Lecture 2.1. Height anomaly, geoid height, gravity anomaly+disturbance, deflections of the vertical. T/M, Ch. 4.

Astronomical system: Φ_A, Λ_A

astronomial longitude and latitude.

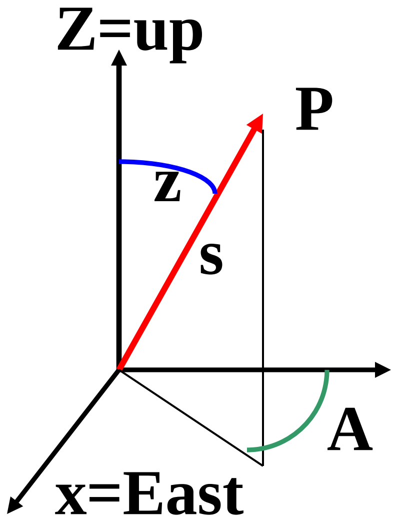
W: value of potential, g= $|\nabla W|$

$$\mathbf{g} = \nabla \mathbf{W} = -\mathbf{g} \cdot \mathbf{n} = -g \begin{Bmatrix} \mathbf{\cos\Phi_A \cos\Lambda_A} \\ \mathbf{\cos\Phi_A \sin\Lambda_A} \\ \mathbf{\sin\Phi_A} \end{Bmatrix}$$

$$\Phi_A = \arctan \frac{-W_z}{\sqrt{W_x^2 + W_y^2}}, \Lambda_A = \arctan \frac{W_y}{W_x}$$

Local astronomical system

Plumb-line-oriented.



z=zenith-distance

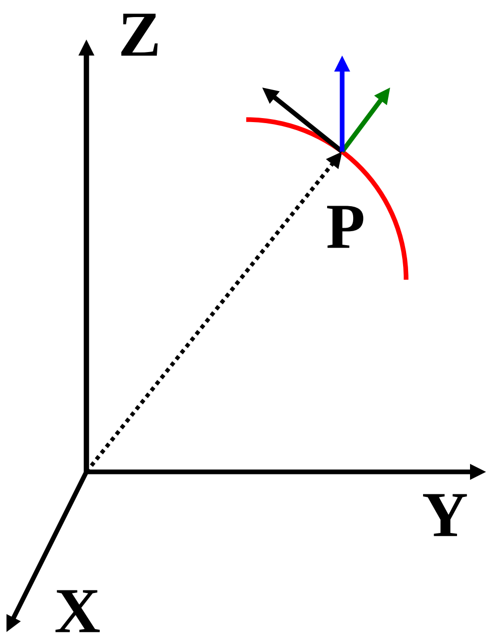
A=azimuth,

positive with the clock, from north

y=North

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = s \begin{Bmatrix} \sin z \sin A \\ \sin z \cos A \\ \cos z \end{Bmatrix}$$

Local astronomical system II,



y - North

x - East

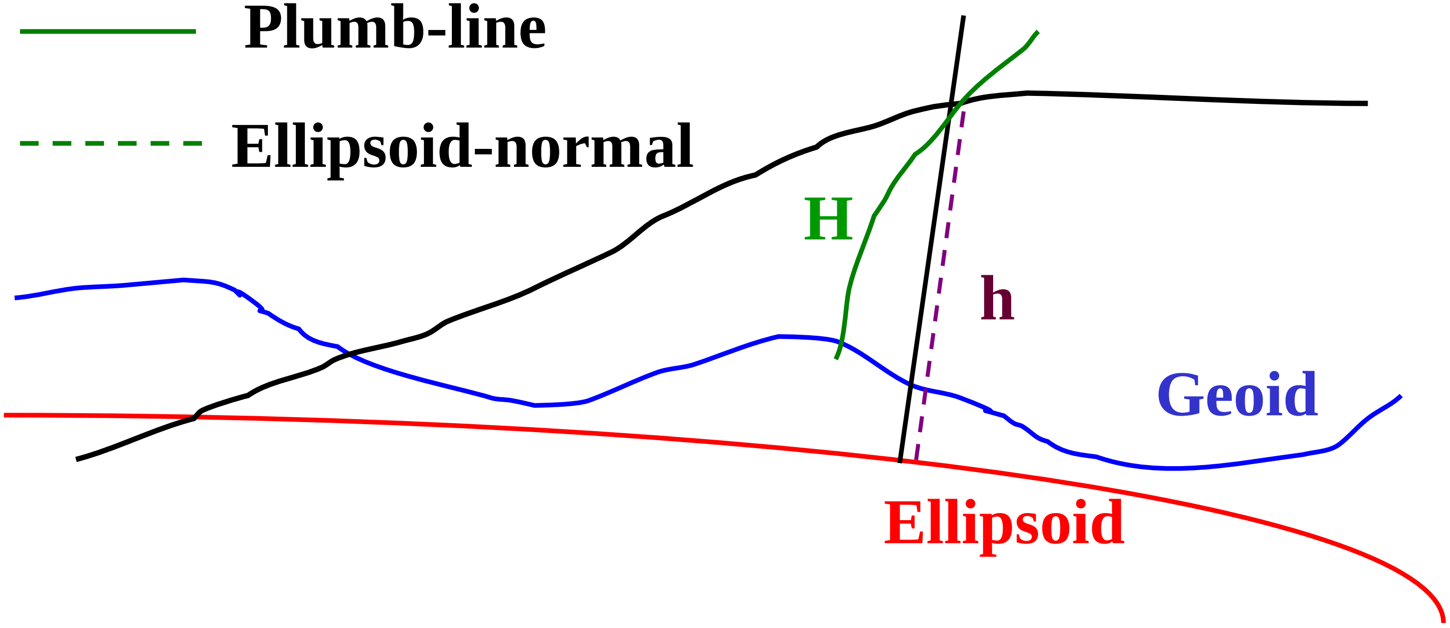
z - Up

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = A^{-1} \begin{Bmatrix} X - X_P \\ Y - Y_P \\ Z - Z_P \end{Bmatrix}$$

$$A^{-1} = \begin{Bmatrix} -\sin\Lambda & \cos\Lambda & 0 \\ -\sin\Phi\cos\Lambda & -\sin\Phi\sin\Lambda & \cos\Phi \\ \cos\Phi\cos\Lambda & \cos\Phi\sin\Lambda & \sin\Phi \end{Bmatrix}$$

The Geoid as a reference-surface.

As altitude must be used C= $W_0 - W_P = -\int_0^P dW = \int g dn$



Geopotential number: Unit gpu, 100 m^2/s^2=kgal x m

Normal-potential, U., p. 101

Approximation U to W, which

(1) – represent the "normal" gravity variation as a function of latitude and altitude.

(2) - T=U-W, anomalous potential, enhances geophysically interesting mass anomalies.

(3) - U generated of a "nice" mass-distribution with correct GM

(4) – has an equipotential surface $U=U_0$, which is the ellipsoid.

The Normal-gravity.

Normal gravity on Equator: γ_a

…………………….. Poles : γ_b

Pizetti showed: $$2\frac{\gamma_a}{a} + \frac{\gamma_b}{b} = \frac{3GM}{a^2 b} - 2\omega^2$$

Clairout: $$\beta = \frac{\gamma_b - \gamma_a}{\gamma_a} \quad \textit{gravity - flattening}$$

$$f + \beta = F(\omega, a, b, \gamma_a)$$

Geometry/gravity, p. 104.

Show interconnection between the flattening of the Earth and the change of gravity.

Or from $\gamma_a, \gamma_b, \omega, a$

Is it possible (iteratively) to find f and through this b (semi-minor axis).

Helmert (1901) found from 1400 gravity values

$$\gamma_a = 9.7803 m/s^2, \beta = 0.005302$$

$$f = 1/298.3$$

Normal-potential in spherical harmonics.

(USED FOR HEIGHTS above 1 km !)

$$U(\beta,\lambda,u) = U(\overline{\varphi},\lambda,r) =$$

$$\frac{GM}{r}\left(1 - \sum_{i=1}^{\infty} \left(\frac{a}{r}\right)^{i} J_{2i} P_{2i}(\sin\overline{\varphi})\right)$$

$$+ \frac{\omega^2}{2} r^2 \cos\overline{\varphi}^2$$

$$J_2 = -C_{20}$$

From this we easily calculate $\vec{\gamma} = \nabla U$,

Normalgravity, p. 104

$$\gamma = \left| \nabla U \right|$$

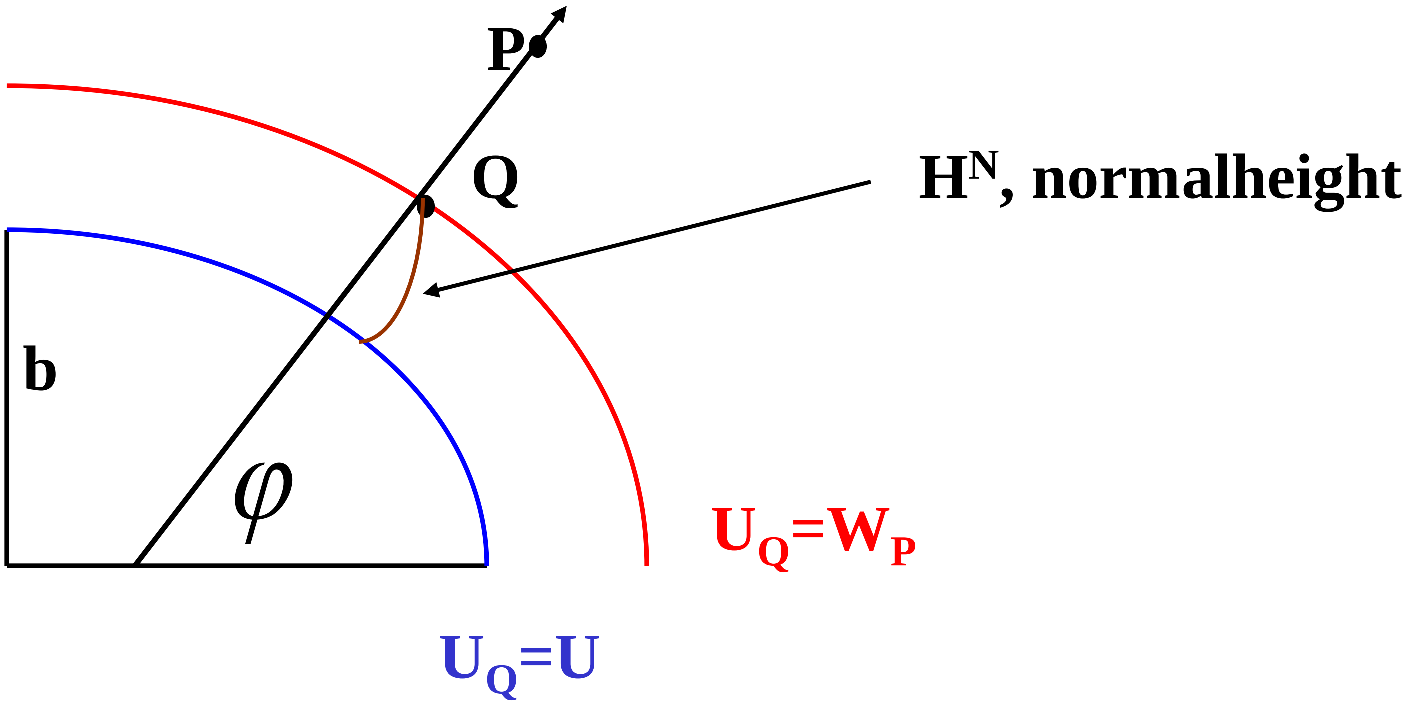
Approximative expression (Not to be used if h > 1 km !)

$$\gamma_0(\varphi, h = 0) = \gamma_a (1 + \beta \sin^2 \varphi + \beta_1 \sin^4 \varphi \ldots)$$

$$\gamma(\varphi, h) = \gamma_0 (1 + \frac{2}{a}(1 + f + m - 2f \sin^2 \varphi) h + \ldots)$$

$$m = \frac{\omega^2 a^2 b}{GM}, \beta_1 = \frac{1}{8} f^2 - \frac{5}{8} fm$$

Geometry of Normal-Gravity. , p. 105



Geodetic referencesystem (GRS), p. 108.

Set af parameters, which determine the ellipsoide and the normal-gravityfield.

GRS80:

$a=6378137$ m,

$GM=3.986004 \times 10^{14}\ m^3/s^2$

$J_2=-C_{20}=0.00108263$

$\omega=7.292115 \times 10^{-5}$ rad/s

Older systems (may be selected in GRAVSOFT), p. 209.

Hayford=International Ellipsoide:

a=6378388 m, 1/f=297,

International gravity formula 1928

Krassowsky (USSR, nu Rusland):

a=6378245.0 m, 1/f=298.3

Bessel: a=6377397.0, 1/f=299.15

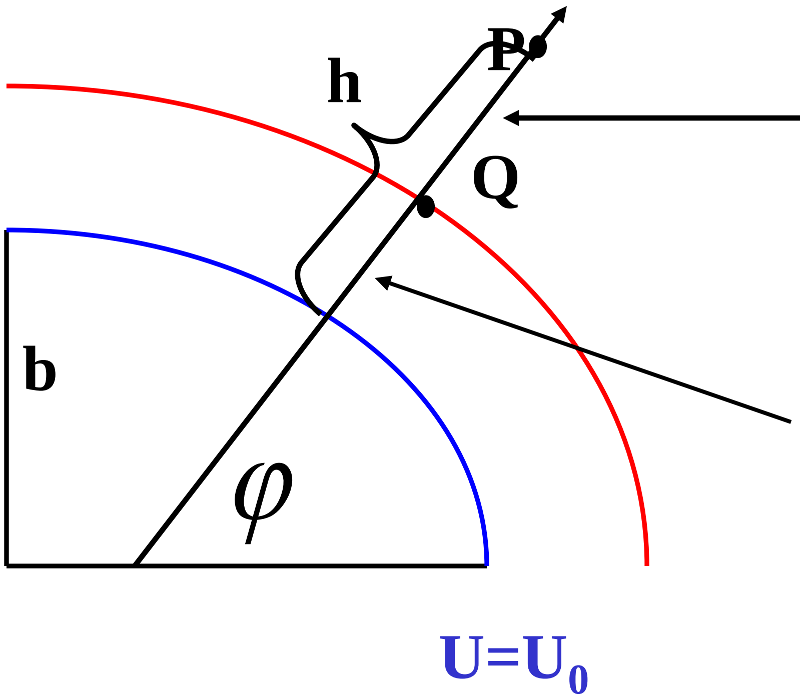
Clark: a=6378249 m, 1/f=293.46 (1880)

a=6378206 m, 1/f=294.979 (1866),

Centers of ellipsoids may be several 100 m wrong.

Height anomaly.

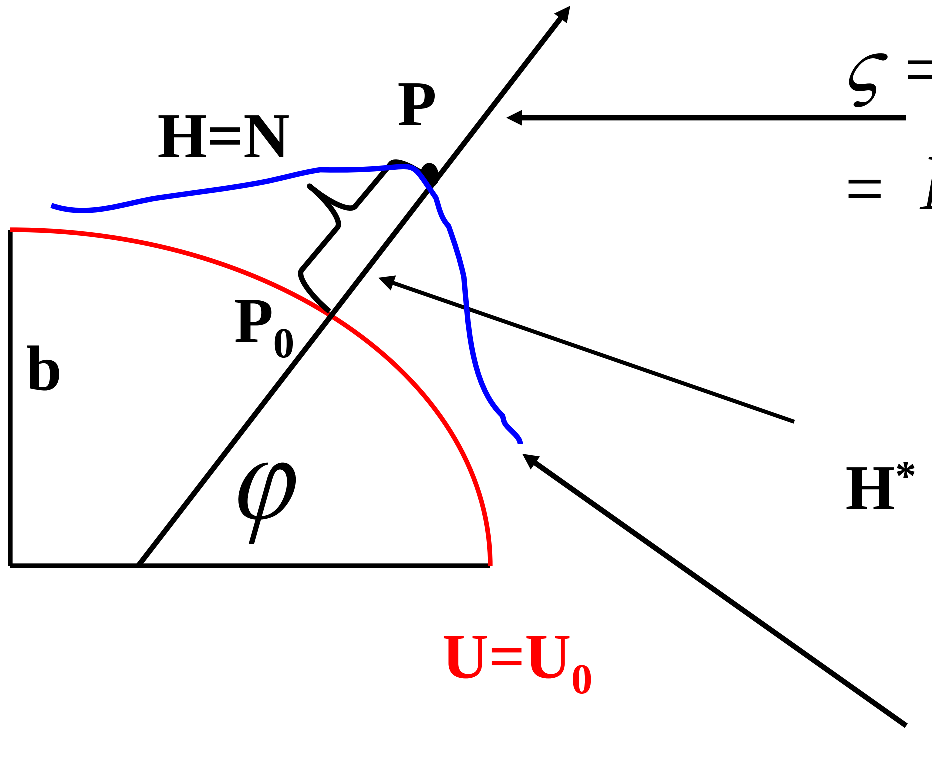
p. 225.



$$\zeta = \textit{height anomaly},$$

$$= h - H^*$$

Geoidheight, N, point P on the geoide.



$$\varsigma = \textit{height anomali},$$
$$= N, \textit{geoidheight}$$

Bruns equation (linearizing), p. 260.

$$On \quad the\ geoid\ is$$

$$W(P) = W_0 = U_0 = U(P_0)$$

$$U(P) = U(P_0) + \frac{\partial U}{\partial h} N + \ldots$$

$$U(P) - W(P) = -\gamma \cdot N$$

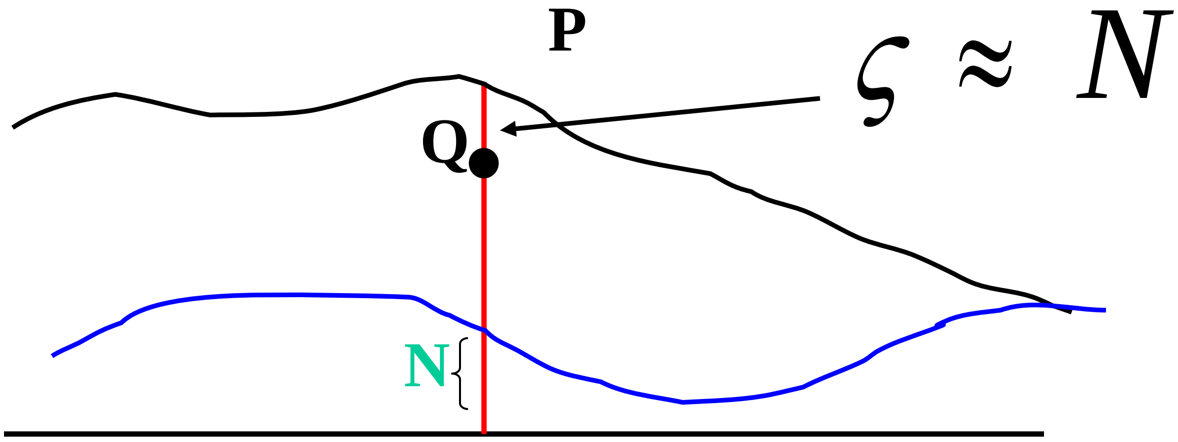
$$N = (W(P) - U(P)) / \gamma = T / \gamma$$

Height anomaly, p. 259.

Geoid height may be generalized to an arbitrary point:

Distance between a point P and a point Q, where

W(P)=U(Q), on the same ellipsoid normal



Generalised Bruns equation.

$$\zeta(P) = (W(P) - U(Q)) / \gamma(Q)$$

$$\varsigma(P) = T / \gamma$$

If local height system different from global, then the difference (bias), N_0 must be added.

Gravity disturbance, p.260.

We find the point P on the ellipsoid-normal, which have ellipsoidal height h, above the ellipsoid. Normal gravity is computed in the same point as P !

$$\delta g = g(P) - \gamma(P)$$

Gravity anomaly, p. 260.

We find in practice Q as the point on the ellipsoid-normal, which have ellipsidal height equal to P's height above the geoide, H.

$$\Delta g = g(P) - \gamma(Q)$$

Gravity anomaly, linearized., p. 261.

$$g(P) \approx -\frac{\partial W}{\partial r}\Big|_P$$

$$\gamma(Q) \approx -\frac{\partial U}{\partial r}\Big|_Q = -\frac{\partial U}{\partial r}\Big|_P - \frac{\partial^2 U}{\partial r^2} N$$

$$= -\frac{\partial U}{\partial r}\Big| - \frac{\partial \gamma}{\partial r}\frac{T}{\gamma} \quad where\ \gamma \approx \frac{GM}{r^2}, \frac{\partial \gamma}{\partial r} = -2\frac{GM}{r^3}$$

$$\Delta g = g(P) - \gamma(Q) =$$

$$-\frac{\partial}{\partial r}T - \frac{\partial \gamma}{\partial r}\frac{1}{\gamma}T = -\frac{\partial}{\partial r}T - \frac{2}{r}T$$