Lecture 1.2: **Gravity field of the Earth**, (T/M Cph. 3).

Gravity direction and magnitude **g** (m/s^2)

Acceleration

Gravity-potential (W): ability to execute work:

$$Work = \int_A^B |g| ds = W(B) - W(A)$$

The undisturbed sea-surface is a surface where W is constant (we disregard tides etc.)

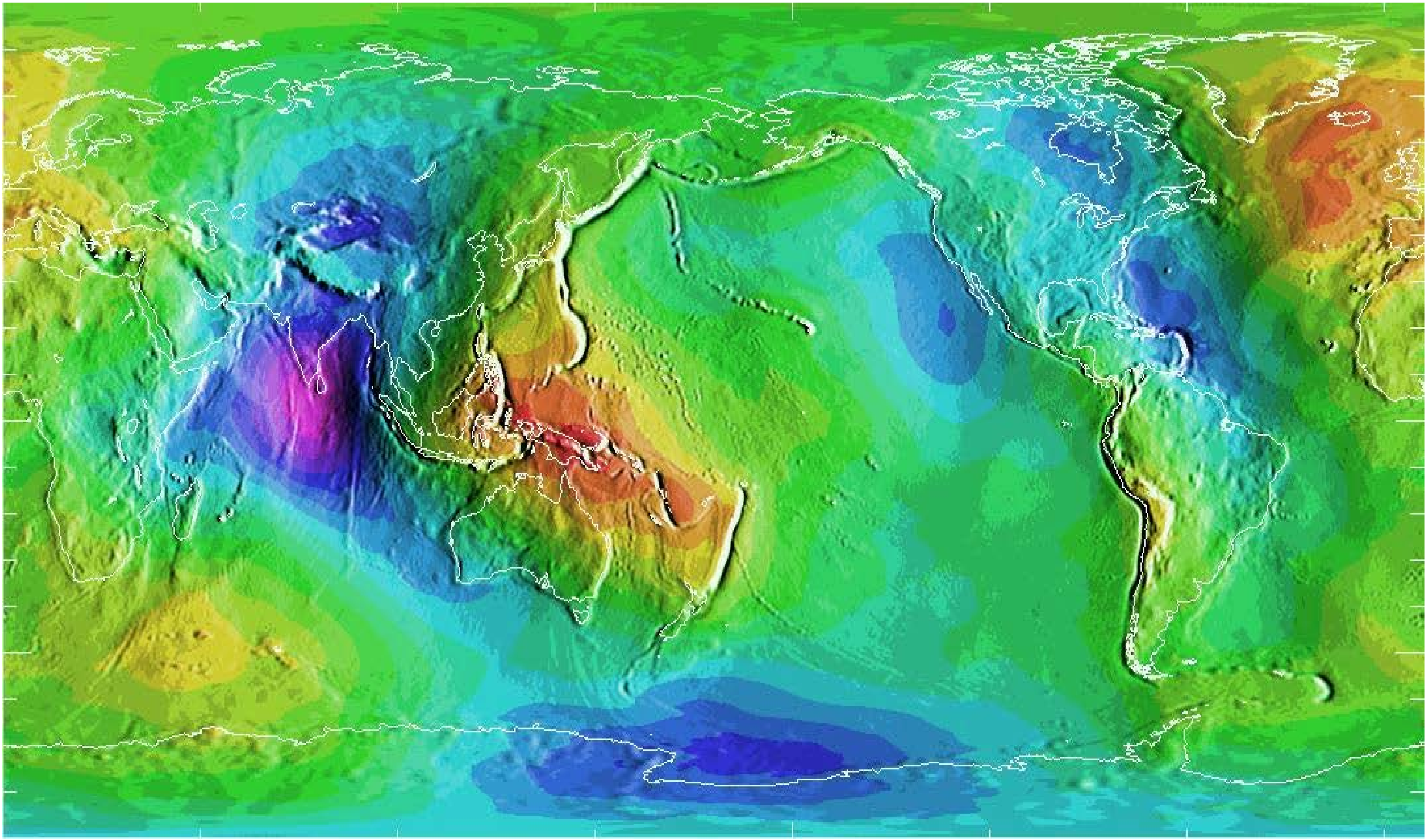
The geoid I

Gravity does not do any work when we move horisontally !

The surface is named the **Geoid.**

If we can find this, then we know most of the shape of the Earth because most of the Earth is the Ocean.

Global Geoid, min=-107 m, max=86 m



Geoid II

Altitudes, determined by levelling (and gravity measurements) are potential-differences

W and **g**: Important information on Earth's interior mass-distribution.

Enables levelling with GPS

Important for inertial navigation:

$$\frac{d^2 s}{dt^2} = a - g$$

Units

mgal: 10^{-5} m/s^2

μgal: $10^{-8}\ m/s^2$

μms: $10^{-6} m/s^2$

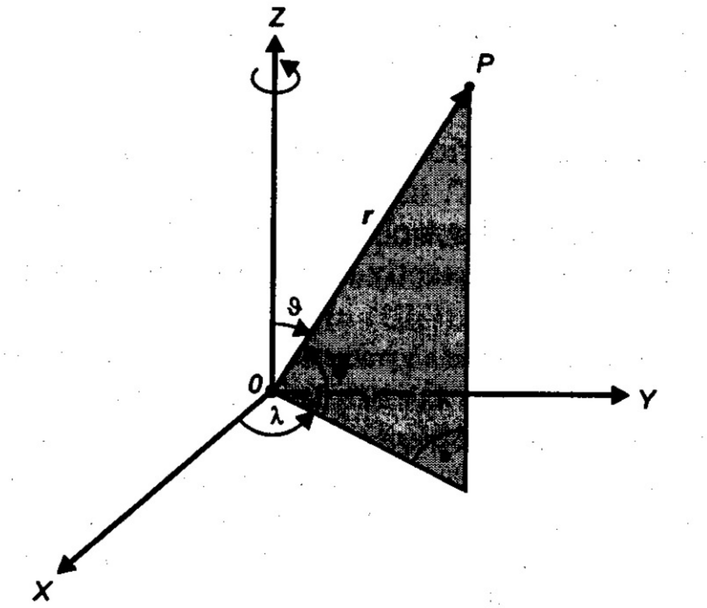
nms: $10^{-9} m/s^2$

Spherical coordinates

p. 48

Center in Gravity center

X-Y plane = Equator.



X - axis: Greenwich meridian

Y - axis: 90^{o}

Z - axis: Rotation axis

$$X = r\cos(\overline{\varphi})\cos(\lambda)$$

$$Y = r\cos(\overline{\varphi})\sin(\lambda)$$

$$Z = r\sin(\overline{\varphi})$$

Local spherical coordinates

We want to express the gradient of W in a simpel way:

$$\left.\begin{array}{l}(1)\text{ - axis: East}\\(2)\text{ - axis: North}\\(3)\text{ - axis: Radius - vector}\end{array}\right\} local(x,y,z)$$

Partial derivatives:

$$\left.\begin{array}{l}\dfrac{\partial}{\partial x}=\dfrac{1}{r\cos\overline{\varphi}}\dfrac{\partial}{\partial\lambda}\\[2ex]\dfrac{\partial}{\partial y}=\dfrac{1}{r}\dfrac{\partial}{\partial\overline{\varphi}}\\[2ex]\dfrac{\partial}{\partial z}=\dfrac{\partial}{\partial r}\end{array}\right\}\quad \text{Proof}: \quad use \quad chain-rule$$

$$Volume-measure: \quad d\Omega = r^2\cos\overline{\varphi}\,d\lambda\,d\overline{\varphi}$$

Geodetic coordinate-system

p. 98,99

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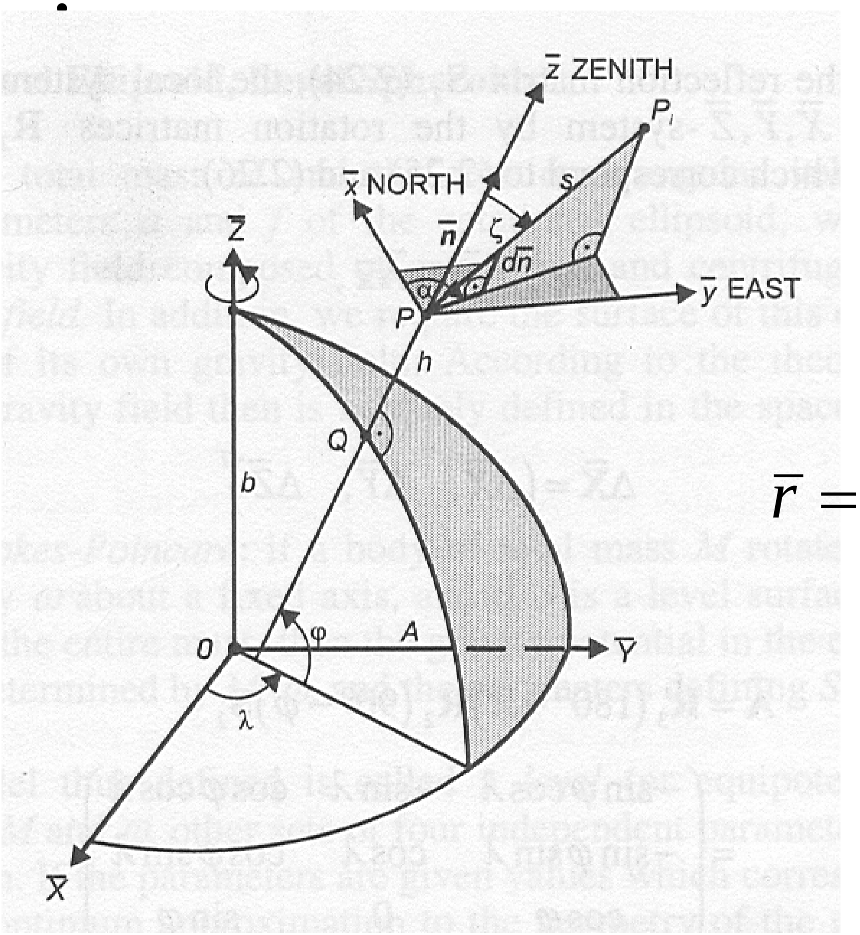


$$\bar{r} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} (N+h)\cos\varphi\cos\lambda \\ (N+h)\cos\varphi\sin\lambda \\ ((1-e^2)N+h)\sin\varphi \end{pmatrix}$$

$$N = \frac{a}{(1-e^2\sin^2\varphi)^{1/2}}$$

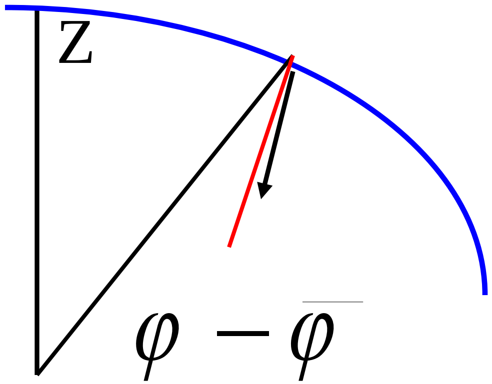
Local geodetic coordinate-system

(1) – axis East

(2) - axis North

(3) - axis: Direction of Ellipsoid-normal

Rotation needed, but small because components of gradientens are small in x and y directions



$\varphi - \overline{\varphi}$

Newtons law of attraction

p. 54

$$K = -G\frac{m_1 m_2}{l^2}\left(\frac{\vec{l}}{l}\right)$$

$$l = distance \quad (length),$$

$$l = ((x - x')^2 + (y - y')^2 + (z - z')^2)^{1/2}$$

$$\vec{l} = \boldsymbol{r} - \boldsymbol{r}' = connection - vector$$

$$G = 6.67259 \times 10^{-11} m^3 kg^{-1} s^{-2}$$

Gravity acceleration.

Gravity acceration: m_2=1

$$\boldsymbol{b} = -G\frac{\boldsymbol{r}-\boldsymbol{r}'}{|\boldsymbol{r}-\boldsymbol{r}'|^3} \quad then$$

$$curl\mathbf{b} = \frac{\partial}{\partial \mathbf{x}}b_1 + \frac{\partial}{\partial y}b_2 + \frac{\partial}{\partial z}b_3 = 0$$

$$\Rightarrow \quad \mathbf{b} \quad gradient\ of\ \ potential$$

$$V = \frac{Gm}{l}(sign\ geophysical \quad convention)$$

Potential, p. 55

For the Earth

$$V = G \iiint_{Earth} \frac{dm}{l}$$

$$= G \iiint_{Earth} \frac{\rho}{l} dv, \quad \rho = massedensity$$

Special case: Spherical symmetric Earth. p. 55

$$Shell, \sigma: \quad thickness \quad dr', density\ \rho, constant$$

$$V' = G\rho dr' \iint_{\sigma} \frac{1}{l}(r')^2 \cos\overline{\varphi} d\overline{\varphi} d\lambda,$$

$$For \quad exterior \quad potential, r > r'$$

$$V' = 4\pi G\rho(r')^2 dr' \frac{1}{r},$$

$$then\ dm' = 4\pi\rho(r')^2 dr'\ (mass\ of\ shell)$$

$$Exterior\ potential \quad V_e = \frac{GM}{r}$$

Potential inside. P. 56,57

Interior potential, constant density T(3.21)

$$V_i = \frac{4}{3}\pi G r^2 \int_0^r \rho r'^2\, dr' + 4\pi G \int_r^R \rho r'\, dr'$$

$$= 2\pi G\rho\left(R^2 - \frac{r^2}{3}\right)$$

Gradient of V.

P.58

$$\frac{\partial V}{\partial x} = V_x = -G\iiint_{Earth} \frac{x - x'}{l^3} dm$$

$$\frac{\partial^2 V}{\partial x^2} = -G\iiint_{Earth} \frac{1}{l^3} dm + 3G\iiint_{Earth} \frac{(x - x')^2}{l^5} dm$$

$$\Rightarrow \Delta V = \sum_{i=1}^{3} \frac{\partial^2 V}{\partial x_i} = V_{xx} + V_{yy} + V_{zz} = 0$$

V is HARMONIC function outside masses

Inside: $\Delta V = -4\pi G\rho,$ *POISOINS equation*

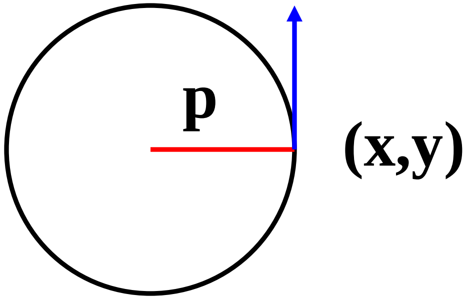
Centrifugal-force

p. 60

We rotate in orbit which is a circle around the Z-axis:

$$x(t) = p \times \cos(\omega(t - t_0))$$

$$y(t) = p \times \sin(\omega(t - t_0)), \quad z = constant$$



Centrifugal potential.

Velocity:

$$\dot{x} = -p \times \omega \times \sin(\omega(t - t_0))$$

$$\dot{y} = \quad p \times \omega \times \cos(\omega(t - t_0)), \quad \dot{z} = 0$$

Acceleration:

$$\ddot{x} = -p \times \omega^2 \cos(\omega(t - t_0))$$

$$\ddot{y} = -p \times \omega^2 \sin(\omega(t - t_0)), \quad \ddot{z} = 0.$$

Gradient of:

$$\Phi(x, y, z) = \frac{\omega^2}{2}(x^2 + y^2) = \frac{\omega^2}{2} r^2 \cos\overline{\varphi}^2$$

Total potential. p. 62

$$W = V + \Phi = G\iiint_{Earth} \frac{\rho}{l} dv + \frac{\omega}{2} p^2$$

$$g = \nabla W,$$

$$|g| = gravity$$

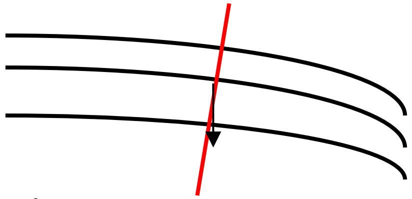
Generalise*d* **Poisson** *Equation*

$$\Delta \mathbf{W} = -\mathbf{4}\pi\mathbf{G}\rho + \mathbf{2}\omega^{\mathbf{2}}$$

Level surfaces.

p. 63

W constant, cut by plumb-line **g** tangent.



Plumb-line coordinat axis, so

dW is a total differential, $dW = -gdn + 0dx + 0dy$

Height difference between 2 points is independent of the path we take between them. Used to adjust levelling.