83

Proceedings of the General Meeting of the IAG, Tokyo, May 7–15, 1982

GEOID DETERMINATION FOR THE NORDIC COUNTRIES USING COLLOCATION

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ABSTRACT

The method of collocation may be used for the determination of an approximation to the anomalous (gravity) potential from heterogeneous observations. This approximation may subsequently be used for the estimation of all the gravity field components (and their standard error) needed for geodetic applications, such as the geoid undulations.

The method requires systems of equations to be solved for, with as many unknowns as the number of observations. This problem may be circumvented by (1) constructing local approximations, (2) by using only parts of the data and (3) by smoothing the field through the subtraction of the potential generated by the topographic masses. This permits the consistent use of the large set of observational data (gravity anomalies, deflections of the vertical, doppler and satellite altimeter determined height anomalies and potential coefficients), which are available in the Nordic countries and their surroundings.

The practical feasibility of this procedure has been tested by computing an approximation to the anomalous potential using potential coefficients and all available deflections of the vertical, and by comparing the result with the approximations computed from the data in 3 slightly overlapping subareas.

1. Introduction

Let W be the gravitational potential of the Earth, from which we have subtracted all time varying effects and the potential of all masses external to the solid earth surface. U is a reference potential of the Somigliana-Pizzetti type, i.e. determined so that an adopted reference ellipsoid will coincide with an equipotential surface of U. The anomalous potential $T = W - U$ is then constant in time, harmonic outside the solid earth surface and regular at infinity. We furthermore presuppose, that the coefficients of degree zero and one in the spherical harmonic expansion of T are all equal to zero.

The geoid is then (per definition) the equipotential surface of W, which is closest to mean sea level. We presuppose, that the value of W, W_0 is equal to U_0, the potential of U on the ellipsoid.

The geoid is used as a reference surface in oceanography (defining sea surface heights) and in cartography (defining the orthometric height, H). In precise geodetic work, orthometric heights can not be used, because of the problems associated with their rigorous computation. (Actual gravity, g, must be known along the part of the plumbline, from the geoid to the point on the surface of the earth being considered). Instead the geopotential number, $C = W_0 - W$, or a derived metric quantity, the normal height $H^* = C/\bar{\gamma}$, is used. $\bar{\gamma}$ is the mean value of the normal gravity $\gamma = |\nabla U|$, along the "plumbline" associate with U. If T (or an approximation, $\tilde{T}$) is known as a function of position, then $C = W_0 - U - T$, and H^* may be computed. This enables us to compute the height anomaly $\zeta = h - H^*$, where h is the height above the ellipsoid. (Note, that ζ is defined everywhere in space).

The difference $N = h - H$, is then (if we disregard a very small error) equal to the geoid height or geoid undulation.

The geoid may be constructed, by marking on the plumbline the point with the distance N from the ellipsoid. If height anomalies evaluated on the earths surface are used instead, the so-called quasi-geoid is constructed. This surface, which may be rigorously computed, should be preferred for the geoid in geodetic practice.

However, the geoid (and the orthometric or mean sea level heights) are still used in some organizations. They will have to determine the difference

$$N - \zeta = H^* - H ,$$

which according to Heiskanen and Moritz (1967, eq. (8-103)) is approximately equal to the product of the Bouguer anomaly, Δg_B, and the height anomaly H. (Δg_B is equal to $g - \gamma - A_T$, where A_T is the total attraction of the topography. γ is computed in the point on the normal plumbline with $h := H^*$).

Geoid heights may therefore be computed with sufficient accuracy for many purposes, if we are able to compute the height anomaly, ζ , the (free -air) gravity anomaly, $\Delta g = g - \gamma$, and the quantity A_T . This will be the case when a numerical approximation, $\tilde{T}$, to T has been determined, and when a digital model of the topography is available.

In this paper we will then regard "geoid determination" as a part of the more general problem of determining an approximation, $\tilde{T}$.

2. The Determination of a Geoid for the Nordic Countries

At its meeting in 1978, the Nordic Geodetic Commission adopted a resolution requesting the Danish Geodetic Institute to execute a determination of a "standard" geoid for the Nordic countries. The institute asked the author of this paper to handle the practical work. Representatives of the major data contributors (Norges geografiske Oppmåling, Swedish Land Survey Office, Finnish Geodetic Institute) was later on appointed in order to facilitate the data collection.

At a meeting in Hønefoss (Norway) in March 1982, with these representatives it was decided, that the first goal was the determination of an approximation $\tilde{T}$, which would permit the computation of height anomaly differences (with respect to a central point) at points on the earths surface with a standard deviation below 0.5 m. This would satisfy the main purposes, which were identified as

(1) Computation of normal heights in points with known ellipsoidal heights (obtained for example from space techniques).

(2) Computation of ellipsoidal heights used in 3-dimensional network adjustment at points with known normal heights.

(3) Computation of gravity disturbances from gravity anomalies (preferred by some geophysisists in large scale gravity interpretation).

A number of different methods are available for the construction of approximations to T or for the direct estimation of height or gravity anomalies. A review of the properties of various methods can be found in

(Tscherning, 1980). The most flexible methods are the methods of least squares or least norm collocation. The methods permits the use of all the different gravity field dependent data types, and will work even in situations, where data lack in some areas (such as lakes) or because the area is politically inaccessible.

Three very important properties of the methods are, that (1) the varying quality of the data may be taken into account, (2) that parameters (such as datum shift parameters) may be determined and (3) standard deviations or error bounds may be computed for values of linear functionals applied on the approximation.

The method of (least squares) collocation was because of these properties selected (by the author) as the most suitable method for the computation of $\tilde{T}$. However, also the practical results obtained in for example Greenland (Forsberg and Madsen, 1981) and by other authors was important for the choice.

This does not mean, that collocation can be used in a push-button fashion. The method has a number of build-in problems, which we briefly will describe in section 3. Practical solutions (and proposals for solutions) are described in section 4. Even though the method permits the simultaneous use of different kinds of data, then $\tilde{T}$ has not (yet) been computed from more than one kind of data (disregarding potential coefficients), namely from deflections of the vertical. This computation has been done as a part of a test of the proposals described in section 4. This test, and the evaluation of the resulting quasigeoid is described in section 5.

The reasons why only one kind of solution is described are three. The data collection is not yet finished, and data must be validated before they can be used in the computations. Furthermore, the computer programs and procedures, needs always to be modified (or to be improved). These problems, which in practice are the most difficult (independent of computational method used), will not be dealt with in this paper.

3. Collocation as a Gravity Field Approximation Techniques

The abstract setting for the use of collocation is a Hilbert space, H, with a reproducing kernel, K(P,Q), of which T is an element (P and Q are points in space). The elements of H are here all functions harmonic at least outside the surface of the earth, and regular at infinity. The observations x_i, $i=1,\dots,n$ are linked to T through linear (or linearized) functionals L_i, i.e. $L_i(T) = x_i$. (We will in this brief description of collocation disregard the fact that observations have errors, and that they may depend on parameters.) Let us suppose, that the functionals are elements of the dual space, H^*, and that they are linearily independent. This implies that the $n \times n$ matrix $\{L_i L_j K(P,Q)\}$ is of full rank.

Collocation is then a method, which permits the determination of an approximation, $\tilde{T}$, which agrees with the observations, i.e. $L_i(T) = L_i(\tilde{T})$, and which has the least possible norm. The solution is a linear combination of the functions $L_i K(P,Q)$, with constants a_i to be determined

$$\tilde{T}(P) = \sum_{i=1}^{n} a_i \, L_i(K(P,Q)) \,. \qquad (3.1)$$

The constants are determined using the condition that the observations must agree with the values computed using $\tilde{T}$, or

$$L_j(\tilde{T}(P)) = \sum_{i=1}^{n} a_i L_j(L_i K(P,Q)) = x_j \qquad (3.2)$$

This is a system of linear equations with n unknowns.

If T is an element of H, then absolute error bounds may be computed.

The application of the methods of collocation then requires (1) the choice of a reproducing kernel K(P,Q) and (2) the solution of the system of equations (3.2).

A solution may also be found in the case where T is not an element of H. The only requirement is that $L_i \in H^*$ and that $\{L_i L_j K(P,Q)\}$ is of full rank. $\tilde{T}$ will then be an element of the space in which K(P,Q) is the reproducing kernel. (The space may for example consist of functions harmonic also down to a sphere totally enclosed in the earth).

One possible choice for a reproducing kernel is the so-called empirical covariance function, K(P,Q) = cov(T(P), T(Q)), the empirical covariance between the value of T in P and the value of T in Q. The use of this function gives a $\tilde{T}$ which is optimal in a certain sense, see (Moritz, 1980), but T will not be an element of the corresponding Hilbert space.

Through the empirical covariance function, one tries to represent the "mean square variation" of the coefficients of the spherical harmonic expansion of T, which have the same degree. If the coefficients are C_n^m and S_n^m (degree n, order m), then

$$\mathrm{cov}(T(P),\ T(Q)) = \sum_{i=2}^{\infty} \left(\frac{R_E^2}{r_P r_Q}\right)^{i+1} \sigma_i^2\, P_i(\cos\varphi_{PQ}) \qquad (3.3)$$

where

$$\sigma_i^2 = \sum_{j=0}^{i} ((C_i^j)^2 + (S_i^j)^2)\left(\frac{GM}{R_E}\right)^2 , \qquad (3.4)$$

R_E is the mean radius of the earth, GM is the product of the gravitational constant and the mass of the earth, P_i the Legendre polynomial of degree i, σ^2 the so-called degree-variances, ψ_{PQ} the spherical distance between P and Q, r_P, r_Q the radial distances of P and Q from the origin.

cov(T(P), T(Q)) may in practice be determined as described in (Tscherning and Rapp, 1974). It may also be adapted so that it reflects the local variation of T, see e.g. Lachapelle and Schwarz, 1980).

The quantities $L_i L_j(\mathrm{cov}(T(P),\ T(Q)))$ are called the covariances between the related quantities $L_i(T) = x_i$, $L_j(T) = x_i$. The functions $L_i K(P,Q) = \mathrm{cov}(L_i(T),\ T(Q))$ used in eq. (3.1) are hence the covariance between the observed quantities and the value of the T in Q.

Error estimates may formally be computed as in the case, where T is an element of H. However, the error estimates becomes here estimates of the mean square error.

Let us finally mention one requirement for the use of collocation, which should not be forgotten, as it causes a lot of problems in practice. All observations must refer to a common reference system and normal potential. The reference system should be geocentric, and the reference ellipsoid should give a best fit to the geoid.

4. Preparations for the Use of Collocation

4.1. Data and Reference Systems

A huge amount of data is available within (or close to) the Nordic countries, see Table 1.

Table 1. Data available for geoid determination in Federal Republic of Germany (D) north of 54°, Denmark (DK), Finland (SF), Norway (N), Sweden (S) and surrounding waters (200 NM zone).

Kind		Number of values	Distribution	Reference systems(s)
Gravity	Δg	> 50000	Land and sea	Potsdam + IGSN71
Deflections of the vertical	ξ η	681 649	Land	ED1950, DHDN
Geoid heights from altimetry	N	> 15000	Sea	GRS1980 and local system
Height anomalies from doppler	ζ	25 (DK,N and SF)	Land and Oil Rigs.	NWL9D or WGS72
Potential coefficients	C_n^m S_n^m	181 x 181	Global	Undefined
Topographic heights and depths	H H*	Mean values of 5'blocks	Land and partly of sea	Mean sea level

Various sets of potential coefficients are available, complete to degree and order 180, GEM10C (Lerch et al. 1981) and two sets computed by Rapp (1978, 1981). The sets have been analysed as described in (Tscherning and Forsberg, 1981) by comparing computed values with observed values of ξ, η, ζ and N, cf. Table 1. The best agreement was found with the potential coefficients (Rapp, 1978).

Using sea-surface heights obtained from (Rapp, 1982), the semi-major axis of a locally best fitting ellipsoid was found to be 6378135.5 m.

The coordinate system NWL9D is known to require a differential scale change of $-0.4 * 10^{-6}$ and a rotation around the Z axis of $-0''.8$, (T. Vincenty, 1980, private communication). However, the doppler derived height anomalies showed, that a shift of $\Delta Z = 2.5$ m was needed or a further scale correction should be applied. Combining these values with the parameters describing the transformation from ED1950 to NWL9D given in (Ordnance Survey, 1980), the transformation from ED1950 to an (approximate) geocentric system was obtained. The transformation from DHDN (Deutsche Haupt Dreiecks Netz) was determined in a similar way.

This permitted the transformation of the geodetic coordinates in ED1950 to the geocentric system. The observatory corrections needed for the astronomical coordinates was estimated to be below the standard deviations of the observations, and such corrections were therefore not applied, even though these are of a systematic character.

The gravity values were in nearly all cases transformed from the Potsdam system to IGSN1971 by subtracting the difference between the national base station values in the two systems, if only one base station was involved. A new adjustment to the IGSN1971 network was not considered to be necessary (or possible in practice). The errors introduced by not performing a new adjustment is probably below 0.5 mgal.

4.2. Choice of Covariance Function

The use of potential coefficients as observed values in collocation is nearly equivalent to using the corresponding spherical harmonic series as a reference potential. The degree-variances σ_i^2 , (cf. eq. (3.3)) will then be approximately equal to the square sum of the standard deviations of the coefficients of degree i multiplied by $(GM/R_E)^2$. These degree- variances are according to Rapp (private communication, 1981) equal to

$$\sigma_i^2 = \frac{c^2}{(i-1)^2} \left(\frac{GM}{R_E}\right)^2 , \quad c = 5.8 \times 10^{-8}, \quad 12 < i < 180 .$$

For $i < 12$ was used values based on error estimates of satellite-determined coefficients. For $i > 180$ the degree-variance model determined in (Tscherning and Rapp, 1974) was used. Auto- and cross-covariance functions based on these degree-variances for points with $r_P = r_Q = R_E$ and having the same longitude are shown in Figures 1-6.

In order to see whether this "global" covariance function agreed with the variation of T in the area, an empirical covariance function based on sea-surface heights determined by SEASAT-A and treated as geoid undulations, N, was determined. (The area used was bounded by $0^\circ < \lambda < 32^\circ$, $52 < \varphi < 72^\circ$). The estimated covariance are shown in Figure 1.

4.3. Data Selection

The main computational problem associated with the method of collocation stems from the fact, that as many equations as the number of observations must be solved, and all the coefficients $\mathrm{cov}(x_i, x_j)$ must be computed.

However, in order to reach the goal of being able to compute height anomaly differences with a standard deviation below ±0.5 m, not all observations are needed. (But this does not mean, that all areas contain the necessary amount of data, unfortunately). The required number of (regularily distributed) observations (n) is related to the local gravity field variation, and may be expressed through the mean square gravity anomaly variation, $C_0^2 = \mathrm{cov}(\Delta g(P), \Delta g(P))$, $r_P = R_E$, and the correlation distance ψ_0 of the covariance function, see (Forsberg and Tscherning, 1981a, eq. (13)). These quantities are here $C_0 = 31$ mgal and $\psi_0 = 7''3 \approx 13.5$ km, see Figure 2.

Let the mean square gravity estimation error for a certain square-shaped area be $\overline{\sigma(\Delta g)^2}$, and let s be the side length. Then

$$n = C_0^2 \cdot s^2 \; 0.09 / (\psi_0^2 \; \overline{\sigma(\Delta g)^2}) \qquad (4.1)$$

In order to obtain height anomalies with a standard deviation of ±0.5 m, deflections must be computable with a standard deviation better than ±1". This corresponds to a standard deviation of the predicted gravity anomalies of ±6 mgal. Hence for a $1^\circ \times 1^\circ$ area we get ($C_0^2 = 982$ mgal²),

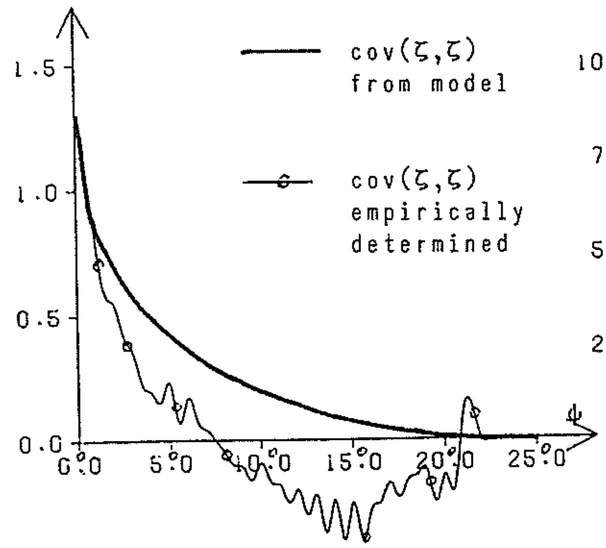

Figure 1. Covariance function of height anomalies (ζ). $r_P=r_Q=R_E$ in all figures.

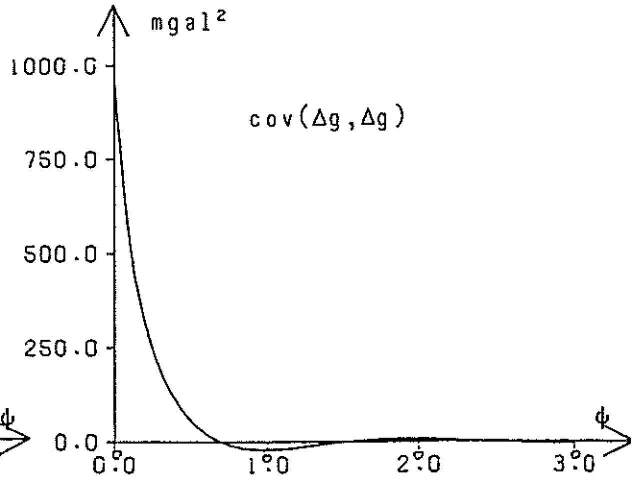

Figure 2. Covariance function of gravity anomalies (Δg).

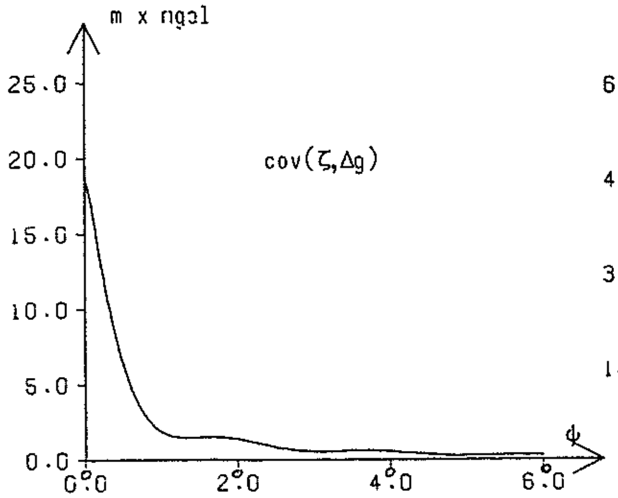

Figure 3. Cross-covariance function of height anomalies and gravity anomalies.

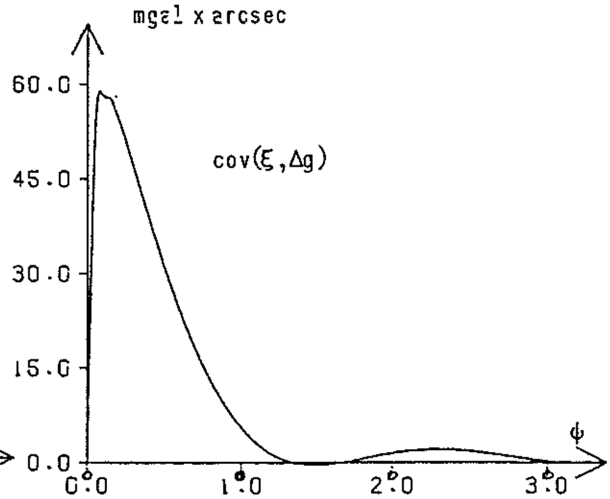

Figure 4. Cross-covariance function of the meridian component of the deflection of the vertical and the gravity anomaly.

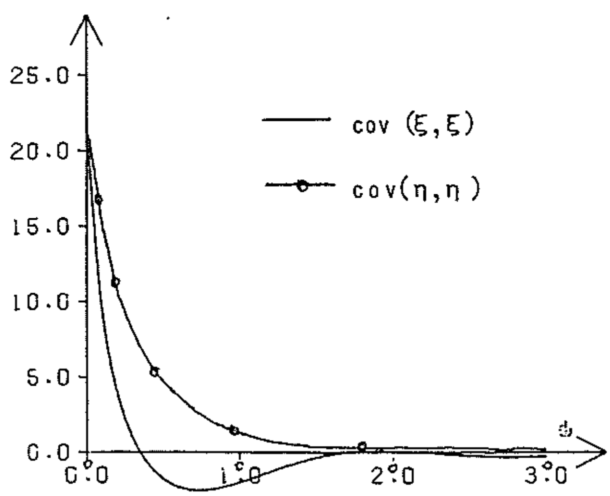

Figure 5. Covariance function of the meridian (ξ) and prime vertical (η) components of the deflection of the vertical.

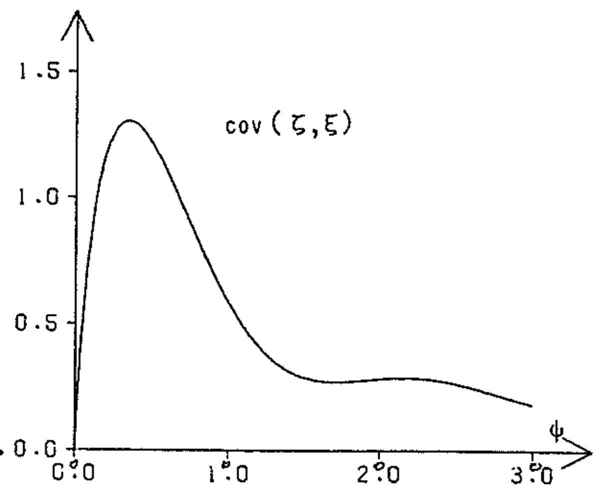

Figure 6. Covariance-function of the height anomaly and the meridian component of the deflection of the vertical.

$$n = \frac{982}{36} \left(\frac{60'}{7!3}\right)^2 \; 0.09 \approx 166 \; .$$

More than 100 such blocks are needed to cover the Nordic countries and their territories of economical interest. We would then need a system of equations with more than 15000 unknowns. Such a system may not be solved using the computer of the Geodetic Institute, and we shall see that it in fact not is necessary to solve such a large system.

Suppose the effect of the isostatically compensated topography is removed from the observations. Then we may cf. (Tscherning, 1979, Table 1) expect the mean square variation C_0^2 to drop 75% to around 245 mgal². In this case only 25% of the observations are needed, if ψ_0 does not change its value. This system is solvable in our computer, but will require many hours of computation.

Inspecting Figures 1-6, it is noted, that the correlations (except these involving the height anomaly) becomes below 1% within $2^o - 3^o$ of spherical distance ψ_{PQ} . These coefficients could then be put equal to zero in the normal equations, eq. (3.2). This would then enable us to use already developed sparce matrix techniques (Mark and Poder, 1981), which are very much in use for the adjustment of geodetic networks.

A nearly equivalent alternative is the partitioning of the data in overlapping blocks. The blocks must then subsequently be "glued" together, in the same manner as done in network adjustment using Helmert blocking. A brute-force method (also used in network adjustment) is to use values in the overlap area predicted by one block as observed values in the adjacent block.

This problem of how to join the blocks is currently being investigated.

Let us finally again consider the data selection problem. The estimated value of 166 gravity anomalies was based on a common value of C_0 . This value will in reality vary considerably from area to area, and this should naturally be taken into account. Based on a locally determined value of C_0 , the value of n may be computed. Observational data as close as possible to n evenly distributed cells (of area s^2/n) should then be selected. These values (together with some values from the neighbouring blocks) should then be used to estimate the remaining observations using collocation. The observations, which differ most from the computed values are then candidates for a second choice of supplementary observations. This process may be repeated until the required level (here 6 mgal) is reached. This procedure is also very useful, because erroneous values may be revealed this way. It has been applied successfully in the southern part of the area. and will later be used in the rest of the area.

5. Determination and Evaluation of an Astrogeodetic Quasigeoid

In order to test the computational procedure discussed in section 4.2, it was decided to carry out a computational experiment, which could be compared to older results, and for which a complete dataset existed.

Astrogeodetic geoid determination has been carried out for the Nordic area, as a whole or in parts, see Bomford (1971), Lachapelle (1973), Monka et al. (1979), Tscherning (1970) and Ussiso (1975), and give an excellent possibility for a check of the computational result.

First all astrogeodetic deflections were used for the computation of an approximation $\tilde{T}$ valid for the whole area. Contour lines for the quasigeoid (evaluated on the ellipsoid), referring to GRS 1980 are found in Figure 7. The values transformed to ED1950 are shown in Figure 8. Error

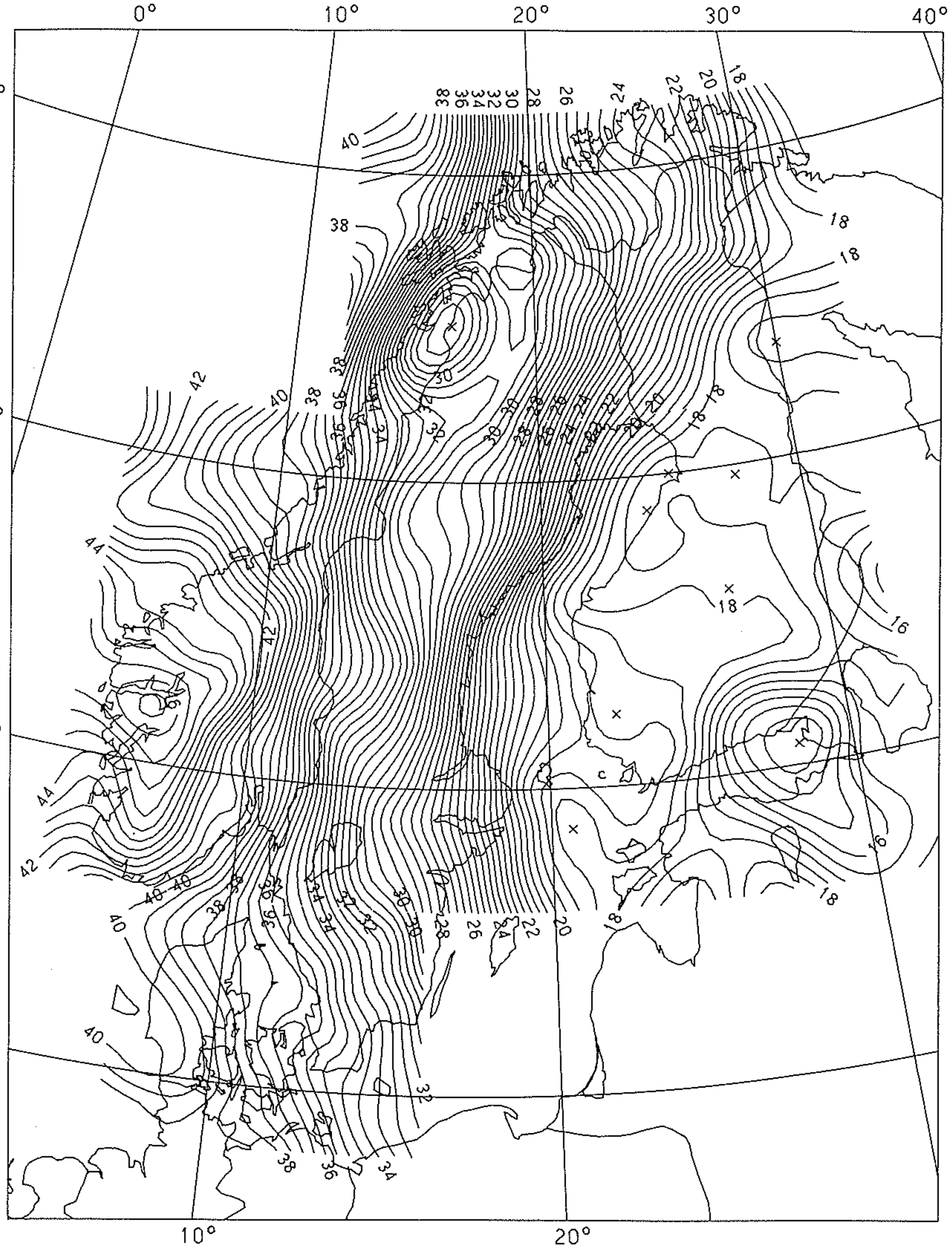

Figure 7. Height anomalies in GRS1980 evaluated on the ellipsoid. Contour interval 0.5 m. x indicates local minimum.

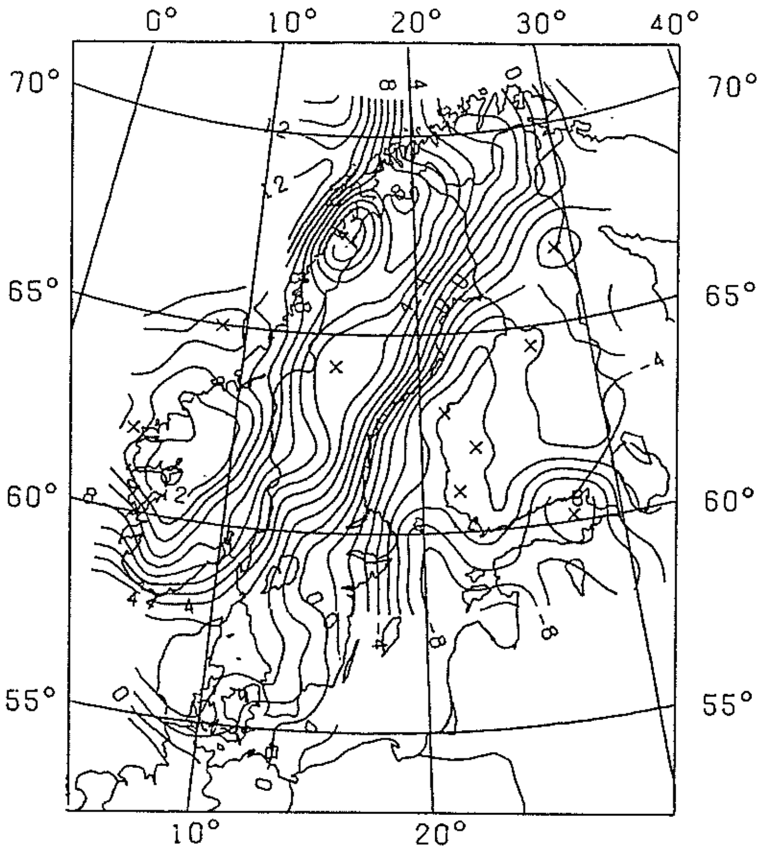

Figure 8. Height anomalies in ED1950. Zero level fixed to 1.5 m in Landskrona. Contour interval 1 m.

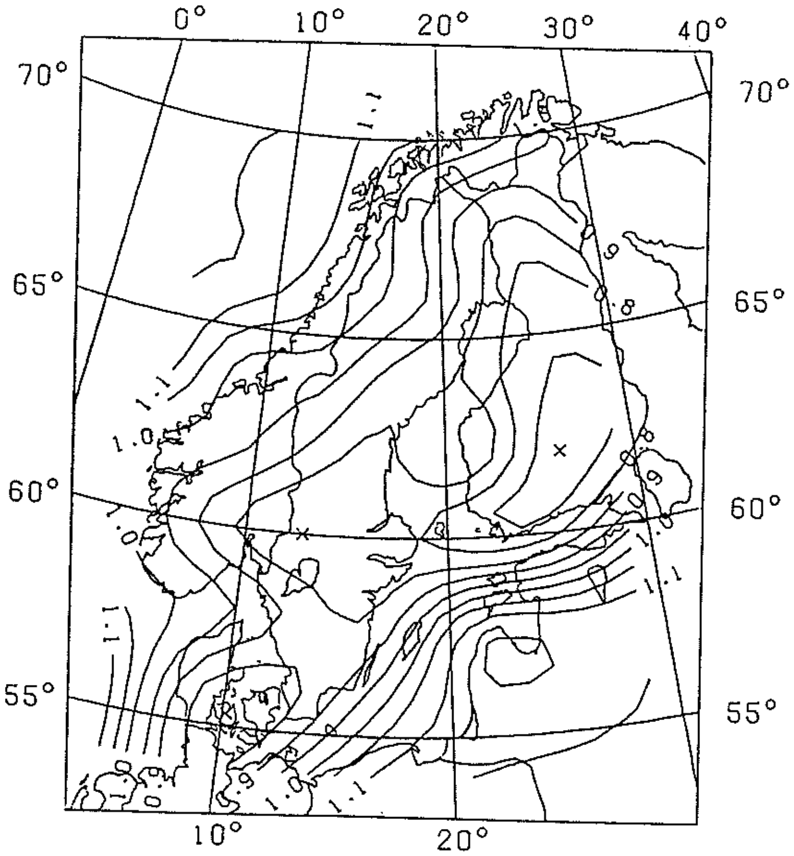

Figure 9. Estimated standard deviations of height anomalies shown in Figure 7. Contour interval 0.05 m.

estimates are shown in Figure 9. Note, that the areas covered with data clearly show up. Also note, that Finland is the area with the smallest errors, due to the dense net of astronomical stations and the high quality of the data.

The observations were the split in 3 sets, along the lines $\varphi = 57.^{\circ}5$ and $\lambda = 20.^{\circ}5$ with a $0.^{\circ}5$ area of overlap. The solutions were "glued" together very primitively, by using the values computed in the area north of $\varphi = 57.^{\circ}5$ and west of $\lambda = 20.^{\circ}5$ in the two points with ($\varphi = 60^{\circ}$, $\lambda = 20^{\circ}$) and ($\varphi = 58^{\circ}$, $\lambda = 11^{\circ}$). The height anomalies were then computed in the same points as done using the first solution. This showed an agreement of ±0.3 m and always better than 0.5 m in the land area. Differences as large as 0.65 m occurred just south of Norway in Skagerrak (as could be expected).

In order to test the quality of the result, the values were compared with doppler derived height anomalies and with sea-surface heights regarded as geoid heights. The results are summarized in Table 2. The results show, that the astrogeodetic determined geoid agrees slightly better with the altimeter geoid heights, than if Rapp's potential coefficients we used alone. The doppler results show the opposite, however these heights are probably not better than ± 1 m, while the satellite geoid error is estimated to be below ±0.3 m.

Table 2. Mean and standard deviations of differences between observed and computed values.

Data	SEASAT geoid		SEASAT geoid		Doppler	
Source	(Rapp, 1982)		* (Andersson et al, 1981)		National organization	
Number of values	110		98		23	
Data used for geoid	Potential coefficients	Deflections	Potential coefficients	Deflections	Potential coefficients	Deflections
Mean St. dev. R.m.s.v.	0.47 m ±0.80 ±0.93	-0.02 m ±0.67 ±0.67	0.64 m ±0.61	0.0 m ±0.51	0.22 m ±0.86 ±0.87	-0.13 m ±0.93 ±0.93

* Bias removed

Based on these results, the error estimates shown in Figure 9 seems very reasonable.

Also a comparison with older results confirm this result, however a more detailed analysis need still to be done.

The executed computations has shown the potential of the collocation method, and the feasibility of computing the quasigeoid using data from overlapping areas. This is also comfirmed by (unpublished) results obtained in the New Mexico test area, see (Forsberg and Tscherning, 1981).

6. Conclusion

We have here described the types of preparations and the types of results which may be obtained using collocation. We have shown, that the method of computing local, but overlapping solutions, is feasible.

Results better than 0.5 m may be obtained if data of other kinds are included, or if the topography is taken into account. This presupposes, that data are available, validated and in an appropriate physical form. It is in practice here the major problems occur, and where most human effort will be spend in the future.

Acknowledgement: Data used in this paper have been furnished by the following persons and organisations: Defence Mapping Agency Aerospace Center (gravity), R.H.Rapp (potential coefficients and altimeter data), S.Klosko (same) and H.-G. Wenzel (deflections of the vertical).

References

Anderson,A.J. and H.G.Scherneck: The Geoid in the Baltic and Gulf of Bothnia obtained from SEASAT Altimeter Data. University of Uppsala Institute of Geophysics, Department of Geodesy Report No. 10, 1981.

Bomford, G.: The astrogeodetic geoid in Europe and connected areas 1971. Appendix to Mrs. I.Fishers Report for SSG V-29, IAG, 1971.

Forsberg,R. and F.Madsen: Geoid Prediction in Northern Greenland using Collocation and Digital Terrain Models. Annales de Geophysique, 37, 31-36, 1981.

Forsberg,R. and C.C.Tscherning: The use of Height Data in Gravity Field Approximation by Collocation. J.Geophys.Res., 86, 7843-7854, 1981.

Forsberg,R. and C.C.Tscherning: Deflection and Gravity anomaly prediction for Inertial Positioning using Collocation. Proceedings Second Int. Symp. on Inertial Technology for Surveying and Geodesy, Banff, June 1-5, 1981, (edited by K.-P.Schwarz), 89-105, 1981.

Heiskanen,W.A. and H.Moritz: Physical Geodesy. W.H.Freeman & Co., San Francisco, 1967.

Lachapelle,G.: Estimation of the Accuracy of the Astrogeodetic Geoid. Diss. University of Helsinki, 1973.

Mark,A.N. and K.Poder: Ordering and Dissection of Geodetic Least Squares Equations. Presented Symposium on Geodetic Networks and Computations. München, Aug. 31 - Sept. 5, 1981.

Monka,F.M., W.Torge, G.Weber and H.-G.Wenzel: Improved Vertical Deflection and Geoid Determination in the North Sea Region. Wiss.Arb. der Fach. Verm. der Univ. Hannover, 94, 1979.

Moritz,H.: Advanced Physical Geodesy. H.Wichmann Verlag, Karlsruhe, 1980.

Lerch,F.J., B.Putney, S.Klosko and C.Wagner: Goddard Earth Models for Oceanographic Applications (GEM 10B and 10C). Marine Geodesy, 5, 145-187, 1981.

Ordnance Survey: Report of Investigations into the Use of Satellite Doppler Positioning to Provide Coordinates on European Datum 1950 in the Area of the North Sea. Professional Papers (New Series) 30, 1981.

Rapp, R.H.: Global Anomaly and Undulation Recovery using Geos-3 Altimeter Data. Dep. of Geodetic Science, rep. no. 285, The Ohio State University, Columbus, 1979.

Rapp, R.H.: The Earths Gravity Field to Degree and Order 180 using SEASAT Altimeter Data, Terrestrial Gravity Data, and other Data. Dep. of Geod. Sc. rep.no.322, 1982.

Rapp, R.H.: The determination of Geoid Undulations and Gravity Anomalies from SEASAT Altimeter Data. Submitted J.Geophys.Res., 1982.

Schwarz, K-P. and G.Lachapelle: Local Characteristics of the Gravity Anomaly Covariance Function. Bulletin Geodesique, 54, 21-36, 1980.

Tscherning, C.C.: Geoidebestemmelse ved Kollokation. (Geoidedetermination by Collocation, in Danish), Nordiska Kommissionen för Geodesi. Helsinki, 1971.

Tscherning,C.C.: Gravity prediction using collocation and taking known mass density anomalies into account. Geophys. J. R. astr. Soc., 59, 147-153, 1979.

Tscherning,C.C.: A comparison of some methods for the detailled representation of the Earth's gravity field. Rev. Geophys. Space Phys., 19, 213-221, 1981.

Tscherning,C.C. and R.Forsberg: Geoid-determination in the Noewegian-Greenland Sea - An Assessment of Recent Results. Submitted to Geoevolution, 1981.

Tscherning,C.C. and R.H.Rapp: Closed Covariance Expressions for Gravity Anomalies, Geoid Undulations and Deflections of the Vertical implied by Anomaly Degree Variance Models. Dep. of Geodetic Science, Rep. no. 208, The Ohio State University, Columbus, 1974.

Ussiso,I.: GEOID RAK 1970. Rikets Allmänna Kartverk, Meddelande D 28, 1975.