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Riyadh workshop. Exercise 1.5.

1.5.1. The centrifugal potential is given by

$$\Phi(r,\overline{\varphi}) = \frac{\omega^2}{2}(r\ \cos(\overline{\varphi}))^2 = \frac{\omega^2}{2}(X^2 + Y^2)$$

Compute the value at Equator expressed in m^2/s^2. Use T=7.292115x10^{-5} radians/s and semi-major axis a=6378137.0 m.

1.5.2. A point has the geodetic coordinates ν = 56°, 8 = 10°, h = 0 m, where (X,Y,Z),

$$\overline{r} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 3520532.589\ m \\ 620764.882\ m \\ 5264442.236\ m \end{pmatrix}$$

r= 6363477.886 m, $\overline{\varphi} = 55.82$ degrees.

What is the value of the centrifugal potential M ?

1.5.3. Calculate the gradient of M in the point expressed in local spherical coordinates.

1.5.4. The Earth is regarded as spherical and homogeneous with GM = 3.986005 * 10^{14} m^3/s^2. Calculate the **gradient** in a point with spherical coordinates as used in exercise 1.5.2.

1.5.5. Calculate the value of the Laplace operator applied on *M.*

1.5.6. A simple approximation to the potential W of the Earth is given by

$$W(r,\overline{\varphi},\lambda) = \frac{GM}{r}(1 - J_2\frac{a^2}{r^2}P_2(\sin\overline{\varphi})) + \frac{\omega^2}{2}\left(r\cos\overline{\varphi}\right)^2$$

with constants (GM, a, J_2, T) from GRS80,

$$GM = 3.986005\times 10^{14}\ [m^3/s^2]$$
$$a = 6378137\ m$$
$$b = 6356752.314\ m$$
$$J_2 = 1.08263\times 10^{-3}$$

$\omega = 7.19115 \times 10^{-5} \ m$

Calculate the potential at the following points, all having the height, h, above the ellipsoid equal to zero: North pole, Equator with 8 = 0 and the point in exercise 2 .

Note the order of magnitude in the variations of W between the 3 points. What is the corresponding heights, N, of the geoid in GRS80 ? Use that the GRS80 normal potential U_0 at the ellipsoid is equal to 62636860.850 m^2/s^2 and Bruns formula $N=(W-U_0)/\gamma$, with $\gamma=9.8$ m/s^2.