A local study of the influence of sampling rate, number of observed components and instrument noise on 1 deg. mean geoid and gravity anomalies determined from satellite gravity gradiometer measurements.

by

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Abstract: The error of 1 deg. mean geoid undulations and gravity anomalies determined from satellite gravity gradiometry has been estimated as a function of various instrument and mission parameters. The method of least-squares collocation was used with a covariance function representing a typical area in the North Atlantic, and supposing the spherical harmonic coefficients up to degree 60 to be well known. The errors have been estimated using signal noise values between 0.04 and 0.005 E.U. (10**-9 /s**2), data sampling between 1/16 and 1/4 degrees and either 1, 3 or 5 independent gravity gradient components. All estimates have been made for 160, 180 and 200 km satellite altitude. Two different mission durations were also studied, corresponding to a track spacing of 1 and 0.5 degrees.

The results give a good indication of the relative improvements which may be achieved by changing the parameters. Within the studied range of the parameters it is seen that the use of 5 components instead of 1 gives an improvement corresponding to a 3 times increase of the sampling rate. An increase in the precision a factor 2 corresponds to 20 km decrease in the satellite altitude.

1. Introduction.

The results of a satellite gravity gradiometer mission will depend on several parameters such as satellite altitude, mission duration, instrument resolution and number of gravity gradient components, which can be observed. It is clear that the longer the mission, the lower the orbit and the better the instrument, then improved gravity field quantities may be determined. However, especially fuel consumptions put a limit on the mission duration and the altitude. But if such mission limitations could be counteracted by instrument improvements, this would be an important factor in deciding where the emphasis should be put in the further development of the whole mission concept, i.e. on instrument improvement, on the reduction of fuel consumption or on other mission parameters.

In order to study this problem in its full extend, a global data coverage should be used, and the (geographically) varying characteristics of the gravity field should be taken into account. A global investigation has recently been presented by Colombo (1987), which, however, used a fixed value for the instrument noise and the data sampling rate, but looked at varying mission length and satellite altitude.

The influence of the instrument noise and the data sampling as well as the use of one or several gravity gradient components may also be studied locally, see eg. Robbins(1985), and Tscherning(1987). In a local simulation, however, only the relative influence of the various parameters may be seen. In the following we will use the same method as used in both of the just mentioned papers, but the statistical gravity field model will be the same as in the second one mentioned. Here the gravity field variation is like in a typical area of the North Atlantic, and the contribution from the spherical harmonic coefficients up to degree 60 is supposed to be well known.

The method is least-squares collocation, which has the draw-back that a system of equations with the number of unknowns equal to the number of observations has to be solved. For simulation studies involving satellite data, the regular distribution of the data may frequently be used in order to speed up the solution of the (full) system of equations. However, the collocation program system GEOCOL used in this study does not have this possibility at present.

This has put some restrictions on the present study, i.e. maximally 1250 unknowns have been used. On the other hand, it has been possible to use already reduced parts of the normal equations (subroutine NES), so that partial studies, involving only subsets of the data could be made without any noteworthy extra costs in terms of cpu time.

In the following section the data used will be described, and in section 3 the results will be presented. The final section 4 contains the conclusion The paper contains only a few references and for further literature the reader is referred to the already mentioned earlier paper.

2. Data selection.

A test data set was generated using the OSU86F field to degree 250, (Rapp and Cruz, 1986). Totally 5 tracks was used with 50 points on each, spaced 1/16 degree apart. The distance between the tracks was 1/2 degrees. Up to 5 independent components were supposed to be associated with each point, giving a maximal total number of observations equal to 1250. 3 sets of points, located at altitudes 160, 180 and 200 km were generated.

From this basic dataset, subsets were then generated by taking every 2, 3 and 4'th observation, and by leaving out 2 of the 5 tracks (track 2 and 4). This last modification corresponds to using a mission duration of only half the total length.

No crossing tracks were used. This would probably have given slightly improved results, but not changed the relation between the results obtained from the different data configurations.

The various data combinations were then used to predict 1 degree mean geoid undulations and gravity anomalies. In order to limit the computational effort, they were computed as point values at 10 km altitude. The justification for this is given in Tscherning and Rapp (1973).

Simulations were then carried out with 1, 3 and 5 components, where each set consisted of the following configuration:

(a) Tzz,
(b) Tzz, Txz, Tyz,
(c) Tzz, Txz, Tyz, Txy, Tdelta.

Tdelta is Txx - Tyy. The letters x, y and z added to T indicates differentiation of the anomalous gravity potential, T, with respect to the local cartesian coordinates x, y and z, where x is east, y north and z up (radial direction).

3. Results:

First the error estimates of the 1 deg. mean values were carried out with data in 3 altitudes using 4 different data spacings on all 5 tracks, namely 1/16, 1/8, 3/16 and 1/4 degree. The results are found in Table 1 for all the configurations (a) - (c). A signal noise standard deviation of 0.01 E.U. (= 10**-9/s**2) was used.

Table 1. Standard deviation of estimated 1 degree mean geoid undulations (meters) (1) and 1 degree mean gravity anomalies (2) (mgal), as a function of data spacing, altitude and data configuration (a) -(c). Signal noise standard deviation fixed to 0.01 e.u.

	Estimation standard deviation						
	(1)			(2)			spacing
Data config. Altitude (km)	(a)	(b)	(c)	(a)	(b)	(c)	deg.
160	0.10	0.09	0.09	5.5	5.3	5.1	1/16
	0.11	0.10	0.10	5.9	5.5	5.3	2/16
	0.12	0.10	0.10	6.0	5.6	5.5	3/16
	0.13	0.11	0.10	6.1	5.7	5.6	4/16
180	0.13	0.12	0.10	6.2	6.0	5.8	1/16
	0.14	0.12	0.12	6.5	6.1	6.0	2/16
	0.15	0.13	0.12	6.6	6.3	6.1	3/16
	0.15	0.13	0.13	6.7	6.4	6.2	4/16
200	0.15	0.13	0.13	6.8	6.5	6.3	1/16
	0.16	0.14	0.14	7.1	6.7	6.6	2/16
	0.17	0.15	0.14	7.2	6.8	6.7	3/16
	0.18	0.16	0.15	7.3	6.9	6.8	4/16

Also the influence of the signal noise was simulated, however, only using Tzz data in 160 and 200 km altitude. Values of the noise between 0.005 and 0.04 E.U. were used. The results are given in Table 2.

Table 2. Standard deviations of estimated 1 degree mean geoid undulations (1) in meters and 1 degree mean gravity anomalies (2) using data configuration (c) (tzz only) and 1/16 degree data spacing as a function of the signal noise standard deviation (E.U.).

Altitude km	Data noise e.u.	Estimation standard deviation (1) m	 (2) mgal
160	0.02	0.12	6.1
	0.01	0.10	5.5
	0.005	0.09	5.0
200	0.04	0.21	7.9
	0.02	0.18	7.4
	0.01	0.15	6.8
	0.005	0.13	6.3

Finally computations using only 3 of the 5 tracks were made. In this case the tracks were 1 deg. apart. We only carried out the computations with data spaced 1/16 and 1/8 degree apart, because already these values gave the nesessary information, which is presented in Table 3.

Table 3. Standard deviations of estimated 1 deg. mean geoid undulations (1) in meters and 1 deg. mean gravity anomalies (2) in mgal, obtained using tracks space 1 deg. apart. Noise standard deviation of 0.01 E.U. used.

	Estimation standard deviation						
		(1)			(2)		Spacing
Data config.	(a)	(b)	(c)	(a)	(b)	(c)	
Altitude (km)		m			mgal		deg.
160	0.13	0.10	0.10	6.2	5.5	5.4	1/16
	0.13	0.11	0.10	6.3	5.9	5.6	2/16
180	0.15	0.12	0.12	6.7	6.1	6.0	1/16
	0.15	0.13	0.12	6.8	6.3	6.2	2/16
200	0.17	0.14	0.14	7.2	6.7	6.5	1/16
	0.18	0.16	0.15	7.3	6.9	6.8	2/16

4. Conclusions.

Smaller signal noise and higher data sampling improves especially the determination of the high frequency part of the gravity spectrum, here 1 deg. mean gravity anomalies. A factor of 4 in the resolution is for a 1-component gradiometer equivalent to a change in altitude from 160 km to 200 km. A decrease in data spacing a factor 1, 2, and 3 times corresponds to the use of 1, 3 and 5 components of the tensor, cf. table 1.

An increase in the mission duration corresponding to an increase in the track spacing from 1 deg. to 0.5 deg. corresponds to a 3 times increase in the data sampling. Conversely, the negative effect of a shorter mission duration can be counteracted by an increased sampling rate.

In general it seem possible to eliminate the effect of having to increase the altitude or shortening the mission duration by improving the data sampling rate, the number of gravity gradient components observed and decreasing the noise standard deviation. Tables 1 - 3 express this in relative numbers, and a global simulation is necessary in order to obtain estimates of the absolute magnitudes.

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