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Deflection and gravity anomaly prediction for inertial positioning using collocation

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Abstract
The accuracy of positioning using inertial techniques depends primarily on the performance of the inertial measuring unit and on the knowledge of the gravity field. This knowledge may be obtained from observed values of the gravity field and from information on the topographic heights.

Collocation may be used as an adequate tool for the computations of the accelerations due to the gravity field along the track in space of the inertial sensors, thus providing, if the approximation is sufficiently good, the necessary separation between the ficticious accelerations (due to the system movements) and the gravity vector. In the paper the quality of the computed gravity accelerations are described as a function of the spacing of the used gravity field data and topographic heights. From the results the improvement of the inertially determined positions due to the better knowledge of the gravity field variations may be estimated

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1. INTRODUCTION

With the development of inertial survey systems and procedures towards higher and higher accuracy, the lack of knowledge of the detailed structure of the earth's gravity field has evolved more and more as being a primary error source for inertial surveying. By its very nature inertial surveying relies on a knowledge of the gravity field, the accelerations sensed by the accelerometers being

$$f_i = \ddot{r}_i + g_i , \tag{1}$$

where $\ddot{r}_i$ are the accelerations of the inertial measuring unit in an inertial reference system and g_i are the gravity vector components. In all nonexperimental inertial surveying operations equation (1) is "solved" by a (usually real-time) double integration along the track of the vehicle, assuming g_i to be given as the normal gravity field vector γ_i . In this way the gravity disturbance vector

$$\delta g_i = g_i - \gamma_i = (\nabla T)_i , \tag{2}$$

where T is the anomalous potential, is left to produce systematic navigation errors ε_i. (Geodetic terms used here will be explained in section (2)).

If the anomalous gravity field were sufficiently well known, ε_i could be evaluated straightforward as an integral

$$\varepsilon_i(t) = \int_{t_0}^{t} \int_{t_0}^{t} \delta g_i \, dt' dt'', \tag{3}$$

along the path of the vehicle. Using unaided inertial navigation, the error ε_i grows rapidly with time and may become very large, see e.g., (Chatfield et al., 1975) and section 3 of this paper.

In surveying applications, the build up of the gravity-induced error and errors due to instrumental imperfections (gyro drifts, misalignments, etc.) are kept limited by the use of zero velocity updates - ZUPT - where the vehicle carrying the inertial survey system is stopped and the size of the system computed velocity at the stop is utilized in a Kalman filter estimation of position errors, velocity errors, instrumental errors, etc. The state model of the system may also include means of values of gravity field quantities between the stops, see e.g., Schwarz (1980).

In spite of the ZUPT procedure significant gravity field induced position errors may still build up, especially in mountainous areas, where the variation of the gravity field is largest. (An example is given in section 3). In actual inertial surveying applications, profiles are traversed both forward and backward, and in mountainous areas shorter time intervals between ZUPT-stops are utilized. In this way the gravity induced errors are suppressed as much as possible. However, if we have a good gravity model for the area, we would have been able to calculate the errors ε_i directly. Besides allowing longer time between ZUPT's in mountainous areas, we would be able to investigate the behavior of the instrumental errors in a better way, as a major systematic error has been almost cancelled.

A (3-dimensional) gravity field model useful for inertial surveying should be able to utilize all types of existing gravity field data in the area of operation, including topographic information (gridded heights) in mountainous areas. It should also be able to utilize data acquired by the system at each ZUPT-stop where the accelerations sensed are the gravity vector components, ($\ddot{r}_i = 0$ in eq. (1) if we disregard earth rotation).

The method of collocation fulfills all these demands, and is thus very well suited for post-mission inertial survey analysis. The method permits the use of all kinds of gravity field data such as, for example, gravity anomalies, deflections of the vertical and geoid undulations. In mountainous areas, topographic data may be used to improve the approximation accuracy.

Once a collocation approximation is constructed for a given area, any number of predictions (e.g., of the gravity disturbance vector along the spatial track of a vehicle) may be easily calculated. The method is thus especially well suited for use in inertial survey test areas, where different data types are available, such as e.g., the White Sands test area of New Mexico (which besides being an inertial survey test area (Hannah and Paulis, 1980), (Todd, 1981), the area also has been used to test gravity prediction methods (see Tscherning and Forsberg, 1978), Schwarz (1978), Lachapelle and Mainville (1980), Forsberg and Tscherning (1981)).

In section 2 of this paper, we will review the method of collocation and discuss the approximation accuracy obtainable for the gravity disturbance vector as a function of the density of gravity field data and height information. In section 3, the errors in position due to gravity field uncertainty are briefly discussed based on a simulated inertial survey vehicle pass in the New Mexico test area.

2. GRAVITY FIELD MODELING USING COLLOCATION

2.1 Some Basic Definitions

The gravitational potential of the Earth, W, is equal to the sum of the gravity potential, V, and the rotational potential, Φ. The potential V is a harmonic function in space. (We disregard all masses external to the solid earth's surface). V may then be approximated by a series in solid sperical harmonics,

$$\tilde{V}_o(\bar{\phi},\lambda,r) = \sum_{i=0}^{N} \frac{GM}{r} \left(\frac{a}{r}\right)^i \sum_{j=0}^{i} P_{ij}(\sin\bar{\phi})\{C_{ij}\cos j\lambda + S_{ij}\sin j\lambda\}, \quad (4)$$

where $\bar{\phi}$ is geocentric latitude, λ the longitude, r the distance from the origin, GM the product of the gravitational constant and the mass of the Earth, a the semi-major axis of the reference ellipsoid, P_{ij} the associated Legendre - functions, C_{ij}, S_{ij} are constants, the so-called potential coefficients. (The tilde symbol on the top of V denotes here and in the following "approximation to" or "estimate of").

The normal potential, U, mentioned in section 1, is an approximation to W, defined by GM, a, C_{20}, and Φ. The difference T=W-U, will be a harmonic function in space, because the rotational potential contributions from W and U cancels.

2.2 Collocation and the Use of Height Data

In order to determine the gravity disturbance vector $\{\delta g_i\}$, we will determine an estimate $\tilde{T}$ of the anomalous potential. Here we are facing the old geodetic problem of determining a solution to a partial differential equation (the Laplace equation, $\Delta T = 0$) using data at discrete points.

A review of different methods for the determination of $\tilde{T}$ are given in (Tscherning, 1981). However, only approximation techniques like the method of collocation gives us the possibility of combining different data types and a suitable method for computing δg_i at an arbitrary point. And collocation is the only method, which permits the easy update of $\tilde{T}$ when new data are available. It also permits us to compute error estimates and the error covariances of estimated quantities.

Before giving more details about collocation, we will describe how two important data sources, the topographic heights, and sets of potential coefficients C_{ij}, S_{ij} can be taken into account.

The topographic masses (contingently with their isostatic compensation) produces a potential, T_M, which we may add to U, or equivalently subtract from T getting

$$T^c = T - T_M . \qquad (5)$$

This function will be much smoother than T and it will still be a harmonic function, hence, it may be approximated using exactly the same methods as used when approximating T, but the results are better, due to the smoothing. We then have

$$\tilde{g}_i = \{\nabla\tilde{T}\}_i + \{\nabla T_M\}_i + \gamma_i \qquad (6)$$

We may choose to regard only the "noisy" high-frequency part of the topography, e.g., the difference between a $1/2^\circ - 1^\circ$ mean height surface and the true heights. This gives a number of computational advantages. It limits, for example, the distance out to which we need to consider this "residual terrain model", RTM, see (Forsberg and Tscherning, 1981).

Instead of using the normal potential U, we may use $W_0 = \tilde{V}_0 + \Phi$, where $\tilde{V}_0$ is given by eq. (4). Today, potential coefficient sets complete to degree and order 180 are available, see e.g., Rapp, (1978). The effect of using this kind of reference field, is that data which are not "near" to the area of interest, do not necessarily need to be considered when constructing approximations to

$$T_0 = W - W_0 \text{ or } T_0^c = W - W_0 - T_M .$$

In many cases $\{\nabla T_0^c\}_i$ will be so small, so that $W_0 + T_M$ contains sufficient information for inertial surveying.

Because T, (T^c and T_0^c) is a harmonic function, it will be an element of a so-called reproducing Kernel Hilbert space. The reproducing Kernel, K(P,Q), has a number of properties of which the most important are that it is a function of two-space variables (points) P and Q, and that it is a harmonic function for P or Q fixed.

Let us suppose that our (error-free) observations (m_j) are associated with linear functionals L_j applied on T,

$$L_j(T) = m_j, \; j = 1,\ldots,n, \tag{7}$$

(Example: L_j may be equal to one of the components $\partial/\partial x_i$ evaluated in a point P, $L_j T = \partial T/\partial x_i | P = \delta g_i$). Then an approximation $\tilde{T}$ may be constructed as a linear combination of harmonic functions $L_j K(\cdot,Q)$, where L_j is applied on K for fixed Q. The approximation

$$\tilde{T}(Q) = \sum_{j=1}^{n} a_j \, L_j \, K(\cdot,Q), \tag{8}$$

will agree exactly with error-free observations,

$$L_j(T) = L_j(\tilde{T}) = m_j \;, \tag{9}$$

and it will be the most "smooth" (= has the smallest norm) between all functions fulfilling eq. (9). The coefficients $\{a_j\}$ are determined by solving the "normal equations"

$$\{L_i L_j K(\cdot,\cdot)\} \; \{a_i\} = \{m_j\}. \tag{10}$$

Note, that addition of new observations just mean adding new columns (and rows) to the matrices in eq. (10). This means that the cost of adding new observations (updating $\tilde{T}$) is small if the Cholesky factor of $\{L_i L_j K(\cdot,\cdot)\}$ is kept. Other types of approximation techniques require that all elements of the normal equation matrix are changed.

Data errors or covariances may be taken into account by adding the variance - covariance matrix of data errors to $L_i L_j K$.

Using $\tilde{T}(Q)$, we may then compute estimates $L(\tilde{T})$ of any quantity $L(T)$, as for example, one of the gravity disturbance vector components in a point in space. When the so-called empirical covariance function is used, $(\mathrm{cov}(T(P),\, T(Q)) = K(P,Q))$, the estimates $L(\tilde{T})$ are optimal in a least-squares sense, as discussed e.g., in Heiskanen and Moritz (1967, Chapter 7). The covariance function is determined based on a knowledge of the potential coefficients C_{ij}, S_{ij} and on estimates of their behavious for large i (see Tscherning and Rapp (1973), Lachapelle and Schwarz (1980)).

Root mean square prediction errors can be computed using Heiskanen and Moritz (1967, eq., (7-64)),

$$\sigma(L(T)) = (LLK(\cdot,\cdot) - \{LL_iK(\cdot,\cdot)\}^T \{L_i L_j K(\cdot,\cdot)\}^{-1} \{LL_j K(\cdot,\cdot)\})^{\frac{1}{2}} \tag{11}$$

We may use this equation in order to find the prediction error as a function of the mean data spacing,

$$d = ((\text{area size } (\mathrm{km}^2)/N)^{\frac{1}{2}} \;, \tag{12}$$

where N is the number of points.

For gravity anomalies at the same height, the gravity anomaly covariance function, (C), will be a function of the spherical distance, ψ, between P and Q. Let us denote the mean square variation $C(0)=C_0$ and the correlation length by ψ_o, (where $C(\psi_o)=\frac{1}{2}C_0$). The covariance function may then for small ψ ($< \psi_o$) be approximated by

$$C(\psi) = C_0(1-\tfrac{1}{2}(\psi/\psi_o)^2). \quad (13)$$

The mean over the area of the root mean square prediction error, $\overline{\sigma(\Delta g)}$, is then to a good approximation

$$\overline{\sigma(\Delta g)} = C_0^{\frac{1}{2}} \cdot d \cdot 0.3/\psi_0 \quad (14)$$

or if the area is a square with side length s,

$$\overline{\sigma(\Delta g)} = C_0^{\frac{1}{2}} \cdot s/\sqrt{N} \cdot 0.3/\psi_0. \quad (15)$$

This relation is clearly seen in Figure 1, where $s = \frac{1}{2}^{\circ}$ and $\overline{\sigma(\Delta g)}$ was computed using collocation in 50 mountaneous areas of the United States. (In these areas the mean value of ψ_o was 6'.).

The equations also enable us to compute the mean prediction error for deflection of the vertical $\overline{\sigma(\xi,\eta)}$ because we from models of the empirical covariance functions (see Tscherning and Rapp, 1973), know that there is an approximately constant ratio equal to 6.6 arcsec/mgal between $C_0^{\frac{1}{2}}$ and the root mean square variation of the deflection of the vertical. (A gravity prediction error of ± 5 mgal will correspond to an error in predicted deflections of ± 0".8, cf. table 1).

2.3 Results of Least Squares Collocation

The method of least squares collocation has been extensively tested (and used in practice, see e.g., (Forsberg and Madsen, 1981)). Results of a number of tests are given in (Lachapelle and Tscherning, 1978).

In lowlands, the gravity variation generally is so small that the gravity disturbance vector can be computed everywhere with a precision satisfactory for inertial surveying (see section 3). The real difficulties occur in mountainous areas, where gravity is scarcer, its variation (C_o) is large and ψ_o is small.

Fortunately, we may, at least partially, make up for this by removing the short wavelength topographic gravity field noise, i.e., constructing approximations to T^c instead of to T. This is clearly seen in numerical predicition experiments in two $1^{\circ} \times 1^{\circ}$ squares in New Mexico (Forsberg and Tscherning, 1981) where a 2-3 times reduction of the error of both horizontal and vertical gravity vector components took place when the influence of the topography was eliminated before using collocation.

In inertial surveying we will, in many cases, be in the situation where very few gravity observations are available, but digitized topographic height information exist. This information may be based on large-scale maps, so it is important to know how much information is contained in differently spaced height information.

We have, therefore, made a new investigation in the northernmost of the two $1^o \times 1^o$ areas used in (Ibid., 1981). This area has the most varying topography, and is shown in Figure 2.

From a data set consisting of 9 deflections of the vertical and ~103 gravity anomalies (spaced approximately 6' apart), an approximation $\tilde{T}_0^c$ was constructed. Rapp's 180 x 180 potential coefficients was used as $\tilde{V}_0$ as discussed above.

The accuracy of the gravity field approximation was evaluated by comparing predicted values with an independent set of observations consisting of 58 deflection pairs and 50 gravity anomalies. The terrain reductions were based on topographic mean heights on a geographic grid using the methods described in Ibid. (1981).

Table 1 shows the prediction result statistics when using a RTM-reduction based on a 30' x 30' mean height grid, i.e., when only topographic irregularities of wavelength less than 30' are taken into account.

From the table, the dramatic gain in approximation accuracy obtained when using the topographic height is clearly seen on all components. When the most detailed topography is used (and gravity anomalies 6' apart), we see that we can reproduce the complete gravity disturbance vector with an accuracy of 4-5 mgal. The adequate data spacing for the digital terrain model seems to be 1' x 1'.

Apart from the spacing of the height data, it is also necessary to know how much height data is needed. If we only took into account the shortest wavelengths of the topography, then only the topographic data most near to the prediction points is needed. Table 2 shows prediction results as a function of "wavelength." It is seen that too short-periodic RTM's should be avoided, and a reasonable compromise between accuracy and the need for data seems to be 30' x 30' RTM-reductions. (Note that topographic information of wavelength longer than 1^o is included in Rapp's 180 x 180 coefficients.)

3. AN EXAMPLE OF GRAVITY ERRORS IN INERTIAL NAVIGATION

Gravity errors in inertial navigation have been studied in a number of papers: (Nash, 1968), (Levine and Gebb, 1969), (Hildebrant et al., 1974), (Chatfield et al., 1975), (Bernstein and Hess, 1976), (Schwarz, 1979).

In order to get more insight into the gravity errors occurring during an inertial navigation mission, we have computed the error along a west-east profile in the New Mexico test area (fig. 2), using a perfect local-level inertial navigation system. (By perfect, we mean a system free of all instrumental errors, such as gyro drifts, etc.)

The navigation errors of such a system would be given as the gravity induced errors ε_i, which can be calculated component-by-component using (3), when the position of the inertial system is known as a function of time. In order to simulate the errors $\varepsilon_i^!$ occurring when ZUPT-procedures are used, we have introduced a very simple update procedure for our ideal inertial navigator. When the navigator is stopped, the velocity error accumulated since the previous ZUPT-stop is supposed to be due to errors δa_i (gravitational or instrumental) in the accelerations sensed by the inertial survey system. If the velocity error at the last ZUPT-stop is $\Delta\upsilon$, and the time since the last ZUPT is Δt, we have

$$\Delta\upsilon_i \approx \overline{\delta a_i} \cdot \Delta t \tag{16}$$

and hence for the position correction $\Delta\varepsilon$,

$$\Delta\varepsilon \approx \tfrac{1}{2}\, \overline{\delta a_i}\, (\Delta t)^2 \approx \Delta\upsilon_i \frac{\Delta t}{2} \tag{17}$$

In the following examples, we have assumed the inertial survey system to move at ground level, at a constant speed ν, along 33.3 N in the test area, and evaluated the unaided position error ε_i and the error ε_i' as it builds up using ZUPT's at time intervals Δt.

Figure 3,4 shows the topography, the gravity disturbance, and the inertial errors occurring for a forward and backward traverse at speed υ = 50 m/s and Δt = 4 min., roughly corresponding to the time interval used in a heliborne survey.

The vertical and the east-west (η) components are shown, evaluated at the basis of $\tilde{W} = \tilde{W}_0 + \tilde{T}_0^{\,c} + T_M$, where $\tilde{T}_0^{\,c}$ is computed using collocation corresponding to the second row of Table 1. From the figures, the need for velocity updates is clearly seen. During the half-hour effective flying time along the traverse, the "unaided" error builds up to several hundred meters, while the ZUPT survey errors stay at the meter level.

Note the high correlation between the "forward" and "backward" errors ε_i'. These errors will be highly systematic, and may only be eliminated by either shortening the ZUPT interval Δt or by using gravity field modeling techniques, e.g., as described in section 2. The effect of shortening Δt is shown in Table 3, as the maximal error ε' for a single forward run.

The example discussed here shows that large gravity-induced systematic errors may occur, and they can not be eliminated by the filter techniques used today. Here collocation may be used in order to reduce this type of error.

4. CONCLUSION

Some potential users of collocation have been discouraged because of the needed computational efforts. This may, however, be very much reduced by constructing local approximations to T, which are only valid for a specific area, e.g., a $1^0 \times 1^0$ block. For such an area around 100-200 observations will be needed in order to recover $(\Delta T)_i$ to a precision of ± 5 mgal, if we take the topography into account.

Collocation is well suited to handle new data collected during vehicle movement using gradiometry or ZUPT gravity data. Considering the capacity of modern computers, it may be feasible to use collocation during a mission. It would, for example, be possible to "compress" a sequence of gradiometer observations into an estimate of a gravity disturbance vector. This could be used immediately during the vehicle movement and also for the update of the approximation $\tilde{T}$.

When we, by using collocation, have established a gravity field model for an area, we can use it for the direct calculation of the gravity field induced errors. Thus, fewer ZUPT-stops will be needed, and we may be in a better position to handle other types of errors. How much we actually are able to reduce the total error requires a more comprehensive (and realistic) investigation than the one presented in this paper.

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Height Data Used	ξ (arcsec)			η (arc sec)			Δg (mgal)		
	$\bar{x}$	σ	max	$\bar{x}$	σ	max	$\bar{x}$	σ	max
None	0.77	2.52	8.26	-0.25	2.37	6.13	-1.03	14.93	74.07
0.5' x 0.5'	0.27	0.73	2.60	0.07	0.91	-2.53	0.47	4.78	-11.40
1' x 1'	0.19	0.73	2.12	0.05	1.02	-2.43	0.52	4.79	11.44
2' x 2'	0.14	0.93	2.33	0.04	1.17	2.46	0.52	4.97	12.53
5' x 5'	0.19	1.62	-8.07	-0.20	1.52	-5.18	0.40	5.24	13.83
10' x 10'	0.18	2.71	-11.97	-0.10	2.30	-7.20	0.59	5.20	13.14
Observations Used in Collocation	-2.53	3.40	-13.10	-5.10	5.11	-16.70	-2.23	32.12	98.80

Table 1.--New Mexico Gravity Component Prediction Statistics as a Function of Spacing of Topographic Mean Height Data. RTM-reduction Based on 30' x 30' Mean Height Surface.

Mean Height Grid Defining Mean Elevation	σ_ξ (arcsec)	σ_η (arcsec)	$\sigma_{\Delta g}$ (mgal)
1° x 1°	0.73	0.83	4.70
30' x 30'	0.73	0.91	4.78
15' x 15'	0.81	1.13	6.75
5' x 5'	0.96	1.35	9.86
None (isostatic reduction in a 6° x 7° area used).	0.76	1.10	4.81

Table 2.--Standard Errors of Gravity Field Prediction as a Function of Mean Elevation Surface Resolution

COMPONENT:				
ZUPT - Interval		Vertical (meter)	East-West (meter)	North-South (meter)
min.	sec.			
10	20	23.5	13.7	10.4
5	10	7.1	7.1	4.1
3	26	3.1	3.0	1.1
2	04	0.9	0.8	0.8
1	02	0.3	0.2	0.1

Table 3.--Maximal Absolute Gravity-induced Inertial Position Errors for V = 50 m/s and Varying ZUPT - Interval. Simulated Forward Heliborne Survey of 33.3°N Profile.

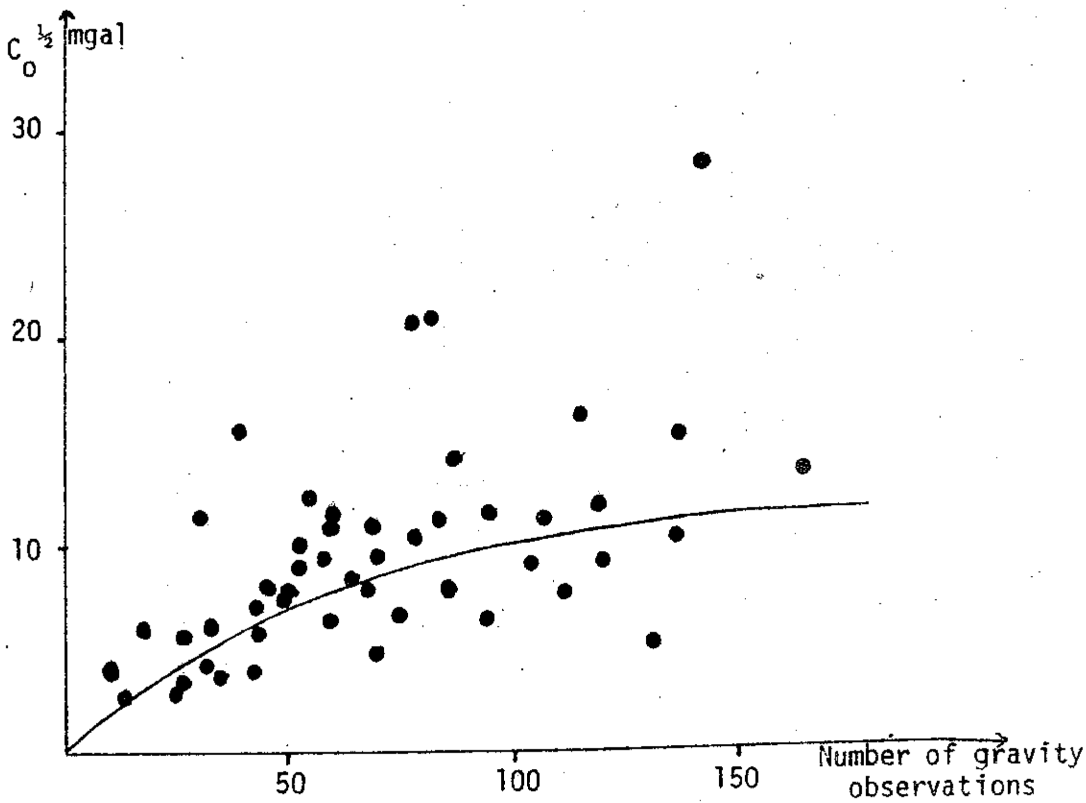

Figure 1. Number of gravity anomalies needed in a $1/2^o \times 1/2^o$ square to get a 1.5 mgal prediction error plotted against the root mean square variation ($C_o^{½}$) of the RTM-reduced gravity anomalies in the square. The (full) curve shows the theoretically needed number of observations for $\psi_0=6'$, see eq. (15).

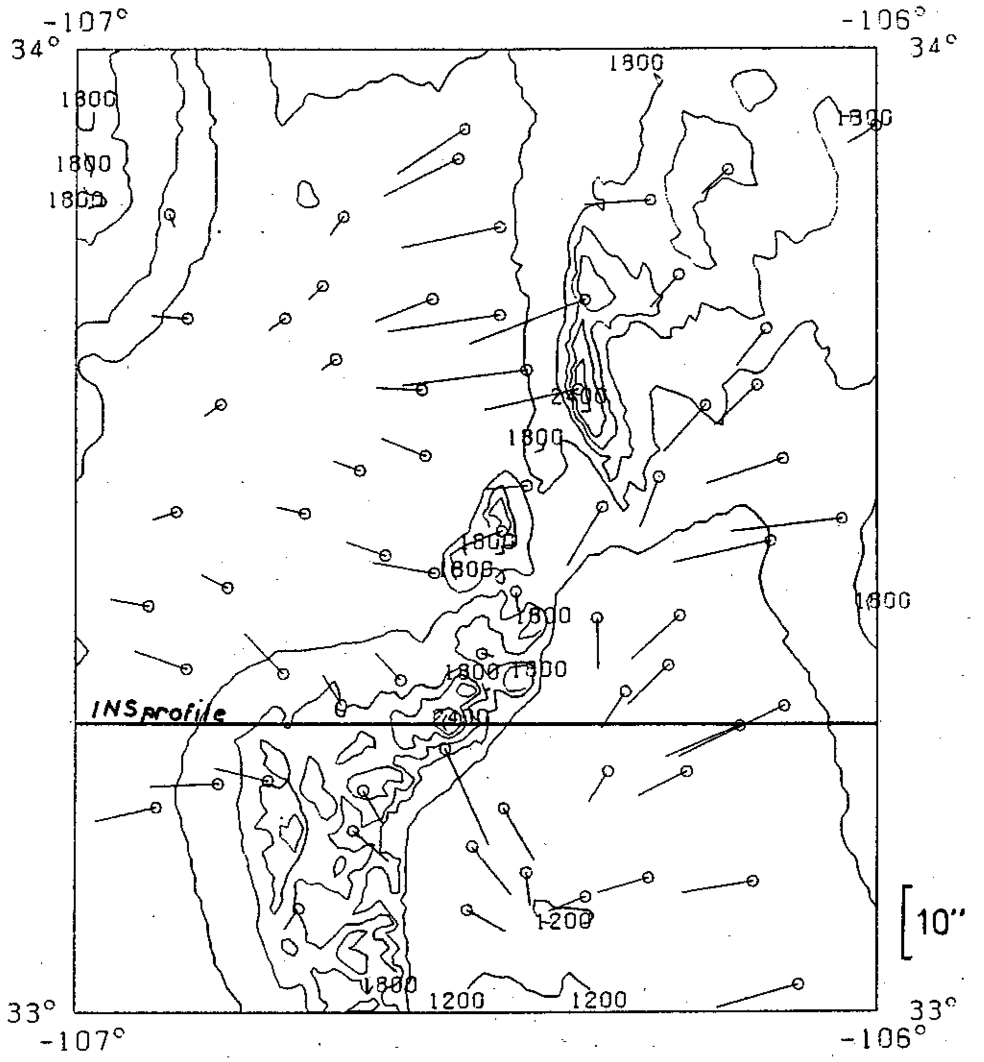

Figure 2 Map of $1^{o} \times 1^{o}$ test area, New Mexico, Contour interval 200 m. Observed deflections of the vertical shown as arrows. Examples of gravity-induced inertial errors along the east-west profile at latitude 33^{o} 18' is given in Section 3.

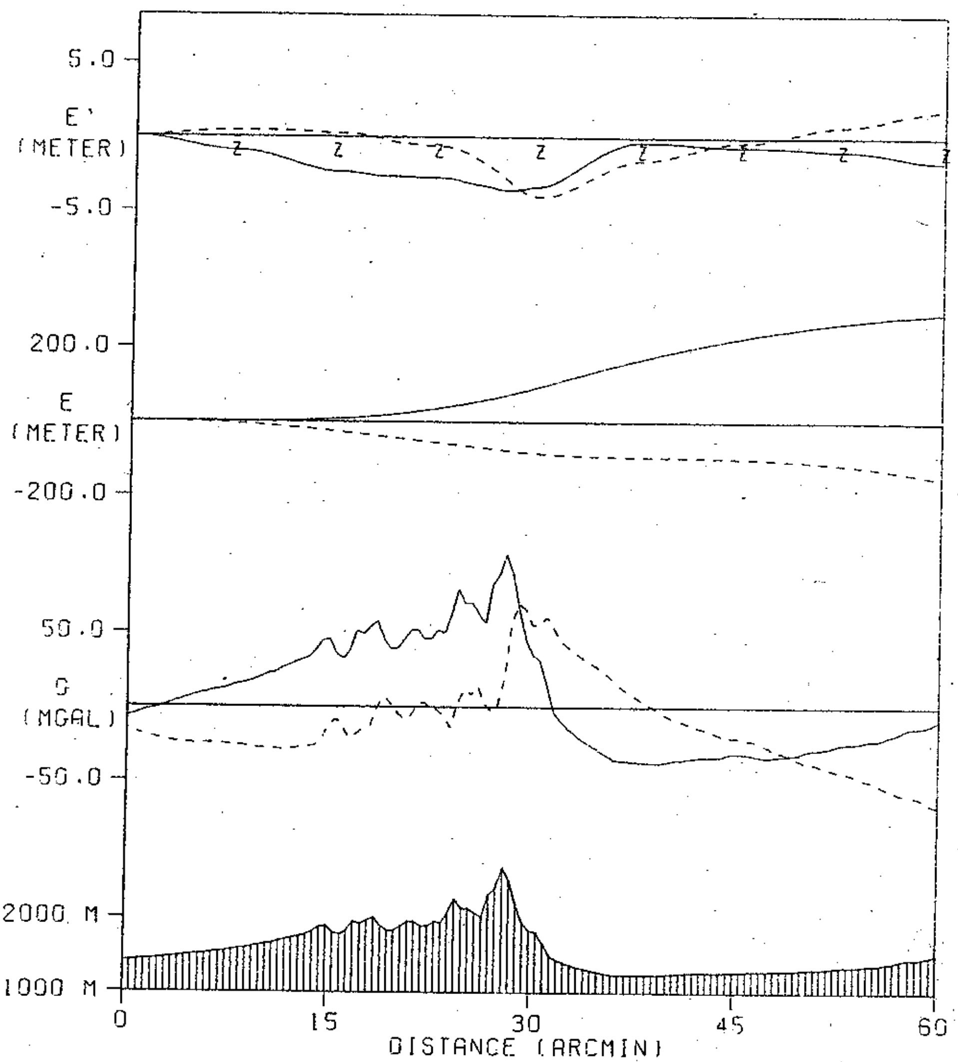

Figure 3 Inertial errors due to the disturbing gravity field along the profile of Figure 2 for a perfect inertial survey system moving from west (left) to east (right) at ground level and speed 50 m/s. From bottom to top is shown: the topographic cross-section, the disturbing gravity acceleration, the unaided inertial navigation error ε and the ZUPT-error ε' for the vertical (δg) component (full curve) and the west-east (η) component (broken curve).