Evaluation of Local Gravity Field Determination Methods

Report of IAG SSG 3.90 for the period 1983-1987.

by

C.C.Tscherning
Geodetic Institute
Gamlehave Alle 22
DK-2920 Charlottenlund
Denmark

Abstract: Local gravity field determination methods have been evaluated by comparing computed gravity vector and geoid heights with observed or synthetic data. The following methods have been used in one or several comparisons: integral methods using classical or optimized kernels, least-squares collocation, combined collocation and integration and Fourier techniques.

For all methods it was found essential that topographic information was taken into account either through remove-restore techniques or as data supporting (gravity) interpolation procedures. Also the use of low-degree spherical harmonic information is important.

In general the investigated methods have different possibilities for handling heterogeneous data or irregularily distributed data. However, it was found that they, when the same data was used, gave results which were not significantly different.

1. Introduction.

IAG SSG 3.90 "Evaluation of local gravity field determination methods" was established by the Executive Committee during the General Assembly in Hamburg, August 1983. A part of the reason for establishing the SSG was to continue and complete work executed by the former SSG 4.70 "Gravity Field Approximation Techniques". This SSG had a rather broad scope, and several study groups, including SSG 3.90, was established to continue its work.

At the General Assembly the scope of Section III was broadned from "gravimetry" to "gravity field determination", while Section IV kept its scope of "theory and metodology". This (somewhat artificial) division of work between sections was purposedly reflected in the formulation of the program of work for the SSG:

> The SSG will study available methods for local gravity field determination, their efficiency, stability, range of applicability and their inherent limitations.

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> Numerical experiments will be executed using common sets of known and artificially generated values of gravity anomalies, deflections of the vertical, quasi-geoidal heights and gravity gradients. The methods will not only be compared with respect their accuracy and stability, but also such topics as related error-estimation procedures and possibilities for incorporating topographic, geophysical and geological information will be treated.

Many distinguished geodesists asked to participate in the work of the SSG, but the IAG rules did only permit 20 members. In order to circumvent this, a category of associate members has been invented. The ordinary members are: D.Arabelos (Greece), F.Barthelmes (DDR), B.Benciolini (Italy), S.C.Bose (USA), J.Chen (PRC), C.C.Goad (USA), H.Hauck (FRG), G.Hein (FRG), C.Jekeli (USA), S.K.Jordan (USA), W.Kearsley (Australia), G.Lachapelle, A.Mainville (both Canada), C.Merry (South Africa), Yu.M.Neyman (USSR), K.P.Schwarz (Canada), R.H.Rapp (USA), L.Sjøberg (Sweden), G. Strang van Hees (Netherlands), H.Süenkel (Austria), K.Tait (USA), M.Vermeer (Finland). The associate members are: R.Forsberg, T.Krarup (both Denmark), D.Lelgemann (FRG), R.Rummel (Netherlands), L.Wilcox (USA), A.N.Marchenko (USSR), J.Ning (PRC).
In the following sections the work of the SSG will be described. Only a few of the goals defined in the program have been achieved, but as the reader will see, has very much significant work been executed. I hereby thank all the (full and associate) members of the SSG and their collaborators for their fine work.

In the following section 2, numerical experiments with the computation of deflections of the vertical will be described. These experiments are a direct continuation of the work executed by SSG 3.70. Meanwhile new computational tasks have gained importance such as the precise computation of geoid undulation (height anomaly) differences and of the gravity vector using satellite altimetry or gravity gradiometry.The SSG has created and distributed data sets which may be used to test methods to handle these tasks. Results obtained using these datasets as well as results obtained using other datasets are reported in sections 3 to 5. Naturally also scientists outside the SSG have worked with the problems described in the program of the SSG. Their work will not be reviewed here but just recorded for the sake of completeness. Here I apologize for probably not having mentioned several important contributions. Finally I have tried to point out topics for further research.

2. White Sands Revisited.

As already mentioned has SSG 4.70 carried out a comprehensive comparison of methods for gravity and deflection computation (Schwarz, 1983). As data were used gravity anomalies, deflections of the vertical and 30" x 30" height data from the White Sands test area, New Mexico, USA. The description of the height data contained an error, which made the original results unreliable. In fact, the method which gave the best results (collocation) was the only method, which had escaped this error.

Due to lucky circumstances several SSG-members or their associates were visiting Division of Surveying Engineering, University of Calgary, at the same time during the fall of 1984. They decided to repeat the test of methods for the computation of deflections of the vertical with the corrected height data. It was also decided to test an old method in a new and more sofisticated implementation and a completely new method. These methods are the numerical integration of Vening Meninesz formula using Rice rings (RINT) and the solution of Stoke's integral by fast Fourier transform techniques (FFT). The other methods tested were the terrain effect integration and collocation method (TEIC) and the combined collocation and integration method (CINT).

In a first comparison were the height data not used explicitly, and it was found that collocation gave the worst results. However, a detailled analysis (Kearsley et al, 1985, p. 123) shows that the difference is due to the fact that the other methods use height data in an interpolation procedure to fill out data gaps.

In a second comparison all methods took full advantage of the height data. This gave improved and nearly identical results. The standard deviation of the differences between observed and predicted values were between 1.2" and 0.9" compared to standard deviations of 2.7" and 6.2" for the observed meridian and prime vertical components, respectively.

The results have been analyzed separatly for 8 half degree blocks in the test area. The difference in smothness of the topography and in gravity coverage was used in an attempt to derive an empirical relationship between these factors and the attainable deflection prediction error. This attempt was only partly successful (the gravity variation with and without topographic effects included would have been a better parameter), but the analysis demonstrated very clearly the importance of using the height data for all methods.

The TEIC method uses the OSU78 field (Rapp, 1978), which is complete to degree 180, while the other methods use GEM10B (Lerch et al., 1981), which is complete to degree 36. This permitted the use of a small data collection area when using TEIC, while the other methods used data distributed over comparably larger areas. The result was that all methods gave results with small biases, with the largest bias in the prime vertical component. This is probably caused by the uncertainty in the longitude origin of the NAD1927 coordinate system.

A comparison of computer processing time has also been made. It showed that the FFT method is much faster than any of the other methods, at least as they were implemented at the time of the comparison. Performance improvements are possible for all methods, using e.g. tabulated covariance functions in TEIC.

The publication by Kearsley et al.(1985) contains many other interesting results, and the reader is strongly recommended to study it in detail. The publication has been analyzed and evaluated by a group at the Analytic Sciences Corporation (TASC), which include one SSG member, K.Tait (Comer et al., 1986). They arrive to nearly the same conclusions, and add information concerning the GEOFAST algorithm (Tait, 1979).

The use of real data, as used in the White Sands test, has the limitation that one is not able to test whether the methods have some inherent limitations with respect to e.g. precision. This should not be the case for the methods analyzed in Kearsley et al.(1985) according to the theoretical foundations of the methods. However, some approximations are used, such as spherical or even planar approximation, which could have some unexpected side effects. Here the TASC group has executed a test with the GEOFAST algorithm, which uses several "strong" approximations, like the planar approximation. In their test deflections are recovered using synthetic data with standard deviations below 0.3". This means that if the other methods use similar "clean" data, they should be able to achieve at least this level of precision.

3. Geoid undulation difference computation.

The computation of geoid or height anomaly differences has gained new importance due to the need of converting precise GPS-derived ellipsoidal height differences into differences of orthometric or normal heights. A first set of such data was made available in the White Sands area by C.C.Goad. Only one SSG member (Kearsley, 1985) has reported results using the data. The results are excellent, and showed for the first time that geoid undulation differences may be computed (for short distances) with cm-accuracy. The test data set only includes 4 points, and this limits its usefulness.

Another test data set was collected by R.H.Rapp for the Central Ohio Area (USA). This area is rather flat, and there exist a good distribution of gravity data (Heiskanen and Uotila, 1956). Here a comparison was executed as described in Engelis et al.(1984) between numerical integration of Stoke's formula and least squares collocation. When using Stoke's equation, 30" x 30" height data was used to support the interpolation of free-air gravity anomalies, while this information was not used in the collocation procedure. The test, which included 13 height differences, showed that there is no significant difference between the two methods if the integration process is supported by auxilliar interpolated gravity values, and that a 5 cm accuracy could be achieved.

A comparison between so-called modified astronomical and gravimetric levelling in Austria (Erker, 1987) also confirms that a 5 cm accuracy may be achieved. However, the longest distance between the points was in both tests only 35 km, so also these tests had only value for rather short distances.

Fortunatly the results have been confirmed for distances up to 125 km by Schwarz et al.(1987) and to a certain extend for even longer distances by Ning and Qui (1987). In Schwarz et al.(1987) the FFT, CINT and TEIC methods are compared. The FFT method gives the best agreement with observed geoid undulation differences. The reason that TEIC does not give so good results is probably that data distributed over a relatively small area had to be used in order to limit the size of the system of equations to be solved. The reason why CINT, which was used succesfully for deflectioncomputation in the White Sands area, does not give equally good results,is not clear. The important fact is,however, that results of 2 - 3 ppm precision (height difference relative to distance) has been achieved, corresponding to 0.5" in the deflections of the vertical.

A similar test with the less precise Doppler derived geoid undulation differences has been executed by Ning and Qui (1987), using collocation, optimixed integral kernels (Wenzel, 1982) and kernels modified according to a proposal by Meissl. They find that the optimized integral kernel gives the best result. The reason for this seems again to be the simple fact that data covering a large area may be taken into account and its effect balanced properly with respect to the information contained in the set of potential coefficients used.

4. Southern Ohio torsion balance test computations.

In order to test the computation of the gravity vector from gravity gradiometer data, a similar kind of data set was collected and distributed to members of the SSG. These data are torsion balance data observed in the 60'ties in Southern Ohio (Badekas, 1967). Here are also available deflections of the vertical in the nodes of the grid formed by the torsion balance data points (Mueller and Preuss, 1965) and gravity anomalies as mentioned in section 3. The data have been used for deflection interpolation (Badekas and Mueller, 1968) giving excellent results close to the noise level of the deflection data.

Only one new test has been executed as described in Arabelos and Tscherning (1987) using collocation, and this test gives the same results as obtained in 1968 using a simple numerical integration procedure. The main new result is that also error estimates may be computed, and that the gravity vector components may be computed at points not close to the data points. This gave the possibility for a comparison with gravity anomalies distributed in the area. The results agree with the results obtained for the deflections (0.4"), namely 2 mgal standard deviation of observed minus predicted gravity anomalies.

For the test of the use of gravity gradiometer data collected by an aircraft TASC has developed a test data set (White, 1984). This should be used in futher tests, also here using sets of data which correspond to the expected data collection strategy.

5. Geoid and gravity vector from satellite altimetry.

Satellite altimeter measurements may for a local area be viewed as linearily biased geoid undulations. Used as such the data has been used in numerous tests, especially carried out by R.H.Rapp and he's associates, for the computation of the gravity vector and of geoid heights in the areas between the tracks. It is a challenge to extract the most precise derived quantities, and to compare methods which are able to handle these tasks. In order to support such investigations a test data set was made available by R.H.rapp covering the New England sea mount area, and a sea-gravity data set was received from A.B.Watts.

The altimetry data set had originally been used in Rapp(1984), and a new investigation is reported in Tscherning and Knudsen (1986). In both investigations collocation was used, and similar results were obtained. However, in the last mentioned investigation a local adjustment of the altimeter data was incorporated, and quite significant differences were observed. Unfortunatly the gravity data could not be used to check whether an improved or a worse solution had been found, because this data contained 10 to 20 mgal biases, different for each ship track.

A realistic evaluation of methods (including e.g. the so-called inverse Stoke's method) should therefore take place in an area with high quality sea-gravity data, preferably with a gravity signal as large as the one observed in the New England Sea mount area (more than 100 mgal gravity anomaly).

6. Other evaluation studies.

The importance of providing gravity vector information at flight altitude has inspired several tests, Cruz and Laskowski (1984), Cruz (1985) and Jiang and Cao (1985). Since these tests have not included the comparison with any (synthetic) data, they will not be futher analyzed here. However, it should be noted that there in the two first mentioned studies are found conclusions corresponding to these obtained by Kearsley et al.(1985), namely that the use of topographic information and of spherical harmonic coefficients is very important.

7. Conclusion and recommendations for future work.

The SSG has studied **available** methods for local gravity field determination. The word available has here the significance that it actually means that software exist in a "production" ready form. And it is an important conclusion from the work of the SSG that such software exist for the CINT, FFT, TEIC, RINT and GEO-FAST methods. There do exist other methods (like generalized point mass modelling) for which (operational ?) software exist, and it would be very interesting to test these methods e.g. in the White Sands area. Here the publication by Kearsley et al.(1985) will be extremely helpfull, since the already obtained results are documented here in full detail.

The methods clearly differ in their range of applicability. Only TEIC and CINT are able to use heterogeneous data. On the other hand are these methods the only ones for which instability problems seems to occur, because their use involve the solution of systems of equations. It is on the other hand well known how these problems may be avoided in practice, see e.g. Tscherning (1986, p. 42). Instability problems due to changing data selection patterns seems, however, not to play a role (Barzaghi and Benciolini, 1987). But this problem also need further investigations; especially a comparison between methods will be important.

Especially for the computation of geoid undualtion differences for points separated long distances, it seems to be important to have a large data collection area (Sideris and Schwarz, 1987). This means that the use of collocation is not possible if only one step is used, because a too large system of equations will have to be solved in order to obtain the needed accuracy. However, the existing software permit multi-step solutions and this should be tried. Also new developments with covariance functions set to zero at some distance (see e.g. Tscherning et al. (1986)) may help improving the performance of the collocation method.

Numerical experiments which involve the comparison of several different methods have been carried out succesfully as described in section 2 and 3. Maybe the most interesting outcome of these comparisons have been the success of the FFT methods. Their possibilities seems not yet to have been fully exploited, and a new SSG should be created to pursue these possibilities. Also the upward continuation problems deserves a greather attention, and here could an artificially generated data set be useful. In fact, H.Strang van Hees has kindly provided the SSG with tools to generate such data, but no applications have been reported as yet.

The error-estimation procedures associated with the methods (if any) have not been compared. This is an important area with several delicate problems see e.g. Tscherning (1986). For the moment it seems like error estimation procedures have only been implemented for the collocation method.

All methods have to a certain degree the possibility for incorporating topographic, geophysical and geological information. At least this is possible through remove-restore techniques. For

RINT and FFT it may also be done in the pre-processing stage where gravity values are interpolated. The significance of using the topographic information is clearly demonstrated in Kearsley et al.(1985), Arabelos(1985) and Arabelos and Tscherning (1987a). However, S.Jordan has reported (in a private communication) that the use of geological information in the White Sands area did not improve the results. Here maybe more detailled topographic information will give an improvement.

Geophysical information in terms of knowledge of the (approximate) location of known density discontinuities may not be used directly in existing implementations. Point density anomaly data may be used in collocation (Eisfeller et al. 1986, Tscherning and Strykowski, 1987) but the results need further analysis. Also here a new SSG could do important investigations into the problems of using topographic and geophysical information for gravity field determination.

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