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The Role and Computation of Gravity for the Processing of Levelling Data

by

C.C. Tscherning
Geodætisk Institut
Gamlehave Allé 22
DK-2920 Charlottenlund

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ABSTRACT

The computation of geopotential differences from levelling data requires the knowledge of gravity values with a spacing Δs between the points of observation depending on the gravity prediction root mean square error μ_g and the height variations $\Delta z_{\Delta s_1}$ along the levelling line. It is observed that μ_g is approximately $0.022\ C_o^{\frac{1}{2}}\ \Delta s_{km}$, where C_o is the mean square variation of the (contingently reduced) gravity anomalies. Following Ramsayer, the height variations can be expressed as $\Delta z_1\ \Delta s^v$, where Δz_1 is the mean height difference for $\Delta s = 1$ km. Then it is shown that the needed spacing Δs may be determined from

$$\Delta s^v \cdot \Delta z_1 \cdot 0.022 \cdot C_o^{\frac{1}{2}} \cdot \Delta s = \epsilon \cdot \sqrt{\Delta s}\ , \qquad (*)$$

where ϵ is the maximally permitted error per km caused by only knowing the gravity in points of the levelling line with distance Δs.

Various gravity prediction techniques are briefly reviewed and compared. Some of the techniques, such as least squares collocation, have the theoretical capability of furnishing error estimates, which may be very useful when checking that ϵ is not exceeded. Tests carried out in Ohio and New Mexico show, however, that the "true" errors may be up to 5 times larger than the estimated error. This should be taken into account when using equations such as (*) for the planning of gravity surveys in support of levelling operations.

1. INTRODUCTION

In the levelling process the levelling increments Δz_i are observed using an instrument, which is placed in a point P_i with its axis tangent to the plumb line. (See e.g. Heiskanen & Moritz (1967, Fig. 4-7 and 4-2)). Due to the nonparallelism of the level surfaces, the height difference between two points A, B can not be determined by adding the levelling increments.

However, if we know the gravity intensity g_i, we may compute the product $g_i \ \Delta z_i$, which is equal to the potential difference ΔW_i, where W is the gravitational potential of the Earth. (The differences are between points P_i' and P_i'' at the surface of the Earth on which the levelling rods have been placed).

Our new observations

$$\Delta W_i = g_i \ \Delta z_i \ , \tag{1}$$

can then be used for the numerical evaluation of the curve integral

$$W_B - W_A = - \int_A^B g(P) \ \bar{d}z_P \approx - \sum_{i=1}^{n} g_i \ \Delta z_i \ , \tag{2}$$

by which the <u>geopotential difference</u> between A and B is determined. (The bar occurring on dz indicates that we do not deal with a perfect differential, but with a one-form, see e.g. (Grafarend, 1975), (Grossman, 1979, section 4.11)).

The distance between the points P_i are in precise levelling less than 100 m. It is, however, unnecessary to observe g_i in all points P_i. We therefore have a number of questions to be answered:

(1) How well do we need to know g_i ?

(2) How densely do we need to space the points with known gravity values ?

(3) How do we determine g_i in points where no observations have been executed ?

(4) How can we estimate whether the required precision has been obtained ?

In section 2 we will mainly be concerned with (1) and (2). The answers to these questions will naturally depend on the method used for computing (or predicting) g. Fortunately will most reasonable methods predict g with comparable results using the same set of data. The methods do, however, have some important differences, which we will discuss in section 3. Here we will also consider question no. (4), namely the problem of obtaining reliable error estimates.

Units: Gravity will be given in mgal = 10^{-5} m/s^2, potential in geopotential units, 1 g.p.u. = 10 m^2/s^2, and gravity gradients in E.U., 1 E.U. = 10^{-9} s^{-2}.

Notation: γ - normal gravity, Δg gravity anomaly, Δg_M gravity anomaly computed from topography (contingently with isostatic compensation). $\Delta g^c = \Delta g - \Delta g_M$ the reduced gravity anomaly. $g_o = g - \Delta g_M$, the reduced gravity. Subscript i indicates that the quantity is related to a point P_i.

2. METHODS FOR THE DETERMINATION OF THE NECESSARY SPACING OF THE GRAVITY OBSERVATIONS

2.1 The error model

If we only had to deal with the levelling increments Δz_i, we should only have to be concerned with the errors δz_i originating during the observation process. The multiplication of Δz_i by g_i makes it necessary to consider the errors of the observed or predicted gravity value, δg_i. We also commit an error by approximating the integral eq.(2) by a sum. We will suppose, that this error can be disregarded.

The total error ε will then be

$$\varepsilon = \sum_{i=1}^{n} ((g_i + \delta g_i)(\Delta z_i + \delta z_i) - (g_i \Delta z_i))$$

$$= \sum_{i=1}^{n} (g_i \delta z_i + \delta g_i \Delta z_i + \delta g_i \delta z_i). \quad (3)$$

Because the purpose of the paper is to study the role of gravity in levelling, we will not be concerned with the first term $\Sigma g_i \delta z_i$. We will suppose it is of the order

$$1.0 - 0.5 \times 10^{-3} \text{ g.p.u. } \sqrt{s} ,$$

where s is the distance from A to B in km measured along the levelling line. (The quoted value corresponds to an error in metric units of $1.0 - 0.5$ mm $\sqrt{s}$).

We have in practice the ability to observe g_i everywhere, i.e. of having values of δg_i below 0.1 mgal. It has therefore been agreed internationally, that the influence of the gravity error must not exceed 10^{-4} g.p.u. $\sqrt{s_{km}}$, which is equivalent to 0.1 mm $\sqrt{s_{km}}$.

We will in the following presuppose that the maximal distance considered is 25 km, and that the maximal numerical error δg_i is below 100 mgal. Then, for the last term of eq.(3) we have

$$\left| \sum_{i=1}^{n} \delta g_i \, \delta z_i \right| < 100 \cdot 10^{-6} \sqrt{25} \;\; 10^{-3} = 5 \cdot 10^{-7} \text{ g.p.u.}$$

Hence, we can with good conscience disregard this term.

We are then left with the terms

$$b_i = \delta g_i \;\Delta z_i \qquad (4)$$

the total influence of which we must keep below 10^{-4} g.p.u. $\sqrt{s_{km}}$.

2.2 Review of earlier investigations

The use of potential differences in levelling was adopted in the fifties, see e.g. (IAG, 1959), (Baeschlin, 1960), (Kneissl, 1959). This initiated several theoretical and empirical investigations of which the most important have been executed by prof. K. Ramsayer, see the literature list.

The error model used by Ramsayer had as its basis a linear model for g as a function of height, z, and the use of linear interpolation in between the corresponding reduced gravity values

$$g_o = g + \eta \cdot z \,, \qquad (5)$$

see e.g. (Ramsayer, 1960, eq.(2)).

Ramsayer therefore considered separately errors due to the use of eq.(5). A result of his investigations was, however, that the error term of most importance is the one given by eq.(4), for which he introduced the name b_i.

In his investigations Ramsayer used very long levelling lines in Southern Germany and Switzerland along which gravity values had been observed with a density of about 0.5 km. He then computed geopotential height differences using different spacings of the gravity values. By shifting the starting point of the first piece of the levelling line without observed gravity values, he obtained a large sample of b_i values. The varying magnitude of the error b_i was expressed by computing

$$m_b = (\Sigma_{i=1}^{n} b_i^{\,2}/n)^{\frac{1}{2}} \qquad (6)$$

for varying distances Δs and for varying kinds of topography, see Figure 1, which has been taken from (Ramsayer, 1963). He observed, as expected, that the value of m_b did depend strongly on the variation of Δz, and introduced a classification of the topography depending on the value of

$$\Delta z_{\Delta s} = (\Sigma_{i=1}^{n} \Delta z_i^{\,2}/n)^{\frac{1}{2}} \,, \qquad (7)$$

the mean height variation.

A distinction was also made between high mountains and medium high mountains because the angle of climb of the highways in high mountains does not so often change as in medium high mountains.

In the following we will use the formula $\Delta z_{\Delta s} = \Delta z_1 \Delta s^{v}$, with the constants given in Table 1.

In a similar way the mean gravity prediction error, μ_g, was computed for varying spacings Δs,

$$\mu_g = (\Sigma_{i}^{n} \; \delta g_i^{\,2}/n)^{\frac{1}{2}} \,. \qquad (8)$$

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This error was observed to be different in the Alps and in the Black Forest, but in each area, it was linear. This is not surprising, since the main influence of the topography was taken into account by eq.(5). Expressed as a function of Δs it was found (Ramsayer, 1964, page 23):

$$\mu_g = \alpha_g \, \Delta s_{km} \qquad (\mu_g \text{ and } \alpha_g \text{ in mgal}) \tag{9}$$

with $\alpha_g = 0.36$ in the Alps and $\alpha_g = 0.16$ in the Black Forest.

It was observed, that the coefficients were approximately proportional to the mean variation of the horizontal (reduced) gravity gradients, 17.5 E.U. for the Black Forest and 30 E.U. for the Alps.

Having determined μ_g and $\Delta z_{\Delta s}$ as functions of Δs, it is possible to express m_b as a function of Δs as well:

$$\begin{aligned} m_b &\approx \Delta z_{\Delta s} \, \mu_g = \Delta z_1 \, \Delta s^v \, \alpha_g \, \Delta s \\ &= \Delta z_1 \, \alpha_g \, \Delta s^{(v+1)} \, . \end{aligned} \tag{10}$$

When m_b is expressed in this way as a function of Δs we can easily compute the upper limit Δs_{max} (km) from

$$\begin{aligned} &\Delta s_{max}^{(v+1)} \, \alpha_g \, \Delta z_1 = 10^{-4} \text{ g.p.u.} \sqrt{\Delta s_{max}} \quad \text{or} \\ &\Delta s_{max}^{(v+0.5)} \, \alpha_g \, \Delta z_1 = 10^{-4} \text{ g.p.u.} \end{aligned} \tag{11}$$

The value of Δs_{max} for the different types of topography defined in Table 1 are given in Table 2.

This method described by Ramsayer is quite general, if we know $\Delta z_{\Delta s}$ and μ_g. It will generally not be difficult to determine $\Delta z_{\Delta s}$, but μ_g will depend on the local variation of the gravity field. This seems to have inspired the investigations of (Levallois, 1964), who gave a general formula for the error expressed in terms of the variation of the horizontal and vertical gravity gradient. His results do, however, seem to be in good agreement with eq.(11).

2.3 An alternative procedure for the determination of the mean error of gravity prediction

The expressions eq.(9) for μ_g were obtained based on the use of linear interpolation for the computation of g_o. Theoretical expressions may readily be obtained for μ_g using prediction techniques like least squares interpolation, cf. (Heiskanen and Moritz, 1967, Chapter 7).

Here we must consider (contingently reduced) gravity anomalies Δg, from which g can be computed by adding the normal gravity, γ. Having determined the empirical covariance function C(s) for Δg the root mean square prediction error is

$$\epsilon_{\Delta g} = (C(0) - \{C_{Pi}\}^T \{C_{ij}\}^{-1} \{C_{Pj}\})^{\frac{1}{2}} , \qquad (12)$$

where C_{ij} is the covariance between the i'th and j'th gravity observation, C_{Pi} the covariance between the gravity anomaly in P and the i'th observation and $C_o = C(0)$ is the mean square variation of the gravity anomalies.

This formula was used in (Tscherning, 1975) in order to determine the value of $\epsilon_{\Delta g}$ for varying spacings Δs, of gravity anomalies, and with P in the middle of a square with side length Δs. Using (Tscherning, 1975 , Fig. 5a), we get

$$\epsilon_{\Delta g} = C_o^{\frac{1}{2}} \, 0.044 \, \Delta s \, . \qquad (13)$$

The method of least squares collocation has zero prediction error in the points of observation. We will hence suppose that $\mu_g = \frac{1}{2} \epsilon_{\Delta g}$, or

$$\mu_g = C_o^{\frac{1}{2}} \, 0.022 \, \Delta s_{km} \, . \qquad (14)$$

From (Tscherning, 1979) we have $C_o^{\frac{1}{2}} = 26$ mgal for a $2^o \times 3^o$ area in the Alps and from (Strange, 1979) we have $C_o^{\frac{1}{2}} = 9$ mgal in the Black Forest, if we suppose that the root mean square variation of Δg^c is equal to about half the variation of the free air anomalies. The result is then

μ_g (Alps) $= 0.57 \, \Delta s$

μ_g (Black Forest) $= 0.20 \, \Delta s$.

The value for the Alps is much larger than the value obtained by (Ramsayer, 1964), but this is most likely due to the relatively smooth gravity variation along the levelling line. If we use the formulae for the New Mexico Area, cf. (Tscherning, 1979), where a data spacing of 12 km was used and $C_o^{\frac{1}{2}}$ = 10 mgal we get

$$\mu_g = \pm 2.6 \text{ mgal},$$

which agreed with the result quoted in (Ibid, Table 2).

Therefore I dare to recommend the use of eq.(14), if only the value of C_o is known, but eq.(12) may also be used, if a reasonable estimate of the covariance function C(s) is available. We will, however, return to the use of eq.(12) in the following section.

(P.S.: Values of C_o for different areas can be found in (Dimetrijevich, 1978), (Strange, 1979), (Schwarz and Lachapelle, 1980), (Tscherning, 1979)).

3. COMPUTATION OF GRAVITY VALUES AND ERROR ESTIMATION

3.1 Prediction methods

In section 2.3 we noted, that the computation of gravity values was equivalent to the computation of estimates (predictions) of gravity anomalies, Δg. They would generally be reduced topographic-isostatically, $\Delta g^c = \Delta g - \Delta g_M$.

The prediction technique used e.g. by Ramsayer, linear interpolation, is mathematically very simple and requires very little information besides the gravity values - or anomalies - in the end points of the levelling line interval containing the point. However, the distances to the end points must be known, and the line must be approximately straight. The interpolation errors may be estimated if the variation of the horizontal and vertical gravity gradients are known, see (Levallois, 1964).

Modern gravity prediction techniques regards g, Δg and Δg^c as functions in 2 or 3 variables. The most rigorous techniques use the fact, that the gravity anomaly is the value of a linear(ized) functional applied on the anomalous potential, T = W-U or $T^c = T - T_M$, where T_M is the potential of the topography (contingently compensated isostatically). T or T^c will then be harmonic functions in space, a fact we can take advantage of in the prediction process.

If an approximation to T or T^c ($\tilde{T}$) has been constructed the gravity anomaly can be computed in an arbitrary point using the functional relationship between Δg and T:

$$\tilde{\Delta g}_P = - \frac{\partial \tilde{T}}{\partial r}\Big|_P - \frac{2}{r}\tilde{T}(P) , \tag{15}$$

where r is the distance of P from the origin.

A whole range of methods are available for the computation of approximations $\tilde{T}$ or $\tilde{\Delta g}$, regarded as functions in R^3 or in R^2. Suitable methods are discussed in (Tscherning, 1979a), (Merry, 1979), (Hein and Lenze, 1979), Nash and Jordan, 1978), (Vaniček, Boal and Porter, 1972), (Hardy, 1979), (Moritz, 1980). The methods generally aim at expressing $\tilde{T}$ and $\widetilde{\Delta g}$ as a linear combination of some specific base functions. The base functions are either selected explicitly or implicitly through the adoption of a minimum principle, which $\tilde{T}$ or $\widetilde{\Delta g}$ is required to fulfil.

Methods which have the property that the (errorless) observations agree with the predicted values are denoted collocation techniques. By selecting a suitable minimum principle a set of base functions, φ_k, will be determined and we have e.g.

$$\tilde{T}(P) = \sum_{i=1}^{n} a_i \; \varphi_i(P) , \tag{16}$$

where the coefficients are determined from observations Δg_i by the collocation property

$$\Delta g_i = - \frac{\partial \tilde{T}}{\partial r}\Big|_{P_i} - \frac{2}{r}\tilde{T}(P_i), \qquad i=1, \ldots, n \tag{17}$$

where P_i are the points of observation. The base functions will be the covariance functions mentioned in section 2.3, if we require the minimal mean square prediction error.

If we require the minimalization of the norm of $\tilde{\Delta g}$ or $\tilde{T}$ considered as an element of a reproducing kernel Hilbert space, the functions φ_k will be values of the Δg-functional applied on the reproducing kernel, see (Moritz, 1980). This techniques is denoted minimum norm collocation.

Predefined set of base functions may consist of potentials of point masses, 2-dimensional spline functions or polynomials. The coefficients are generally determined by selecting fewer base functions than observations and then solving the overdetermined system of equations (17) using a least squares principle, e.g.

$$\{\Delta g_i - \sum_{k=1}^{K} a_k(-\frac{\partial \varphi_k}{\partial r} - \frac{2}{r}\varphi_k)_i\}^T \{P_{ij}\}^{-1} \{\Delta g_j - \sum_{k=1}^{K} a_k(-\frac{\partial \varphi_k}{\partial r} - \frac{2}{r}\varphi_k)_j\} = \min \qquad (18)$$

where $\{P_{ij}\}$ is a weight matrix.

We will in the following name these techniques according to the type of base functions used, polynomial approximation, spline function approximation etc.

Note, that all the methods require the solution of sets of linear equations, which however in many cases will have only 1, 2 or 3 unknowns. Also when using collocation methods the number of equations may be kept quite small, cf. (Rapp, 1964), because these techniques are used here mainly as interpolation techniques and not as techniques for the approximation of T.

A number of different techniques, which have been used for gravity computation are compared in Table 3. To each method we have put a number of questions. The table should leave no doubt about the techniques which are preferred by the author. A detailed justification of the different questions can be found in (Tscherning, 1979a).

Let us mention, however, that two newly established IAG Special Study Groups are supposed to study and evaluate different methods for gravity field and gravity prediction.

3.2 Possibilities for error estimation

In section 2 we stressed the importance of having possibilities for error estimation. These possibilities could both be used when determinating the needed distance of the gravity observations along a levelling line and for the control of the magnitude of m_b after the gravity survey and the levelling operations had been carried out.

The error estimates related to the use of linear interpolation requires as mentioned earlier the knowledge of the variation of the gravity gradients. This information is generally not available, so we will not consider this method.

The collocation methods give formal error estimates. Using least squares collocation, eq.(12) is valid. We are however faced with the problem of interpreting this error estimate. This is again related to the problem of estimating the covariance function, see e.g. (Schwarz and Lachapelle, 1980).

The question is, how well the covariance function will represent the magnitude of the local gravity field variations. And this again depends on the varying geological conditions within a certain region. (We expect that all variations correlated with the topography have been removed).

Test computations carried out in Ohio (described in (Tscherning, 1975, section 3)) show that between 77 and 83 % of the error estimates were larger than the true error. This is quite satisfactory, but the histogram (Ibid, Fig. 3a) shows some large outliers.

Similar investigations have been carried out in a $1^\circ \times 1^\circ$ area in New Mexico with a strongly varying topography, see (Tscherning and Forsberg, 1978, Fig. 1). A data set of 90 free-air gravity anomalies spaced approximately 6' apart were used to predict gravity anomalies in 100 points with known anomalies. These "test" anomalies were all situated in a 15' x 15' area in the middle of the $1^\circ \times 1^\circ$ block. (The anomalies are different from these used in (Tscherning, 1979)).

For the prediction (and error estimation) a number of different covariance functions were used. They were all on the form

$$\mathrm{cov}(\Delta g_P, \Delta g_Q) = \sum_{i=I+1}^{\infty} \frac{A(i-1)}{(i-2)(i+24)} \left(\frac{R^2}{r_P r_Q}\right)^{i+2} P_i(\cos\psi_{PQ}), \qquad (19)$$

where ψ_{PQ} is the spherical distance between P and Q, r_P, r_Q are the radial distances of P,Q from the origin. The other parameters are given in Table 4. The table show the percentage of the true errors which are between 0-1, 1-2, 2-3 and larger than 3 times the estimated error. The results are similar to these obtained in Ohio considering that more different covariance functions have been used.

The prediction results are of nearly the same quality, with a slightly better result for the covariance function which has correlation length $\psi_{\frac{1}{2}}$ close to the empirical value. But the quality of the error estimates vary very much, with error estimates being up to 5 times smaller than the true error. On the other hand, 80 % of the true errors are smaller than two times the estimated error. This shows, that the error estimates are useful, but must be used with due caution. (See also (Rapp, 1964, Chapter IV)).

An analysis of the points, for which the true errors were larger than 3 times the estimated errors showed that this was not because these anomalies were numerically larger than the other anomalies. On the other hand, it is obvious, that if the variation of the anomalies in an area generally are larger than expected, then this should be taken into account.

Of logistic reasons, it might be worthwhile to densify a gravity network "on the spot". This requires that the gravity observation team is prepared to compute gravity anomalies in the field and the heights of the observation stations must be known approximately.

4. CONCLUSION

In this paper we have reviewed earlier, primarily empirical, investigations of the needed spacing of gravity observations in a levelling network, and we have described a number of different gravity prediction techniques.

We have confirmed the results of (Ramsayer, 1964) and proposed a formula for the estimation of the mean gravity interpolation error, which only requires the knowledge of the local mean square gravity variation.

We recommend the use of least squares collocation for the processing of gravity data in levelling. This has in practice proven feasible, see e.g. (Franke, 1979). We warn the users, that error estimates can be very unreliable and that local gravity variations must be taken into account - contingently it may be necessary to collect more gravity observations.

We furthermore point out, that the use of modern prediction techniques require, that the horizontal coordinates of all main point of the levelling line must be known. This should be taken into account when planning future precise levelling operations.

Table 1. $\Delta z_{\Delta s}$ for different types of topography

Topography class	High mountains (The Alps)	Low mountains (The Black Forest)
I	$10.5\ m\ \Delta s^{0.85}$	$5.4\ m\ \Delta s^{0.43}$
II	$28.8\ m\ \Delta s^{0.91}$	$16.5\ m\ \Delta s^{0.53}$
III	$64.0\ m\ \Delta s^{0.96}$	$37.0\ m\ \Delta s^{0.63}$

(From Ramsayer, 1963, page 19).

Table 2. Δs_{max} (km) for different types of topography

Topography class	High mountains	Low mountains
I	11	165
II	5	34
III	2.7	12

Table 3. Comparison of some methods suitable for gravity anomaly prediction.

Mean square variation of horizontal gravity gradients: G_o Number of observations : n Number of base functions : k	Method						
	Linear interpolation	Graphic (2D) interpolation	Polynomial approximation (2+3D)	Spline function interp. (2D)	Multiquadric interpolation (2D)	Point mass approximation	Least squares or minimum norm collocation
Agrees exactly with error-free observed values	yes	yes	yes, if k=n	yes	yes, if k=n	yes, if k=n	yes
Observation errors can be taken into account	no	no	yes, if k<n	no	yes, if k<n	yes, if k<n	yes
Enables the computation of error estimates	yes, if G_o known	yes, if G_o known	no	no	no	no	yes
Behaves well outside observation area	-	-	no	-	?	(yes)	yes
Stable in clusters of observ.	yes	yes	no	yes	?	(yes)	(yes)
Permits the use of data other than gravity data	no	no	no	no	no	yes	yes
Permits the use of data outside the levelling line	no	yes	yes	yes	yes	yes	yes
Not only gravity values can be predicted	no	no	no	no	no	yes	yes
Permits the simultaneous estimation of parameters	no	no	(yes)	(yes)	(yes)	(yes)	yes
Requires no determination of empirical covariance function or intelligent choice of base functions	yes	yes	no	no	no	no	no
References	Levallois 64	Merry 79	Merry 79	Hein 79	Hein 79	Hardy 79	Moritz 80 Merry 79

(yes) means that the method will have to be modified considerably or that base functions must be selected very carefully.

Table 4. Parameters characterizing 7 different covariance functions used for prediction and error estimation together with the distribution of the true errors, $|\Delta g-\tilde{\Delta g}|$, divided by the estimated errors, $\varepsilon_{\Delta g}$. All covariance functions have $C_o = 852$ mgal^2 equal to the empirically determined value. ψ_o is the spherical distance at which the first zero point of $cov(\Delta g_P, \Delta g_Q) = C(\psi_{PQ})$ occurs and $\psi_{\frac{1}{2}}$ is the spherical distance for which $C(\psi_{\frac{1}{2}}) = \frac{1}{2}\cdot C_o$. The computed values are for points P,Q in the height 1200 m, which is the mean height of the area.

Covariance function parameters (cf.eq.(19))					Prediction result r.m.s.v.	Distribution of $\rho = \|\Delta g_i-\tilde{\Delta g}_i\|/\varepsilon_{\Delta g_i}$ %				
I	A mgal^2	R-6371 km	ψ_o	$\psi_{\frac{1}{2}}$	$\|\Delta g_i-\tilde{\Delta g}_i\|$ mgal	$0\le\rho<1$	$1\le\rho<2$	$2\le\rho<3$	$3\le\rho$	Max.value
200	146.2	-1.1	24'	1!9	± 4.3	97	2	1	0	1.2
200	435.7	0.005	24!5	6!0	4.3	86	11	1	2	2.4
200	622.0	1.25	25'	7!9	3.9	78	11	7	4	3.8
200	819.0	2.5	25'5	9!5	3.8	68	15	6	11	5.1
220	454.1	0.005	22!5	5!7	4.3	77	10	1	2	2.4
220	657.2	1.25	23!0	7!5	3.9	79	10	7	4	3.8
220	875.8	2.5	23!5	8!8	3.8	68	16	5	11	5.0
Empirical values:			21'	7!6						
Distribution of $\|$observed $\Delta g\|/C_o$: %						78	19	3	0	2.3
Distribution of $\|$test $\Delta g_i\|/C_o$: %						70	30	0	0	1.9

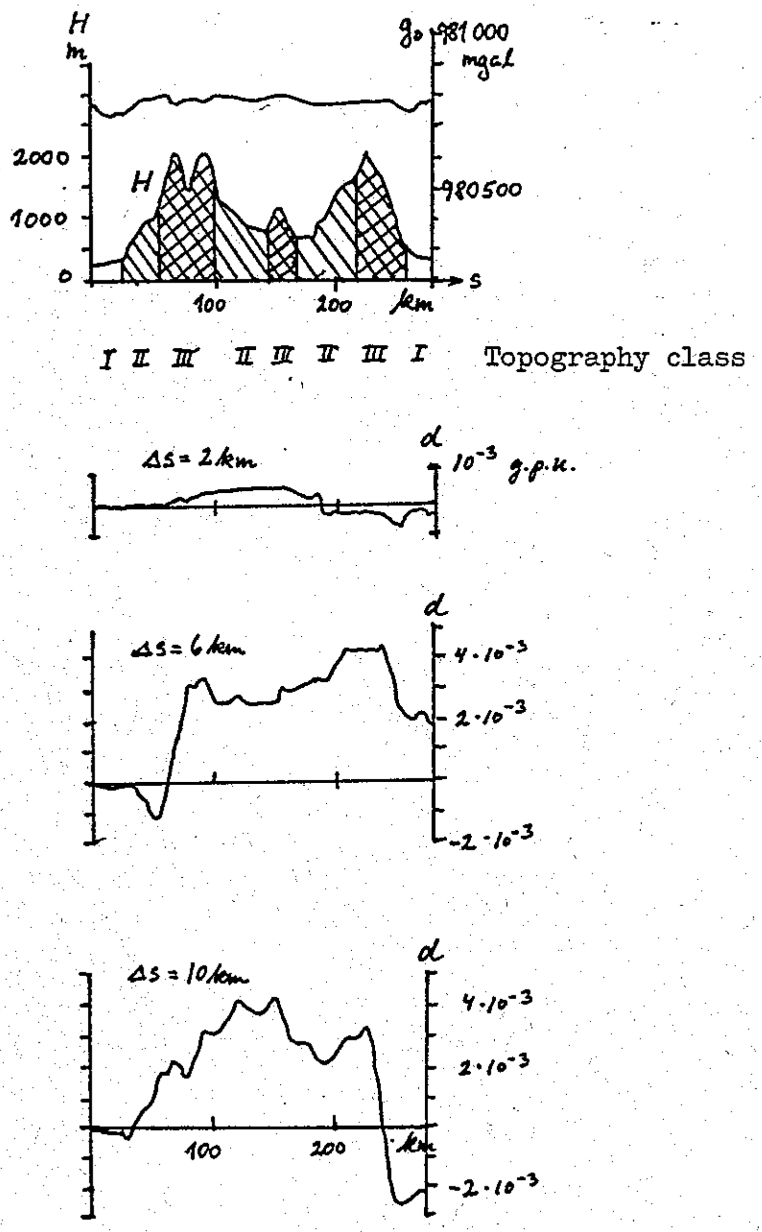

Figure 1. The mean sea level height, H, the reduced gravity g_0 and the total error d for a levelling line in the Alps. The error is not only caused by errors in the gravity interpolation process, but also by errors committed by not observing the height of the gravity station. The value of d is shown for spacings Δs = 2, 6 and 10 km between stations with observed gravity and height. (The figure is based on (Ramsayer, 1963, Fig. 3)).

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PROCEEDINGS

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