

**Exercises in Satellitegeodesy,**

**Prepared by C.C.Tscherning**

**October 2005.**

**University of Copenhagen.**

### Exercise 1.

1.1. A satellite has  $i = 30^\circ$ . The Earth is regarded as spherical with a radius equal to 6371000 m.

Calculate the semi-major axis ( $a$ ), which makes the satellite visible above the horizon for the latitudes  $60^\circ$  and  $90^\circ$ .

1.2. Regard a satellite with a circular orbit and  $i = 60^\circ$ ,  $\Omega = \omega = 0^\circ$ . Compute the velocities for  $a = 7000, 7300$  and  $800$  km. The Earth is as in exercise 1.1 regarded as homogenous and spherical.

Where do the orbits intersect the Equator the first time after one revolution? (All satellites are supposed at time  $t = 0$  to be at point  $(a, 0, 0)$ ).

Where are the satellites after 30 minutes?

1.3.

(1) Calculate the Kepler elements for a central movement around a homogenous, spherical Earth, where we for  $t = 3$  hours have:

:

$X = (3500000 \text{ m}, 3500000 \text{ m}, 4949747.5 \text{ m})$

$v = (-5331.5851 \text{ m/s}, 5331.5851 \text{ m/s}, 0.0 \text{ m/s})$

(2) What is the geocentric longitude of the satellite after precisely one revolution?

### Exercise 2.

2.1. Derive  $[\Omega, e]$ ,  $[a, e]$  og  $[a, M]$  (Kaula, Kap. 3).

2.2 What are the values of the Delaunay-variables in the point in exercise 1.3?

2.3. Derive the expression in Kaula equation (3.78).

2.4. Suppose the potential is given by

$$V(\varphi, \lambda, r) = \frac{GM}{r} \left(1 - J_2 \left(\frac{a}{r}\right)^2 P_2(\sin\varphi)\right)$$

Write down the potential expressed in  $(X, Y, Z)$ .

What is the value of the potential in the point in ex. 1.3 ?

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Use in the following the WGS84 system.

A satellite has on 21 May, 2004 at 0 h (time  $t_0 = 0$ ) the Kepler elements:

$a = 7300000$  m,  $i = 80.0^\circ$ ,  $e = 0.000$  (circular orbit),  
 $\Omega = 45.0449^\circ$ ,  $\omega = 0.1099^\circ$  and  $f = 196.3277^\circ$ .

2.5. How long time will it take until  $\Omega$  has changed  $360^\circ$  ?

2.6. Compute the Kepler-elements at  $t_1 = 3600$  s with  $0.0001^\circ$  precision, taking into account the coefficient  $J_2$ .

2.7. Compute the coordinates and velocity vector of the satellite in the inertial system (CIS= with 10 cm precision at time  $t_1$ .

2.8. A satellite has a circular orbit with  $a = 7070$  km. Which inclination (I) must the orbit have in order to become sun-synchronous. ?

### Exercise 3.

3.1 A point A on the surface of the Earth has geocentric latitude  $40.9238^\circ$ , longitude  $-30.6685^\circ$  and ellipsoidal height  $h = 0$  m in the WGS84 coordinate system (CTS). To this corresponds the Cartesian coordinates

$$A = (X, Y, Z) = (3966985 \text{ m}, -2272853 \text{ m}, 4432312 \text{ m}).$$

On May 21, 1994 are the coordinates of the Pole coordinates  $x_p = 0.1866''$  and  $y_p = 0.2913''$ . Greenwich sidereal time (GAST) is at this moment ( $t_1$ ) 18.0 hours. We will suppose that precession and nutation both are 0. What are the coordinates of A in the inertial system (CIS) ?

3.2 What is mean sidereal time in Greenwich 18 sept. 1998, 00.00 (GMST) ?

What is local mean sidereal time when the longitude is 12.5 degrees ?

What is local sidereal time (LAST) when we suppose it is New Moon, that  $F = 0$  in Seeber equation (2.21) and that the rising node of the Moon has the longitude 45 degrees

### Exercise 4

(1) A signal with the frequency 1.2 GHz. pass vertically through the F layer of the ionosphere. from the altitude of 800 km to 200 km. The electron density is supposed to be equal to the values given in Table 2.5. (Check first whether the values are correct!).

How long time does it take for the signal to propagate in vacuum ?

How long time does it take for the signal to propagate during daytime and at night considering the electron-density.

What are the corresponding distance differences ?

Repeat the calculations for the frequency  $f = 5.0$  GHz.

(2) Suppose a signal has the frequency  $f = 1.2$  GHz., which is received at a point with altitude  $h = 0$  m. Here the temperature is 293.0 K, the pressure 1020.0 HPa, the partial vapour pressure 20 HPa.

What is  $H_D$  ? (Seeber, Eq. (2.112)).

What is the distance correction for the height-angles 90 degrees and 45 degrees ?

### Exercise 5.

5.1. Suppose the Moon and a satellite are both on a straight line through the gravity center of the Earth in the Equatorial plane. The orbital radius is  $r_0 = 6678137$  m. The distance to the Moon is set to 60 earth-radia. What is the relative acceleration of the satellite due to the attraction of the Moon ?

Suppose the orbit is circular, and that the inclination of both the satellite and the Moon is 18 degrees. What are then the changes of the Kepler-elements of the satellite  $\Omega$   $og$   $\omega$  .

In the following we regard the same satellite as in ex. 5.1 and with the same position.

5.2 What is the magnitude of the acceleration due to the part of the earth-tide originating from the Moon ? We put the Love number  $k_2 = 0.3$ .

5.3. What is the state-vector (position and velocity) ? What is the velocity vector of the atmosphere ?

We suppose that the satellite is a ball with mass 200 kg and a radius of 1 m. The density of the atmosphere is put equal to  $30 \text{ g/m}^3$ . What is the drag-acceleration vector ?

5.4. The reflectivity of the satellite is 2.0. We are at the equinox and the Sun is in the direction  $90^\circ$  from the direction from the satellite to the Earth. What is the effect of the solar pressure on the satellite ?

### Exercise 6, not included.

### Exercise 7. Examples of the application of Least Squares Adjustment, LSQ

In the following we will suppose all errors to be normally distributed..

7.1. Adjustment of levelling network.

The following altitudes have been observed::

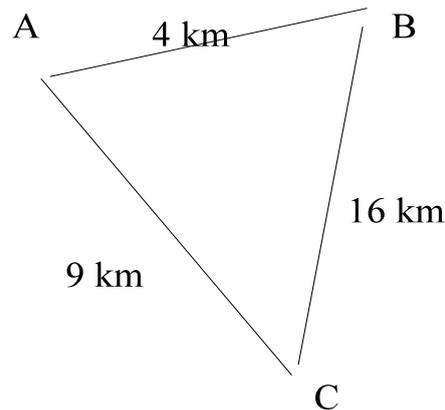
$$\begin{aligned} h_B - h_A &= 1371 \text{ mm} \\ h_C - h_B &= 2521 \text{ mm} \\ h_A - h_C &= -3888 \text{ mm} \\ h_A - h_C &= -3890 \text{ mm} \\ h_A &= 10031 \text{ mm} \end{aligned}$$

The errors of all observations will first be supposed to have a standard-deviation of 1 mm.

Write down the observation equations

$$A x = b,$$

where  $x$  is a vector with the 3 altitudes as elements. Use LSQ for the calculation of estimates of the 3 altitudes, their standard-deviation and the correlation between the errors.



What is the estimate of the errors of the observations ?

Now, suppose the observations have a standard-deviation of  $0.5 \text{ mm} * \sqrt{\text{distance in km}}$ , where the distances in km can be seen on the figure. Repeat the adjustment with the new weights (i.e. calculate the adjusted altitudes and their standard-deviations).

7.2

Suppose we have 3 points A, B, C, which all are in the same plane. A has the coordinates (100 cm, 200 cm), and B has the coordinates (100 cm, 600 cm). Determine using LSQ the coordinates of C. We have a preliminary estimate of (410 cm, 590 cm). A and B's coordinates may be regarded as exact. There have been measured 2 distances  $D(A,C) = 500 \text{ cm}$ ,  $D(B,C) = 300 \text{ cm}$ , both with a standard-deviation of 1 cm.

Compute an estimate of the coordinates of C as well as of the standard-deviations of the estimates.

### Exercise 8.

8.1. Use the position and velocity vector from a satellite as calculated in exercise 2.7

A point P has the coordinates in a CTS:

$$(X,Y,Z) = (3966985 \text{ m}, -2272853 \text{ m}, 4432312 \text{ m})$$

In exercise 3.1 the coordinates of the point has been calculated in a CIS.

What is the vector between P and the satellite in the CIS ? What is the angle between the velocity vector and the connection vector ?

8.2. The satellite transmit a signal on 400 MHz. What is the Doppler effect on the signal received in P ?

8.3. Easier:

The satellite transmit signals on 400 Mhz and 150 Mhz.

We are on the North-pole ! A satellite has inclination  $I = 90^\circ$ , a circular orbit and velocity  $v = 7.5$  km/s. The Earth is regarded as spherical with radius 6371 km.

What is the semi-major axis of the satellite ?

What is the Doppler shift observed at rise and set ? What does it correspond to as a distance-change per minute ?

8.4. The satellite transmit signals with the frequency as in exercise 9.3. The semi-major axis is 26500 km, and the satellite is in zenith.. What is the distance difference after 30 sec. ? (Suppose the angular velocity  $I = 0$ ).

## Exercise 9.

Exercise 9.1:

The exercise illustrates point-positioning with GPS.

Preliminary coordinates (in m):

(X,Y,Z)        3517250.0    784660.0    5244920.0

Observations:

Sat	Tid	X	Y	Z	Distance
16	468262.6	19882818	-4007732	17137390	20790130.3
23	468262.6	-8318660	-15394931	20196264	25008215.1
26	468262.6	9358156	-18567258	16719423	23243865.2
27	468262.6	14687857	7259303	20670310	20115810.1

Calculate improved coordinates with 1 m precision (significant digits) as well as the time-correction. Use a pocket calculator.

9.2:

The exercise illustrates GPS point-positioning from 2 sets of data, with different start values and error-estimates.

Now use the program dixyz on the Unix-system. (/cct/cctf/dixyz ).

Observations are found in the file /cct/cctf/station620 .

Besides these data will the program request error-estimates as input. Use 1.0, 0.5, 0.1 og 0.01 m as error estimates and compare the result.

The programme will producer a file, where the new coordinate are found. They may be used as a new input in order to check the linearity. Run the program with these new values as input with error-estimates set to 0.1 m.

Delete 2 observations, and check how the results change (error estimates 0.1 m).

Make a new file with name station620.new, where the initial coordinates are 1000 m different in x, y, z, and the error-estimates are 0.1 m. The program produce a file, where the new coordinates are found. Use this as new input in order to check the linearity.

9.2.

Given GPS distance-observations from 4 satellittes to 2 stations (Buddinge and Nikolai Kirke), to 2 different times. Data are in the file /cct/cctf/station620 and station41. The error is estimated to 0.1 m.

Preliminary coordinates (X,Y,Z) in m for Buddinge:

```
3513648.63 778953.56 5248202.81
SRN time XS YS ZS /Dist
16 468262.6510 19882818.33 -4007732.61 17137390.13
20789777.704
16 468292.6510 19996259.77 -3943021.90 17019719.27
20797647.028
23 468262.6510 -8318660.05 -15394931.96 20196264.65
25000854.787
23 468292.6510 -8167330.60 -15381072.91 20267446.59
24963363.171
26 468262.6510 9358156.02 -18567258.83 16719423.93
23238398.488
26 468292.6510 9385677.53 -18444555.04 16841759.75
23204193.990
27 468262.6510 14687857.19 7259303.26 20670310.06
20117138.378
27 468292.6510 14562396.87 7361029.17 20720638.02
20119513.020
```

Preliminary coordinates for Nicolai Kirke:

```
3517254.92 784665.16 5244923.62
SRN time XS YS ZS /Dist
16 468262.6510 19882818.33 -4007732.61 17137390.13
20790130.361
16 468292.6510 19996259.77 -3943021.90 17019719.27
20797943.579
23 468262.6510 -8318660.05 -15394931.96 20196264.65
25008215.119
23 468292.6510 -8167330.60 -15381072.91 20267446.59
24970718.883
```

26 468262.6510 9358156.02 -18567258.83 16719423.93  
 23243865.265  
 26 468292.6510 9385677.53 -18444555.04 16841759.75  
 23209651.627  
 27 468262.6510 14687857.19 7259303.26 20670310.06  
 20115810.169  
 27 468292.6510 14562396.87 7361029.17 20720638.02  
 20118186.762

Use the programme dixyz for the calculation of coordinates for Nicolai Kirke (Buddinge was determined in exercise 8).

What is the single and the double differences for the 2 stations and the satellite 16, 23, 26 and 27 ?

Calculate the coefficients in the observation-equations in the two cases.

How does the normal-equations look? What is the dimension of the solution-vector

May they be used for the determination of the coordinate improvements ?

What must we require before they can be used ? What would a phase-measurement have been in Buddinge to satellite 16 on the L1 frequency ? Errors from Ionosphere and Troposphere are supposed to be 0, and the clock-error also 0.

## Exercise 10

### GPS data processing.

#### 10.1 Data collected with GPS-receivers in the stations:

1-13-00001 602 Buddinge  
 2-04-00871 Oksbjerget  
 K-01-00125 Bro over Hareskovvejen samt  
 Tårnet på Rockefeller-komplekset

are to be processed using Trimbles standard program TotalControl. The processeing will be done under Windows using a PC. Instructions will be distributed..

Data can be found in the directory G:\group\fast\gps2005

Station 620 Buddinge is used as a reference station. The coordinates ini WGS84 are

Latitude: 55° 44' 19.74589", Longitude: 12°44'19.74589", altitude = 50.663 m,  
 and the geoidheight 35.909 m. (What is the ellipsoidheight ?).

Use TotalControl for the determination of the other statione coordinats in WGS84 as well as the distances between the stationern.

What is the estimated error of the coordinat differences, and how are they correlated ?

10.2 The coordinates in UTM for the 3 points in ED1950 will be distributed during the lectures.

Compare the distances calculated from the coordinates and from the GPS observations.

10.3. What is the height above mean sea level in the 2 points, where we know that the geoid heights in the two points are

2-04-00871 36.007 m

K-01-00125 35.909 m.

10.4. Use the programme TRANS13 for the calculation of the latitude and the longitude in ED1950. Here is the geoid height in all points 3.000 m.

10.5. We now have 3 points, where we know the coordinates in WGS84 og ED1950. What is the datum-shift between the two systems expressed as a translation vector ? How well does it fit in the stations ?

Would it be possible to express the datum-shift as a translation and 3 rotation-angles ?

The datum-shift may be expressed as a change of latitude and longitude. How big is the error if it is expressed in this manner ?

### **Exercise 11. Altimeter data.**

Select an area which has oceanographic interest, e.g. close to a major current.

Altimeter data from NASA's Pathfinder data-set from the area must be picked-up from the database on ikos. It is possible to use data from TOPEX/Poseidon or from ERS-1/2. Initially the data are extracted.

Data consist of an altitude (of the ocean), and the deviation from a mean surface for each measurement. Make a color contour-plot using GMT. Template for a job can be found in /cct/geod . The output from GMT is stored as a post-script file. Use names for the files so that they are ordered sequentially, (alt01, alt02, ..., alt77).

Show the files as an animation using the program "amin".

The following steps must be done:

- (1) change directory (cd) to /cct/geod/pathfinder and start the programme readtpxc.
- (2) select an area and extract Topex data from 12 or more consecutive cycles (> 10), not 21 or 118.

The programme give output-data on the format

Cycle-number, latitude, longitude, (degrees), h, h-mean (m), time (s).

- (3) Name data from each cycle as e.g. tpx23.dat so that the cycle number is a part of the name..

- (4) Prepare, and run the job which use GMT (area boundaries and input-file name must be changed) /cct/geod/altimetry.gmt,. Check that the contour interval is correct.

- (5) Change the standard name altimetry.ps to e.g. tpx23.ps

When all files have been created show an animation using "amin".

Is it possible to see a current ? Is there an effect of un-modelled tide ? Is it possible to see temperature effects ?

For the more advanced:

Use the program /cct/geocol17 to subtract the contribution from EGM96. A template to do this is found as /cct/geod/geocol.egm. The name of your own files must be inserted.

Make a new contour-plot, and change the figure legend and the contour interval.

## Exercise 12. SST and SGG.

12.1. Regard the term (2,0) in the spherical harmonic expansion for the gravity potential of the Earth. What is the largest difference between the contribution of this term to the radial and north-going components of the 2 order derivatives of the potential for a satellite (GOCE) with an inclination  $82^\circ$  and a circular orbit with radius 6630000 m. What does this say about the measurement capability or measurement area of the GOCE gradiometer ?

12.2. Suppose we have 2 accelerometers with a distance of 0.5 m in a satellite as in 12.1. It is situated above the Equator. The noise of the two accelerometers is uncorrelated. (This is not realistic).

With which precision must the accelerometers measure the gravity gradient in order to achieve a precision of  $1 \cdot 10^{-12} \text{ s}^{-2}$  ?

12.3. In exercise 2.6 the position of a satellite was computed supposing that its orbit was determined by the terms 0,0 og 2,0 in the spherical harmonic development. What would the (X,Y,Z) position has been if we disregarded the (2,0)-term ?

12.4. Data from the Ørsted Satellite are stored in /cct/MARTIN for one single day with 1 s sampling. Select 5 positions and velocities with a 4 hours interval and use the associated Kepler-elements for the computation of  $C_{20}$ . How big are the differences between the values and what can we say from this about the uncertainty of the quantity ?

The programme /cct/geod/cake may be used in the computations.

12.5 What is the gravity and the radial gravity gradient for a sphere with the same mass as the Earth and radius 6371000 m in the altitudes 0 m, 100000 m, 250000 m and 700000 m ?

Suppose we have a sphere with radius  $r$  and center on the surface of the Earth with mass-density  $d$ , and suppose we in a satellite are able to measure the radial gravity gradient with a precision  $e$ . Compute the radius of the sphere,  $r$ , which is necessary in order that the gravity gradient in the satellite is larger than  $e$ .

Compute values of  $r$  for  $e = 0.01 \text{ EU}$ ,  $e = 0.001 \text{ EU}$ ,  $d = 0.1 \text{ g/cm}^3$ , og  $r = 10 \text{ km}$ ,  $100 \text{ km}$ ,  $250 \text{ km}$  and  $700 \text{ km}$ .