Sonderdruck

DEUTSCHE GEODÄTISCHE KOMMISSION
bei der Bayerischen Akademie der Wissenschaften

Reihe B Angewandte Geodäsie Heft Nr. 287

Festschrift

Rudolf Sigl

zum 60. Geburtstag

München 1988

Verlag der Bayerischen Akademie der Wissenschaften
in Kommission bei der C.H.Beck'schen Verlagsbuchhandlung München

ISSN 0065-5317 ISBN 3 7696 8572 5

A STUDY OF SATELLITE ALTITUDE INFLUENCE ON THE SENSITIVITY OF GRAVITY GRADIOMETER MEASUREMENTS

Christian C. Tscherning

Abstract: The sensitivity of gravity gradiometer measurements has been expressed in terms of the standard deviations of derived 1 deg. and 0.5 deg. mean gravity anomalies and mean geoid undulations. Such standard deviations have been computed for satellite altitudes 160 km, 180 km and 200 km, using 1 and 3 component gradient data spaced 0.125 deg. and 0.25 deg. apart. Totally 400 points were used with a supposed measurement noise of 0.01 E.U. (stdv.). Expressed in terms of the ratio between the signal and prediction standard deviations the following results were obtained:

Altitude km	Geoid undulation 0.5 deg.	1 deg.	Gravity anomaly 0.5 deg.	1 deg.
160	0.15	0.11	0.49	0.40
180	0.18	0.15	0.54	0.44
200	0.22	0.17	0.58	0.49

The statistical model used had standard deviations of the 1 deg. mean gravity anomalies and geoid heights at the Earth's surface equal to 16 mgal and 0.81 m, respectively, and the potential coefficients up to degree 60 were supposed to be well known.

Global studies by other authors show an up to 120 percent decrease in the sensitivity with respect to 1 deg. mean geoid undulations when changing altitude from 160 km to 200 km, corresponding to an increase in the mean 1 deg. geoid error from 3.4 cm to 7.8 cm. This shows that a 160 km altitude is essential for obtaining the for oceanographic applications needed accuracy of 5 cm.

1. Introduction

The sensitivity of gravity gradiometer measurements and of satellite-to-satellite tracking has been studied either by computing formal error estimates of derived quantities or by creating artificial test data sets and then comparing the estimated quantities with the test data. This has been done using as test data: potential coefficients, mean gravity anomalies and mean geoid undulations, typically of blocksizes 2, 1 and 0.5 deg., see Hajela (1981), Jekeli and Rapp(1980), Kahn(1987),Robbins(1985), Rummel(1979, 1980), Rummel and Colombo (1985) and Wichiencharoen (1985). For the statistical error analysis as well as for the simulation studies, a model for the spectral behaviour of the gravity field is necessary. Here most authors have used the T/R-model (Tscherning and Rapp, 1974) or slightly modified models, see e.g. Robbins (1985). These models have the deficit that the gradient variance is too large (standard deviation (stdv.) equal to 60 E.U. (10 exp(-9) sec exp(-2))), or too small, 15 E.U..

In the following section 2 some new statistics for the gravity field will be given, which are typical for an oceanic area. The standard deviations of the various quantities show that a gradiometer sensitivity at 0.01 E.U. is a limiting factor if information corresponding to wavelengths shorter than 1 deg. need to be recovered. In section 3 is reported results of simulation studies, using a very limited data set from a typical oceanic area. Finally in section 4 the results of section 3 are compared with results obtained by other authors.

2. Gravity field statistics at altitude

In a recent investigation (Knudsen, 1987) a covariance function for a part of the North Atlantic has been computed. Its derived gradient standard deviations at altitude show where some of the limiting factors occur for gradiometer measurements with a noise level of 0.01 E.U. It is worth noticing that the correlation of the gradiometer measurements at e.g. 180 km altitude is 95 to 99 percent for horizontal distances in the range from 0.1 to 0.3 deg.

In the following table we have used a degree-variance model of the T/R-type, with B=4, depth to the Bjerhammar-sphere equal to 3.17 km and scale factor on the error-degree variances of GPM2 (Wenzel, 1985) equal to 0.2. An important factor is how many potential coefficients we have supposed to be known. This is expressed as the maximal degree and order, N, for which we use the error degree-variances in the statistical model.

Table 1. Gravity field standard deviations. (1) geoid undulations, (2) gravity anomalies (3) second order radial derivative (4) second order mixed horizontal derivative. Units are m, mgal and E.U. and km for the altitude. N is the maximal degree and order where the error degree variances are used.

Height	N	Standard deviations of the signal			
		(1)	(2)	(3)	(4)
0	20	3.64	30.0	26.4	18.7
160	20	1.74	6.9	0.44	0.31
180	20	1.60	6.2	0.37	0.26
200	20	1.48	5.6	0.33	0.23
0	60	0.95	20.0	21.0	14.8
160	60	0.14	1.37	0.17	0.12
180	60	0.11	1.08	0.13	0.09
200	60	0.09	0.85	0.10	0.07
0	90	0.82	25.0	29.5	20.9
160	90	0.05	0.71	0.12	0.09
180	90	0.03	0.51	0.09	0.06
200	90	0.03	0.37	0.06	0.04
0	180	0.31	16.5	23.5	16.3
160	180	0.00	0.04	0.014	0.009
180	180	0.00	0.02	0.007	0.005
200	180	0.00	0.01	0.004	0.003

The tabel show clearly both that an instrument sensity below 0.01 E.U. is essential for the recovery of any information related to the harmonics above degree 180. Also the influence of changing altitude is clearly seen. For the typical case N=60, an altitude of 160 km is sufficient, but an altitude of 200 km becomes problematic, also considering the already mentioned strong correlation of the signal as a function of horizontal distance. It should be mentioned, that the calculations have been executed using the subroutine COVAX (Tscherning, 1976), modified according to Krarup and Tscherning (1984).

3. Simulation study in the North Atlantic area

Since one of the important application areas for satellite gravity gradiometry is in oceanography, a test area in the central North Atlantic has been selected. Here the first idea was to continue upward the local gravity field using least squares collocation (lsc). This is done by combining a spherical harmonic expansion (in this case GPM2) with the local gravity and satellite altimeter data. However, the contribution from the local data was of the order 0.01 E.U. already at altitude 160 km. It was therefore decided to generate test data using a recent spherical harmonic solution complete to degree and order 360, called OSU86F, see Rapp and Cruz (1986). This has also been used by Rapp(1987) to illustrate gradient variations in areas of tectonic importance.

Second order radial derivatives (Tzz) and mixed radial and horizontal derivatives (Txz, Tyz) were used, but also other quantities were computed, but not used. From these data, spaced 0.125 and 0.25 deg. apart, mean gravity and geoid undulations were computed, as well as the standard deviations of the estimated quantities. The program GEOCOL (Tscherning, 1985) were used in all computations.

The results are summarized in table 2.

Table 2. Results of predictions of mean gravity anomalies of blocksize 0.5 deg. (1) and 1 deg. (2), mean geoid undulations of blocksize 0.5 deg. (3) and 1.0 deg. (4) in terms of standard deviations of the predicted quantities for various altitudes and data spacings and combinations.

Altitude km	Spacing deg.	Data	Resulting stdv. (1) mgal	(2) mgal	(3) m	(4) m
160	0.125	Tzz	7.8	5.3	0.12	0.09
180			8.5	5.9	0.15	0.12
200			9.1	6.6	0.18	0.14
160	0.125	Tzz +	7.5	5.1	0.12	0.09
180		Txz,Tyz	8.2	5.7	0.14	0.11
200			8.9	6.3	0.16	0.13
160	0.25	Tzz	8.1	5.6	0.14	0.11
180			8.8	6.2	0.16	0.13
200			9.3	6.8	0.19	0.15
Signal stdv.			15.8	13.4	0.81	0.80

In the calculations, the test values were also recovered, using GPM2 to degree 60 as a reference field, in consistence with the statistical model.used. Here the agreements observed minus predicted were excellent, see table 3.

Table 3. Results of prediction using simulated data from the OSU86F field. Legend and units as in table 2. 16 values used. Tzz, Txz and Tyz data used with 0.125 deg. spacing.

Altitude	Standard deviation of observed minus predicted.			
km	(1)	(2)	(3)	(4)
160	1.6	1.2	0.04	0.03
180	2.1	1.7	0.05	0.04
200	2.8	2.4	0.06	0.05

The very nice results in Table 3 are only standard deviations. Biases of the order 0.2 - 0.3 m are present, but it indicates a good possibility for the determination of differences between mean gravity anomalies or mean geoid undulations.

4. Results of other investigations

Investigations like the one presented in section 3 have been executed by several authors, using slightly different statistical models for the gravity field. One of the most complete investi- gations is the one by Jekeli and Rapp (1980). From their table 2 the following results have been extrapolated. They also used 0.01 E.U. as their noise standard deviation, while Robbins(1985) used 0.001 E.U.

Table 4. Standard deviations of estimated quantities (see Table 2) obtained by Jekeli and Rapp(1980) and Robbins (1985).

Altitude		Standard deviations			
	Robbins	Jekeli and Rapp			
km	(1)	(1) mgal	(2) mgal	(3) m	(4) m
160	7.6	9.1	2.3	0.165	0.049
180	8.6	10.4	3.1	0.206	0.072
200	9.4	11.6	3.9	0.246	0.096

The influence of the decreased noise stdv. is clearly seen from the table. Robbins(1985) contains several interesting results for even lower noise and higher sampling rate. In all the results quoted until now, a 4 sec. sampling rate has been used. A decrease in the sampling interval to 1 sec. will give a 50 percent lower standard deviation for 0.5 deg. mean gravity anomalies, cf. Robbins(1985, Table 4 and 5).

Results obtained in different altitudes as a function of the noise stdv. are also found in Kahn(1987). Here 0.01 E.U. gives a standard deviation of 1 deg. mean gravity anomalies equal to 2.0 and 2.8 mgal for altitudes of 160 and 200 km, respectively. This corresponds quite well to results in Rapp(1987) of 1.7 and 3.2 mgal and 0.034 and 0.078 m for 1 deg. mean gravity and mean geoid undulations obtained using data at 160 km and 200 km altitude.

However in Wichiencharoen (1985) it is warned that 2.5 mgal may be obtained for an area with a smooth gravity field, while only 30 mgal may be obtained, if the gravity field is rough. On the other hand will the percentage gain in information probably be the same, as long as only 1 deg. mean values are discussed.

5. Conclusion

The synthesis of the results of section 3 is that a significant loss of sensitivity occurs if the altitude is increased from 160 to 200 km. It may be expressed as the change in the ratio of the prediction standard deviation with respect to the signal standard deviation. This ratio increases from 0.11 to 0.17 for the 1 deg. mean geoid undulations, cf. Table 2. The results of Rapp(1987) cf. Table 4, indicate an even larger increase, namely about 120 percent. In absolute numbers the increase is from below 0.05 m to nearly 0.10 m. This means that important oceanographic goals can not be reached, cf. Balmino et al.(1985, Table 0).

The signal standard deviations of Table 1 also show that a change in altitude from 160 to 200 km is critical, since we at 200 km altitude come quite close to the noise stdv. If 200 km should be used, it is probably not enough to decrease the measurement noise, but the sampling rate must also be increased.

This should not be taken as an argument for that a satellite at 200 km altitude would not give a tremendous amount of new information under all circumstances. But at this altitude it is difficult, if not impossible, to obtain some of the most significant results with a 0.01 E.U. noise level and 4 sec. sampling.

Acknowledgement: This paper is also a contribute to a pre-phase A study carried out for ESA.

References

Balmino,G., F.Barlier, F.Bernard, A.Bernard, C.Bouzat, R.Rummel and P.Touboul: Proposal for a Satellite Gravity Gradiometer experiment for the Geosciences. Bureau Gravimetrique International, Toulouse, 1985.

Hajela, D.P.: A simulation study to test the prediction of 1 deg. x 1 deg. mean gravity anomalies using least squares collocation from the gravsat mission. Reports of the Dep. of Geodetic Science and Surveying no. 316, The Ohio State University, Columbus, 1981.

Jekeli, C. and R.H.Rapp: Accuracy of the determination of mean anomalies and mean geoid undulations from a satellite gravity field mapping mission. Reports of the Dep. of Geodetic Science no. 307, The Ohio State University, Columbus, 1980.

Kahn, W.D.: Error analysis studies for spaceborn gravity gradiometers. GRM/Gradío SSG meeting proceedings, pp. 89-97, GSFC, May 1987.

Knudsen, P.: Estimation and Modelling of the Local Empirical Covariance Function using gravity and satellite altimeter data. Bulletin Geodesique, Vol. 61, pp. 145-160, 1987a.

Krarup, T. and C.C.Tscherning: Evaluation of Isotropic Covariance Functions of Torsion Balance Observations. Bulletin Geodesique, Vol. 58, no. 2, pp. 180-192, 1984.

Rapp, R.H.: Gradient tables for Ruerto Rico Trench and Mendocino Fracture Zone. GRM/Gradio SSG meeting proceedings, pp. 111-116, GSFC, May 1987.

Rapp, R.H. and J.Y.Cruz: Spherical harmonic expansions of the Earth's gravitational potential to degree 360 using 30' mean anomalies. Dep. Geodetic Science and Surveying Rep. no. 376, The Ohio State University, Columbus, 1986.

Robbins, J.W.: Least squares collocation applied to local gravimetric solutions from satellite gradiometry data. Reports of the Dep. of Geodetic Science and Surveying no. 368, The Ohio State University, Columbus, 1985.

Rummel, R.: Determination of Short-Wavelength components of the gravity field from satellite-to-satellite tracking or satellite gradiometry - an attempt to an identification of problem areas. Manuscripta Geodaetica, Vol. 4, pp. 107-148, 1979.

Rummel, R.: Geoid heights, Geoid height differences, and mean gravity anomalies from "low-low" satellite-to-satellite tracking - an error analysis. Rep. of the Dep. of Geodetic Science no. 306, The Ohio State University, Columbus, 1980.

Rummel, R. and O.L.Colombo: Gravity field determination from satellite gradiometry. Bulletin Geodesique, Vol. 59, pp. 233-246, 1985.

Tscherning, C.C.: Covariance Expressions for Second and Lower Order Derivatives of the Anomalous Potential. Reports of the Department of Geodetic Science No. 225, The Ohio State University, Columbus, Ohio, 1976b.

Tscherning, C.C.: GEOCOL - A FORTRAN-program for Gravity Field Approximation by Collocation. Technical Note, Geodætisk Institut, 3.ed., 25 MAR 1985.

Tscherning, C.C. and R.H.Rapp: Closed Covariance Expressions for Gravity Anomalies, Geoid Undulations, and Deflections of the Vertical Implied by Anomaly Degree-Variance Models. Reports of the Department of Geodetic Science No. 208, The Ohio State University, Columbus, Ohio, 1974.

Wenzel,H.-G.: Hochaufloesende Kugelfunktionsmodelle fuer das Gravitationspotential der Erde. Wiss. Arb. Fachrichtung Vermessungswesen der Universitaet Hannover, no. 137, 1985.

Wichiencharoen, C.: Recovery of 1 deg.-mean anomalies in a local region from a low-low satellite to satellite tracking mission. Reports of the Dep. of Geodetic Science and Surveying No. 363, The Ohio State University, Columbus, 1985.