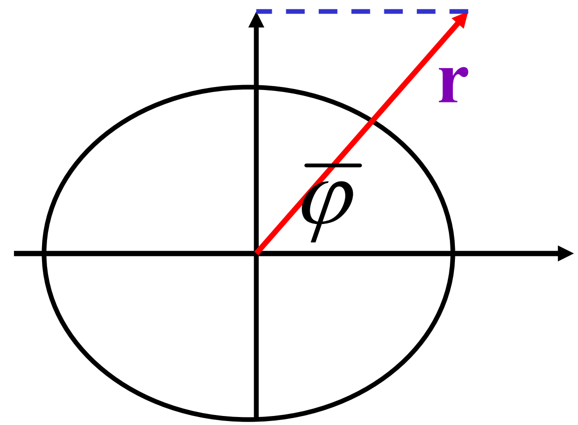
$$\mathrm{C}_{\mathrm{nm}} \rightarrow \overline{\mathrm{C}}_{\mathrm{nm}} \text{ and } P_{nm} \rightarrow \overline{P}_{nm}$$

Centrifugal potential

- **On the surface of the Earth we also measure the centrifugál acceleration,**

$$W = V + \frac{1}{2}\omega^2 r^2 \cos^2 \overline{\varphi} =$$

$$V + \frac{1}{2}\omega^2 (x^2 + y^2)$$



$\omega =$ rotational velocity (in inertial space).

Normal potential, U

- Good approximation to potential of ideal Earth
- Reference ellipsoid is equipotential surface, U=U0, ideal geoid.
- It has correct total mass, M.
- It has correct centrifugal potential

$$U = \left[1 - J_2 (\frac{a}{r})^2 P_{20}(\sin \overline{\varphi}) - J_4 (\frac{a}{r})^4 P_{40}(\sin \overline{\varphi}) + ... \right] + \frac{\omega^2}{2} r^2 \cos^2 \overline{\varphi}$$

$$J_2 = \frac{2}{3} f (1 - f/2) - m/3(1 - \frac{3}{2} m - \frac{2}{7} f) + ...$$

$$m = \omega^2 a / g_e, \text{(gravity at Equator)}$$

- Knowledge of the series development of the gravity potential can be used to derive the flattening of the Earth !

Anomalous potential,T

- T=W-U,
- same mass and gravity center.
- Makes all quantities small,gives base for linearisation.

Gravity disturbance: $\delta g = g_P - \gamma_P \approx \frac{\partial T}{\partial r}$

gravity minus normal gravity in P, with ellipsoidal height h.

Gravity anomaly: $\delta g = g_P - \gamma_Q \approx -\frac{\partial T}{\partial r} - \frac{2}{\mathrm{r}} T$

normal gravity in point with height H.

Height anomaly: $\zeta = T / \gamma$

Geoid height = height anomaly on the ocean.