Merging Regional Geoids Preliminary Considerations and Experiences (*)

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Summary. – Geoids of continental, or regional extent have been computed by different groups or individuals. It is desirable to be able to merge these geoids together into one common geoid, taking advantage of for example local information of high quality and of satellite altimetry at sea.

However, the geoid-pieces may be given in different datums or refers to different ellipsoids. One possibility, for determining both the necessary datum shifts and the common geoid is least squares collocation. The use of this method requires a system of equations to be solved with as many unknowns as number of data points. However, the computational burden may be reduced by using covariance functions set to zero outside a given distance, and by taking into account the repetitive pattern of the gridded geoid values. It is also shown that a continuos form of collocation may lead to a practical solution in case neighbouring geoid-parts have similar error characteristics.

In order to test the proposed procedure, we have tried to merge the geoids for Switzerland, Italy and Greece. While a reasonable agreement may be obtained between Italy and Greece, it does not seem possible to reach an agreement between Italy and Switzerland considering the error estimates associated with the computed geoid heights.

SULL' UNIFICAZIONE DI GEOIDI REGIONALI: CONSIDERAZIONI ED ESPERIENZE PRELIMINARI.

Sommario. – Si pone il problema di unificare geoidi regionali o continentali, calcolati da diversi gruppi, in un unico geoide. Si mostra come si possa utilizzare la collocazione a tale scopo includendo anche una parametrizzazione dei possibili cambiamenti del sistema di riferimento o di altri bias.

Il problema è analizzato anche a livello teorico per mezzo di un modello continuo. Si presentano alcune prove numeriche usando i tre geoidi di Svizzera, Italia e Grecia.

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1. – INTRODUCTION

Geoids for Europe now exist computed based on deflections of the vertical a mean gravity anomalies (Brenneke et., 1983, Torge et al., 1983, 1983a). These geoids probably better than 0.5 m in areas of moderate topography and better than 1.(mountainous areas.

The conversion of GPS ellipsoidal height differences to orthometric (or norn height differences, plus the study of oceanographic phenomena requires improvement of these geoids. The goal should be set for 0.1 m in coastal regions anc areas of moderate topography and 0.3 m in mountains, for geoid height differenc

This goal may probably still be reached by having one or more central comput centres constructing geoids for all of Europe. However, it is difficult to see how s geoids may be improved (updated) locally if an error is found or new data are obtair

This would be possible, if geoids are computed and updated locally (for e country or group of countries for example). The difficult task would then be to comb these solutions to a uniform solution covering all of Europe. However, this ma principle be done, and was for example done by Bomford or Levallois when they fit local geoids togheter preparing the first European geoid maps.

In fact, IAG SSG 3.88 has at a meeting in Paris, 1985, decided to investigate possibility, and this paper is a first contribution to this investigation.

Up to now, geoids have been published for FRG (Lelgeman et al., 1981), I (Benciolini et al., 1984), Greece (Arabelos, 1980, Arabelos and Tziavos, 1985), Scandinavian Countries (Tscherning, 1985), Finland (Vermeer, 1984), Great Brit and Ireland (Olliver, 1980, 1981), Austria (Sünkel, 1983), Netherlands (Willigen, 19 Switzerland (Gürtner, 1978).

However, none of these seem to be as good as we would like. This is due to example:

- lack of data
- use of ED 1950 deflections instead of ED 1979 deflections
- systematic errors in the longitude components of the deflections (Tscherni 1985, 1986)
- systematic errors in the doppler-derived geoid undulations (Tscherni Goad, 1985).

However, looked upon as «biased» geoids they are probably all very gc Therefore it may worthwhile to try to merge the existing local solutions together. T would require, that a number of countries (regions) are willing to participate, and a questionary concerning this was send out in December, 1985.

Positive answers were received from Switzerland, Italy, Greece, the No Countries, Netherlands, Austria, Israel, Turkey, U.K., and Syria. For the firs mentioned areas, digital data were made available. Furthermore satellite radar altimeter data were made available by R.H. Rapp, Ohio State University.

The task of merging local geoids poses several theoretical problems. In section 2 we have given a brute-force solution method, and indicated some other methods, which however have not yet been fully worked out.

The brute-force method east–squares collocation has been used with the data received from Greece, Italy and Switzerland. The result of this both encouraging and discouraging experiment is described in section 3.

Finally in section 4 we draw some conclusions concerning necessary further investigations and data collection.

2. – SOME MERGING METHODS

2.1.- DISCRETE METHODS

Let us suppose the data consist of several sets of height anomalies (signals, s_i), each related to a specific geodetic datum through one or more elements of a parameter vector $\{X_j\}$ of dimension m. Then the observation y_i can be expressed as

$$y_i = s_i + v_i + \{A_{ij}\}\ \{X_j\},\ i = 1,..., n. \tag{2.1}$$

v_i is the error and A_{ij} is a m-vector of constants depending on the coordinates of the i'th data point and on the datum-shift parameters. The observations could be either height-anomalies or geoidal heights. We will for the sake of simplicity suppose that we only deal with geoidal heights. (Otherwise our basic signal should have been the anomalous potential T and the observations would have been related to T trough linear functionals).

There are then (at least) two methods available for the construction of a merged set of geoidal heights:

(1) least squares estimation with suitable base functions,

(2) minimum norm collocation.

In the first method the geoid is represented as a linear combination of given base functions, f_i,

$$\hat{s} = \sum_{i=1}^{k} b_i f_i \tag{2.2}$$

The constants b_i and the vector X is determined by least-squares ($k + m < n$),

$$\{\hat{s}_i - y_i\}^T \{W_{ij}\}\ \{\hat{s}_j - y_j\} = \min \tag{2.3}$$

where $\{W_{ij}\}$ is the inverse variance-covariance matrix of the noise vector v_i.

In the second method the basic idea is that all noise-free observations shoul reproduced exactly, and the solution should fulfil a minimum norm condition. T leads to an estimate $\hat{s}$ like in eq. (2.2) with $k = n$ and base-functions K_i being reproduc kernels (or covariance functions) with one point of evaluation equal to the observation point, P_i.

With $A = \{A_{ij}\}$, $\bar{C} = \{K_i\ (P_j) + W_{ij}\}^{-1}$ and P a contingent a priori weight for parameters we have cf. Moritz (1980),

$$X = (A^T \bar{C}^{-1} A + P)^{-1} A^T \bar{C}^{-1} y$$

$$\{b_i\} = \bar{C}^{-1}\ (y - AX).$$

If we want to preserve the resolution of the original data, then method (1) requ as many base functions as method (2) and it loses its contingent advantages. We therefore only regard methods of type (2). Here the problem is the large numbe equations, which have to be solved, a problem we will discuss further in section 2.3. first we will discuss a continuous equivalent of method (2).

The reason for investigating a continuous approach is the well-known connect between integral methods and collocation, where the integral methods have advantage that a kind of analytic inverse can be used. However, this requires that parameters have been determined first. This on the other hand is not a bad idea, si the quality of their estimation depends more on the areal distribution of the data poi than on the density of the points.

2.2. - A CONTINUOUS COLLOCATION METHOD

In this section we will present some ideas, which have not been worked out in detail, but which on the other hand maybe are not far from being realized in pract considering the success of Fourier transform methods in physical geodesy. We illustrate the basic idea in 1-dimension, see fig. 1.

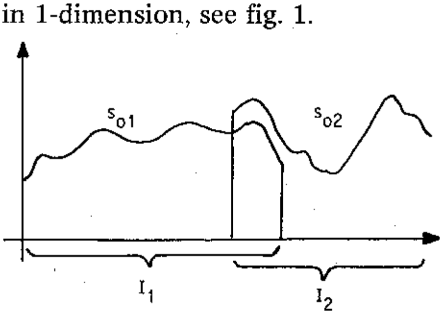

Fig. 1 – Two curves representing the same signal, given on two intervals I_1 and I_2.

Let s_{01}, s_{02} be the «same realization» $s(x)$ of a stochastic process S, with mean value zero and covariance function only dependent on the distance between the points,

$$E(s(x)) = 0, \qquad E(s(x) \cdot s(y)) = C(c - y),$$

plus inconsistencies, which we model as two independent noises

$$\left.\begin{aligned} s_{01} &= s + v_1 \\ s_{02} &= s + v_2 \end{aligned}\right\} \tag{2.6}$$

This implies that s_{01} and s_{02} refers to the same spherical harmonic reference potential and the same datum.

We also compute a unique covariance function $C(x)$ from all the data over the sampling areas I_1 and I_2, see fig. 1. This requires that the two processes are reduced to the same average, as obtained by subtracting first a spherical harmonic reference function containing wavelengths smaller than or equal to the size of the smallest area.

Instead of using a sum as in eq. (2.1) we now want to represent the estimate $\hat{s}$ as an integral.

$$\hat{s}(x) = \int_{I_1} L_1(x, y)\, s_{01}(y)\, dy + \int_{I_2} L_2(x, y)\, s_{02}(y)\, dy, \tag{2.7}$$

where L_1 and L_2 are function to be determined, requiring the error $e = s(x) - \hat{s}(x)$ to have minimum variance. We will need the covariances

$$E(s_{01}(y)\, s_{02}(y')) = C(y - y')$$

$$E(s_{01}(y)\, s_{01}(y')) = C(y - y') + \sigma_1^2\, \delta(y - y')$$

$$E(s_{02}(y)\, s_{02}(y')) = C(y - y') + \sigma_2^2\, \delta(y - y')\,.$$

Then

$$\begin{aligned} E^2(e) = C(0) &- 2\int_{I_1} L_1(x, y)\, C(y - x)\, dy - 2\int_{I_2} L_2(x, y)\, C(y - x)\, dy \\ &+ \int_{I_1 \times I_2} L_1(x, y)\, C(y - y')\, L_1(x, y')\, dy\, dy' \\ &+ \int_{I_2 \times I_2} L_2(x, y)\, C(y - y')\, L_2(x, y')\, dy\, dy' \\ &+ 2\int_{I_1 \times I_2} L_1(x, y)\, C(y - y')\, L_2(x, y')\, dy\, dy' \\ &+ \sigma_1^2 \int_{I_1} L_1(x, y)^2\, dy + \sigma_2^2 \int_{I_2} L_2(x, y)^2\, dy \end{aligned} \tag{2.8}$$

The minimum is obtained by varying each of the two functions

for $y \varepsilon I_1$

$$2 \int_{I_1} C(y-y')\, L_1(x, y')\, dy' + 2\, \sigma_1^2\, L_1(x, y) + 2 \int_{I_2} C(y-y')\, L_2(x, y')\, dy' = 2\, C(y-x) \quad ($$

for $y \varepsilon I_2$

$$2 \int_{I_2} C(y-y')\, L_2(x, y')\, dy' + 2\, \sigma_2^2\, L_2(x, y) + 2 \int_{I_1} C(y-y')\, L_2(x, y')\, dy' = 2\, C(y-x) \quad (2$$

Let as further simplify the problem by supposing that $I_1 =]-\infty, 0]$, $I_2 = [0$ and $\sigma_1^2 = \sigma_2^2 = \sigma_0^2$. Then

$$L_1(x, -y) = L_2(x, y) \qquad y \leq 0$$

for symmetry reasons. Hence we may define a unique function

$$L(x, y) = \begin{cases} L_1(x, y) & y < 0 \\ L_2(x, y) & y > 0 \end{cases} \quad (2$$

satisfying the unique condition

$$\int_{-\infty}^{\infty} C(y-y')\, L(x, y')\, dy' + \sigma_0^2\, L(x, y) = C(y-x). \quad (2$$

We may find a unique solution using Fourier transforms, FC for $C(x)$ and FL for I Then using the convolution theorem,

$$FC(p) \cdot FL(x, p) + \sigma_0^2\, FL(x, p) = e^{ipx}\, FC(p)$$

and then

$$L(x, y) = \frac{1}{2\pi} \int \frac{\cos(p(y-x))FC(p)}{\sigma_0^2 + FC(p)}\, dp$$

$$= \frac{1}{2\pi} \int \frac{\cos(p(y-x))FC(p)}{\sigma_0^2 + FC(p)}\, dp \quad (2$$

because of the symmetry of L around 0. Hence, in this case we have designed a very simple filter eq. (2.7) for s.

The more realistic situation $\sigma_1^2 \neq \sigma_2^2$ may be treated in a similar manner. Introducing

$$\theta_-(y) = \begin{cases} 1 & y < 0 \\ 0 & y > 0 \end{cases}$$

$$\theta_+(y) = \begin{cases} 0 & y < 0 \\ 1 & y > 0 \end{cases}$$

we can again write the two equations (2.9) and (2.10) in a unique way using also

$$L(x, y) = \begin{cases} L_1(x, y) & y < 0 \\ L_2(x, y) & y > 0 \end{cases}$$

so that

$$\int_{-\infty}^{\infty} C(y - y')\, L(x, y')\, dy' + (\sigma_1^2\, \theta_-(y) + \sigma_2^2\, \theta_+(y))\, L(x, y) = C(y - x). \tag{2.14}$$

Now we need the relations

$$\int_{-\infty}^{\infty} e^{\mathrm{i}py}\, dy = \delta_-(p) = A\delta(p) - BP(\frac{1}{p}),$$

$$\int_{0}^{\infty} e^{\mathrm{i}py}\, dy = \delta_+(p) = A\delta(p) + BB(\frac{1}{p})$$

with A and B constants and P is the principal part according to Cauchy. Whence taking the Fourier transform of (2.14) and using the convolution theorem we get

$$\begin{aligned} &(FC)\,(p)\,(FL)\,(x, y) + A(\sigma_1^2 + \sigma_2^2)\,(FL)\,(x, p) \\ &+ B(\sigma_2^2 - \sigma_1^2)\, P \int_{-\infty}^{\infty} \frac{(FL)\,(x, p)}{p - q}\, dq = FC(p)\,. \end{aligned} \tag{2.15}$$

If $(\sigma_2^2 - \sigma_1^2) << \sigma_1^2 + \sigma_2^2$ we can solve eq. (2.15) iteratively

$$(FL_0)\,(x, p) = \frac{FC(p)}{A(\sigma_1^2 + \sigma_2^2) + FC(p)} \tag{2.16}$$

$$(FL_1)\,(x, p) = -\frac{B(\sigma_2^2 - \sigma_1^2)}{A(\sigma_1^2 + \sigma_2^2) + FC(p)}\, P \int_{-\infty}^{\infty} \frac{FL_0\,(x,q)}{p - q}\, dq. \tag{2.17}$$

From $FL(x, p)$ we do not need to go back to $L(x, y)$, since we can directly obtai

$$\hat{s}(x) = \int_{-\infty}^{0} L_1(x, y)\, s_1(y)\, dy + \int_{0}^{\infty} L_2(x, y)\, s_2(y)\, dy$$

$$= \int_{-\infty}^{\infty} L(x, y)\, s(y)\, dy = \int_{-\infty}^{\infty} FL(x, p)\, (Fs)(p)\, dp \qquad (2.$$

Fs is very simply computed, especially if we have gridded data.

2.3. - Solution techniques

The restrictions we put on the data in section 2.2 gave advantages when solving equations, which we also will find in the discrete set up of section 2.1. Let us divide data points in sets P_{i1} P_{i2}. Then

$$\bar{C} = \left\{ \begin{array}{ll} \{K_{i1}(P_{j1}) + \sigma_1^2\} & \{K_{j2}(P_{i1})\} \\ \{K_{i1}(P_{j2})\} & \{K_{j2}(P_{i2}) + \sigma_2^2\} \end{array} \right. \qquad (2.$$

If data are gridded, then each of the submatrices will have a Toepliz structure of same type as used by Colombo (1981).

An even stronger tool to reduce the computational burden is the use of fin covariance functions, see fig. 2.

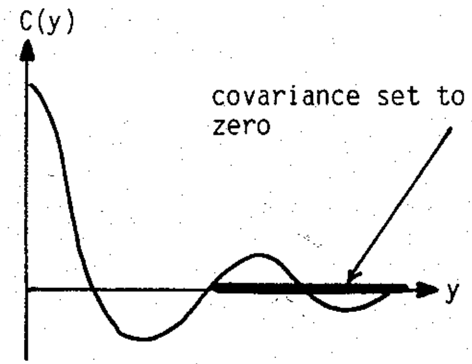

Fig. 2 — Finite covariance function.

The normal equation matrix $\bar{C}$ will have a banded structure if gridded data is us and the equations may hence be solved very easily.

In short, a finite covariance function may be derived from an ordinary non-finite by convolution. Regard two functions with non-zero Fourier transforms, C, φ,

$$(FC)\ (y) \geq 0\,, \qquad (F\varphi)\ (\lambda) \geq 0\,.$$

Then

$$\{FC(C \cdot \varphi)\}\ (\lambda) = (FC) * (F\varphi)\ (\lambda) \geq 0\,,$$

where * is the convolution operation. Then we use as φ the box-car function $R(x)$ folded 4 times with itself,

$$R(x) = \begin{cases} 1 & -x_0 < x < x_0 \\ 0 & x < -x_0,\ x_0 < x \end{cases}$$

$$S_4(x) = R * R * R * R\,.$$

Hence the covariance function multiplied by S_4 is an admissable covariance function, since it has a positive spectrum. A similar construction may probably be carried out on the sphere as well.

3. – COMPUTATIONAL EXPERIMENT FOR SWITZERLAND, ITALY AND GREECE

Despite all the unsolved theoretical problems we decided to carry out a computational experiment using data covering Switzerland (Gürtner, 1978), Italy (Benciolini) et al., 1984) and Greece (Arabelos, 1980).

The Switzerland geoid is based on astrogeodetic data, and the geoid has been transformed to ED 1950. The Greek and Italian geoids are based on gravity data, and are therefore supposed to be given in a geocentric system. However, they refer to different ellipsoids. Arabelos (1980) used the IAG 1975 ellipsoid, while Benciolini et al., (1984) used GRS 1980.

For all areas data were available on gridded form. However we only used a part of the data, namely the $1^\circ \times 1^\circ$ grid values for Italy and Greece and the $0.5^\circ \times 0.5^\circ$ grid values for Switzerland, plus additional values in the overlap regions, see table 1.

TABLE 1

GEOID DATA MINUS GPM2 CONTRIBUTION USED IN COMPUTATIONAL EXPERIMENT

Country	Number of values	Mean m	Standard Deviation m
CH	41		1.68
I	92	0.37	1.78
GR	90	0.09	1.51

As a result only 3 points coincided for the data from Switzerland and Italy and 4 Italy and Greece.

The covariance function was estimated using data from which the contribut from the GPM–sperical harmonic coefficient solution (Wenzel, 1985) had b subtracted. (Also the OSU81–solution (Rapp, 1981) was used, but it did not agree be with the data than GPM2). Unexpected long correlation distances and variances w found, indicating that only the information up to degree and order 90 could considered reliable for this region.

The equations (2.4) and (2.5) was then used with the number of parameters equa 5, namely 3 translation parameters for ED1950, and one bias parameter for Italy and for Greece.

The computations were executed using the GEOCOL program (Tscherning, 198 Besides estimating the parameters, the program was also used to calculate new ge estimates for the area. A comparison of the old and new estimates is found in tabl

TABLE 2

AGREEMENT AND DISAGREEMENT BETWEEN THE GEOIDS FOR CH, GR AND I BEFORE A AFTER THE MERGING USING COLLOCATION. EDOC-2 TRANSFORMATION AND COLLOCATI ESTIMATED DATUM-SHIFT PARAMETERS USED FOR THE ED1050 VALUE FROM CH

Area	CG-I EDOC-2	CH-I collocation	CH- collocation	GR-I	GR- collocatio
Number of points used	83		159	15	70
Mean	−1.61 m	−1.03 m	−0.29	4.17 m	−0.39 m
Standard deviation	±1.97 m	±1.82 m	±0.43	0.68 m	±0.69 m

The new values agreed well with the data from Italy and Greece in the overlap a between the two geoids. The original Greek geoid is shown in fig. 3, the new geoid in 4, and the difference between the old and new values are shown in fig. 5. It is comfort to see that the differences are small – nearly zero in Greece – , and larger at the between the Greece and Italy.

A similar comparison between Switzerland and Italy gave very discouraging resu The old Swiss and the new estimates agreed well, since the old data had been assigne small standard deviation. But the differences with Italy ranged between −4 and +3 i non-linear manner. The non-linearity with respect to latitude and longitude indica that the errors are not due to uncertainties in the datum shift. Also the differences much larger than indicated by the assigned standard deviations. They were much bel

0.5 m for Switzerland and between 0.5 and 1.5 m for Italy. The reasons for these inconsistency is not yet explained. A part of the reason could be that Switzerland uses geoid heights and Italy height anomalies.

So, the main result of the computational experiment is that a merging probably may be done in practice using least squares collocation, but it is necessary that the data are consistent, looked upon using the assigned error restimates.

4. – CONCLUSIONS AND RECOMMENDATIONS

The unification and revision of the existing geoid information is extremely important. Many large engineering projects cross national borders, (cf. the Channel Tunnel project), and a comparison along borders gives checks of national results. The oceanographers need also improved, consistent geoid results for the real-time application of satellite altimetry in the future.

The task of unifying the geoid information can only be carried out through international cooperation, in the manner pioneered by IAG SSG 3.88.

The study group, or its successor after the next IAG general assembly should continue collecting not only basic data like deflection of the vertical on digital form, but also the results should be collected and evaluated. They should preferably be available in a gridded form, with error estimates given for each individual value. Also the geoid heights or height anomalies should be given in only one reference system (datum). This would facilitate the processing of the data considerably.

There is still need for theoretical investigations, and practical implementations. Procedures should be found so that solutions may be rapidly updated, e.g. when new, adjusted satellite altimetry becomes available from the ESA satellite ERS-1 or from the Topex-mission.

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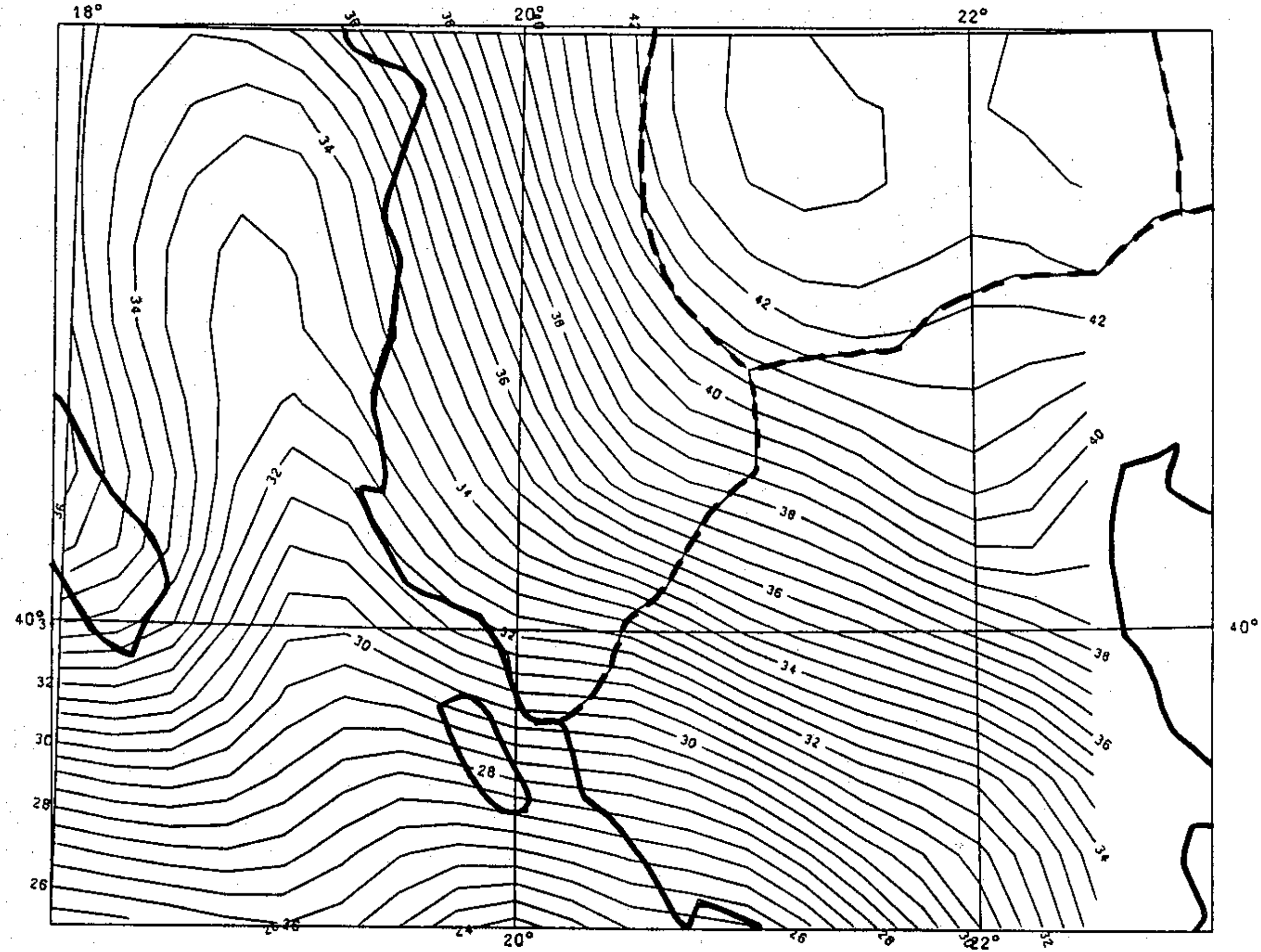
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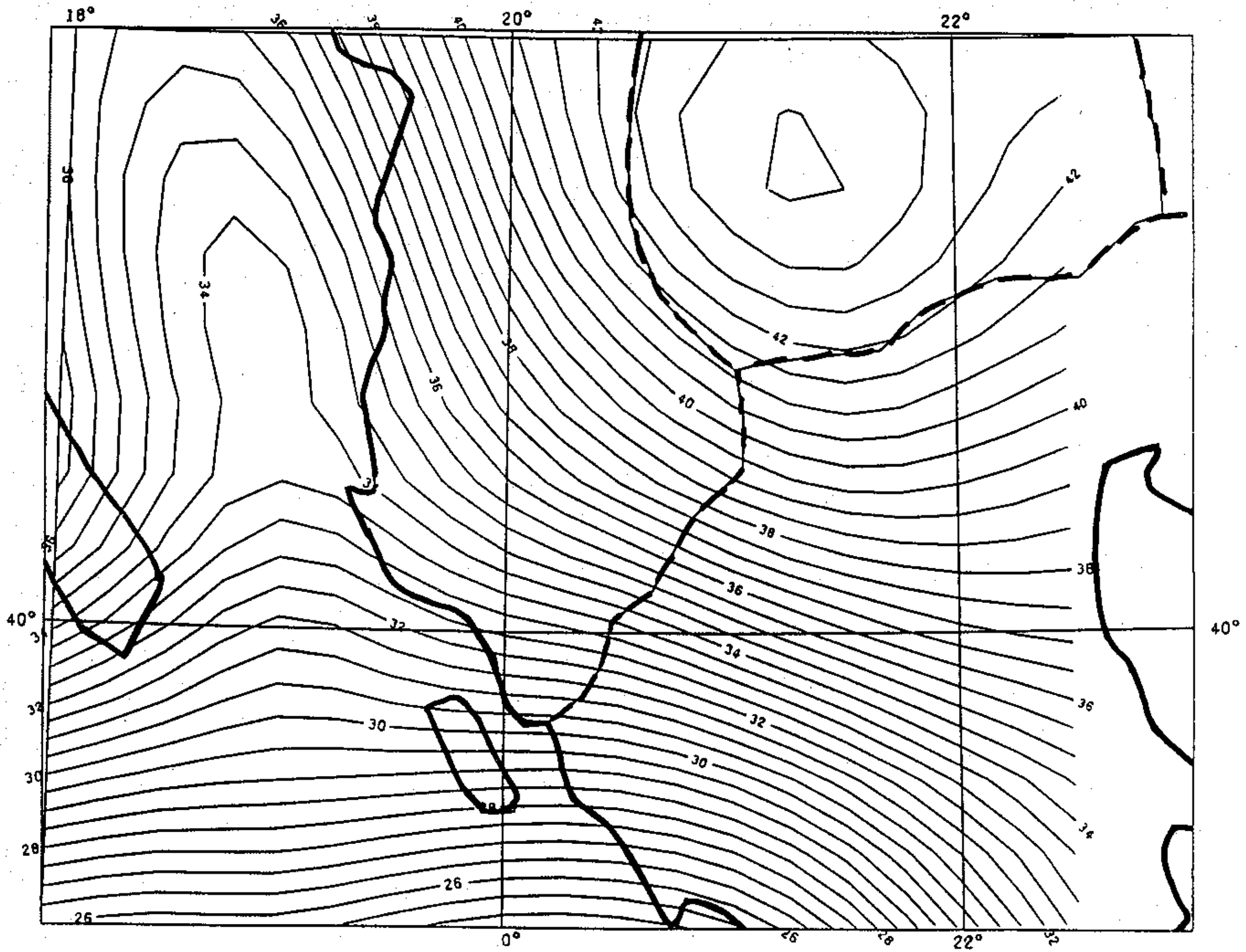
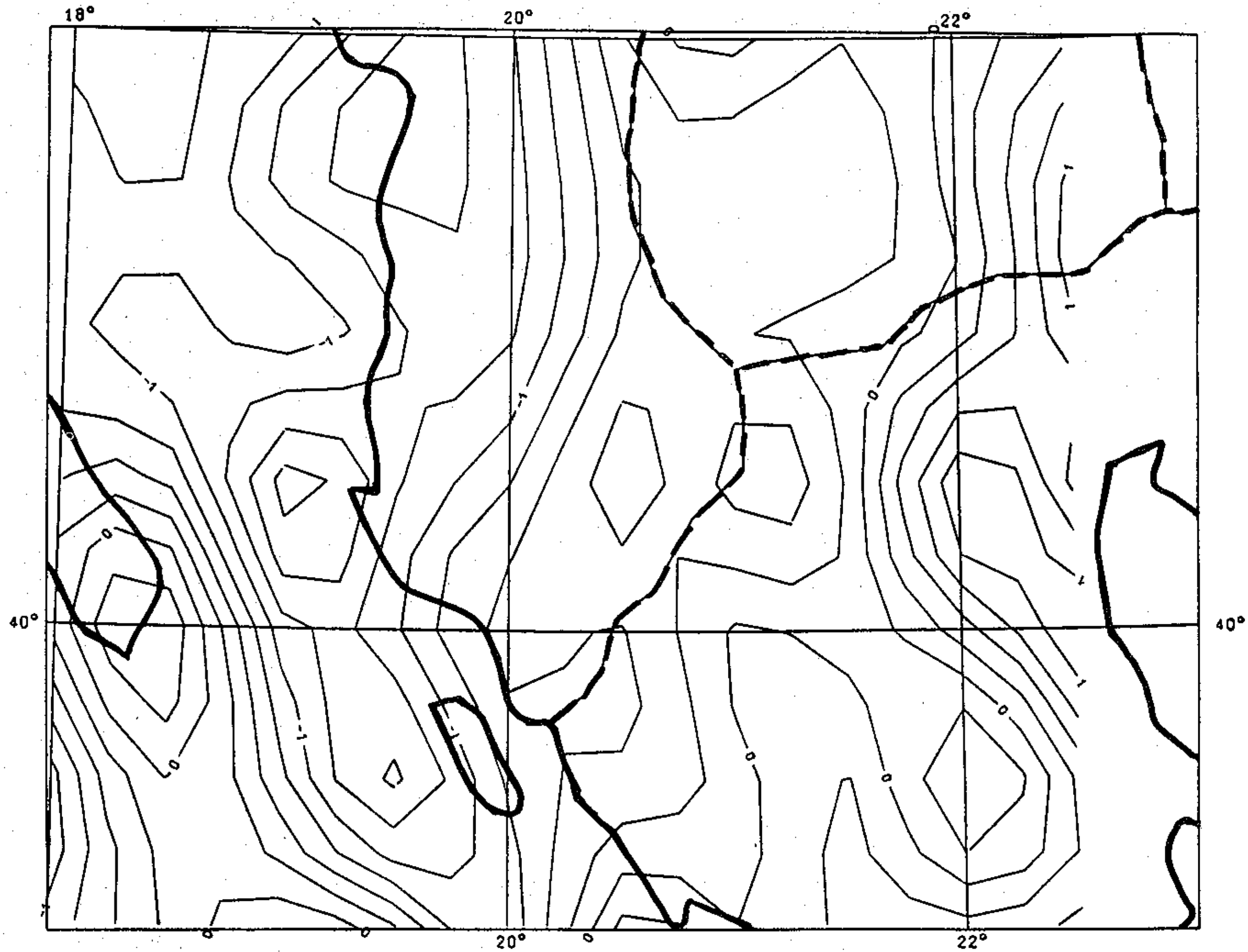


Fig. 4 – Computed geoid from collocation in IAG 1975. Contour interval 0.5 m.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