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Gravity field mapping from satellite altimetry, sea-gravimetry and bathymetry in the Eastern Mediterranean

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SUMMARY

Using data from the Eastern Mediterranean it has been shown how different data types may improve the quality of a gravity field approximation, and a detailed geoid has been computed for the area. The following data types were available: (a) bias-adjusted satellite altimetry, (b) sea-gravimetry, (c) bathymetry and heights and (d) the spherical harmonic coefficient set OSU81, complete to degree 180.

A comparison of the satellite altimetry with geoid heights computed from the OSU81 coefficients with and without the additional use of gravity data revealed substantial errors due to unmodelled orbit tilts. A trend analysis of the altimeter data showed a tilt of 0.0026 $\mathrm{m\,km^{-1}}$ in eastern direction and 0.0067 $\mathrm{m\,km^{-1}}$ in northern direction. The standard deviation (s.d.) of observed altimeter values minus OSU81 geoid heights decreased from ± 1.53 m to ± 1.07 m after a correction for the tilt.

Least-squares collocation was then used to predict subsets of the data types (a) and/or (b) from subsets of the same data types. The results given here are in terms of the s.d. of observed minus computed values. Using every third value of the corrected altimeter data for gravity prediction a s.d. of ± 17 mGal was found. When gravity values spaced 10′ apart were used to predict, (1) gravity values in a grid with the same mesh width shifted 5′ and (2) altimeter observations, a s.d. of ± 10 mGal and ± 0.42 m, respectively, were found. A combination of gravity and altimeter data made the first s.d. of ± 10 mGal decrease to ± 7 mGal, showing the impact of combining the two data types.

The use of topographic information to smooth the locally strongly-varying gravity field gave substantial improvements in the gravity values predicted from either altimeter observations or gravity data. Error estimates of geoid heights predicted in between the altimeter tracks showed that a 65 per cent improvement had been achieved.

Key words: Gravity, geoid, satellite altimetry, collocation, Eastern Mediterranean

1 INTRODUCTION

Satellite altimetry, gravimetry, spherical harmonic coefficients and bathymetry/topographic heights all contain information useful for gravity field mapping. A simultaneous use of all data types makes it possible to reduce the error inherent when using either only satellite altimetry (e.g. orbit errors) or only gravity data (e.g. remote zone effects). Data of one type may also be used to fill in holes where data of another type are missing. Some of these aspects have been treated in Knudsen (1986), using an area in the North Atlantic. However, here the gravity field is rather smooth and the satellite altimetry data have been carefully adjusted (Liang 1983), using both bias and tilt parameters, and taking advantage of a large number of cross-overs and estimates of tidal effects.

In the Mediterranean the situation is different. A cross-over adjustment involves many short segments of tracks, and estimates of tidal effects are generally not available. On the other hand the tidal effects are probably constant along tracks in the open parts of the Mediterranean. These circumstances lead Cruz & Rapp (1982) to adjust the SEASAT data using only bias parameters. Also here, because of the large depth variations, the topographic signal is strong and may be used for a gravity field estimation.

Furthermore, besides extending the results obtained from the North Atlantic, we wanted to compute a precise geoid for an area in the Mediterranean, where this has not been done before, in order also to prove the feasibility of using least-squares collocation (LSC) for such a task. In section 2 we will give a brief introduction to LSC and to the two methods used to account for the topography. The data is described in section 3.

The use of LSC requires the estimation and modelling of a covariance function. This caused some problems which should have warned us. Also our first computational experiments were very discouraging, giving unexpectedly bad results. However, a clear linear increase of the prediction error along the altimeter tracks made us aware of unmodelled orbit tilts, which we subsequently estimated. After having eliminated the effects of these tilts, the estimation and modelling of the covariance function caused no problem. All the details are given in section 4. Having 'cleaned' the data we then carried out several numerical experiments, combining the different data types. The results, which as expected confirm our hypothesis that it is useful to combine data, are described in section 5.

2 LEAST-SQUARES COLLOCATION (LSC) AND THE USE OF TOPOGRAPHIC DATA

The theory and practice of LSC is described in numerous publications. A recent survey is given in Tscherning (1985).

We will suppose that we have given n observations, y_i, related to the anomalous gravity field (T) through n linear or linearized functionals, L_i. Then

$$y_i = L_i(T) + e_i, \qquad i = 1, \ldots, n \tag{1}$$

where e_i is the observation error. If the observations depend on unknown parameters, then (1) will contain one more term representing this dependence. However, we have not used this possibility in the present investigation, because we have been able to estimate the relevant parameters separately.

For a gravity anomaly in a point P (1) becomes

$$\Delta g(P) = -\left.\frac{\partial T}{\partial r}\right|_P - \frac{2}{r}T(P) + e, \tag{2}$$

where r is the distance of P from the origin. For satellite altimetry, where $\zeta(P)$ is regarded as giving the value of the geoid height in the middle point (P) of the satellite footprint, we have

$$\zeta(P) = T(P)/\gamma + e, \tag{3}$$

where γ is normal gravity at P. Equation (3) is the Bruns equation, relating the geoid height and the anomalous potential.

The ijth spherical harmonic coefficient also fits into equation (1). The linear functional is in the spherical approximation the integral over the mean earth sphere of T multiplied by the ijth surface spherical harmonic, also multiplied by a suitable constant. We will not use this, since it can be shown that the use of spherical harmonic coefficient information within the framework of LSC is (nearly) equivalent to subtracting the contribution of the coefficients from the data. We will denote these values by y_i^s.

Then let C_{ij} denote the covariance between $L_i(T)$ and $L_j(T)$, D_{ij} the corresponding error-covariance, and C_{Pi} the covariance between the value of T at the point P and $L_i(T)$. Putting $\bar{C} = \{C_{ij} + D_{ij}\}$, $C_P = \{C_{Pi}\}$ and $y = \{y_i\}$ we then obtain the LSC estimate $\tilde{T}(P)$ of $T(P)$ as

$$\tilde{T}(P) = C_P^t \bar{C}^{-1} y. \tag{4}$$

(The superscript t means transposition). Furthermore an estimate of the mean square error for an arbitrary linear functional L is obtained by

$$\sigma^2(L(\tilde{T}) - L(T)) = C_{LL} - C_L^t \bar{C}^{-1} C_L, \tag{5}$$

where C_L is the vector of covariances between the observations and the quantity $L(T)$ and C_{LL} is the variance of this quantity.

As discussed in Tscherning (1979) it is important that the potential of the topography is a harmonic function. Consequently one cannot subtract the effect of a Bouguer-plate from the gravity anomalies since a different plate is used in each point. The effects of some consistent model of the topographic masses must be subtracted from the observations, and subsequently added.

A slight complication is introduced if the contribution from a spherical harmonic expansion is subtracted, since it also contains topographic effects. There are then two alternatives. The first is to work with a model of the topography equal to the difference between the true topography and mean topographic heights formed as moving averages over blocks of the same size as those used when determining the spherical harmonic coefficients. This is the residual terrain modelling (rtm) method, described in Forsberg & Tscherning (1981) or Forsberg (1985). This method keeps the mean values of the residual quantities close to zero, and thereby gives a small signal variance. This is important when using methods like collocation.

The second alternative is to use topographic–isostatic reduction. In this case we must add back the effect of the topographic–isostatic reduction potential, calculated as described by Lachapelle (1976) or Sünkel (1985), and calculate the effects in the traditional manner using a global integration of the masses. We decided not to do this, i.e. not subtract the contribution from the coefficients, but to calculate the effects locally using integration out to a distance of 600 km. This, however, introduces some quite large biases, see Table 1.

3 THE AVAILABLE DATA

One of the drawbacks of collocation is that one has to solve a system of equations with as many unknowns as the number of observations (cf. equation (4)). We therefore limited ourselves to an area bounded by $31° < \phi < 37°$ and $26° < \lambda < 36°$, where ϕ is the latitude and λ the longitude.

3.1 Gravity data

In this area 3653 (point) sea-gravity anomalies were available (see Fig. 1) from two sources: (a) a free-air anomaly map (Woodside, 1976) including 2856 points in the area, and (b) a set of 1388 points values supplied by Deutsche Hydrographische Institut, containing the result of a gravity survey with the ship *Concrete* in 1961. 797 of these values fell in the area.

The accuracy of the two sources can be expected to be between ±5 and ±7 mGal. Since the error may be reflected in the standard deviation of the anomalies within a certain block size (big differences signifying large errors, and not necessarily a locally strongly-varying gravity field) we used these values in some of our computations. For blocks of size $10' \times 10'$ they ranged between 5 and 30 mGal. However, no

Table 1. Statistical characteristics of the raw and reduced data (2419 SEASAT altimetry and 3653 sea-gravity points)

Data	Altimetry (m) Mean	Altimetry (m) Standard deviation	Gravity (mGal) Mean	Gravity (mGal) Standard deviation
Raw data	14.08	6.62	−45.87	53.27
Data-GPM2	3.15	1.54	−5.14	30.75
Data-OSU81	2.43	1.53	−9.54	33.04
Data-OSU81*	−0.01	1.07	−9.54	33.04
Data-OSU81-RTM*	−0.01	1.17	−0.54	27.31
Data-OSU81-Isostasy*	3.28	1.67	−14.99	29.19

* Tilt removed from altimetry data.

significant difference was found between using these error estimates and using a constant error.

3.2 Altimeter data

Cruz & Rapp (1982) have made a separate adjustment of the SEASAT-A data in the Mediterranean, and these values were made available to us. The data have been adjusted using only bias parameters, and the standard deviation of the cross-over differences is ±0.15 m for the whole Mediterranean, and ±0.08 m for our area. A condition for using LSC is that no observation occurs twice, or that they are not so near that they cause singularities when solving equation (4). We therefore eliminated repeat track data, being left with 2419 points distributed on 32 tracks (see Fig. 2).

In adjustments in the open ocean, the data are first corrected for tidal effects. Tidal information is not available for our area. However, we may consider these effects as constant along a track. This means that the adjusted altimeter measurements should not contain any sea-surface topography effects, and should in theory represent the geoid

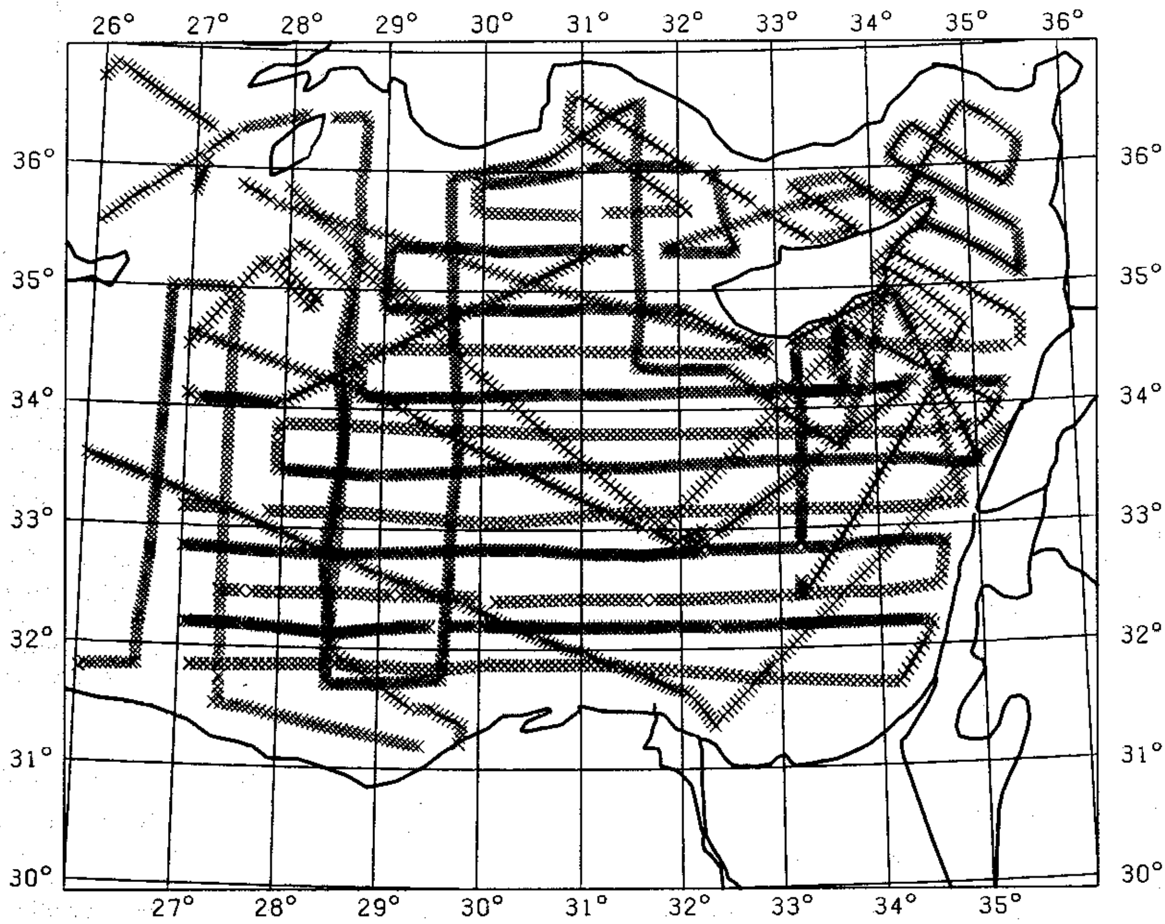

Figure 1. Sea-gravity data available in Eastern Mediterranean.

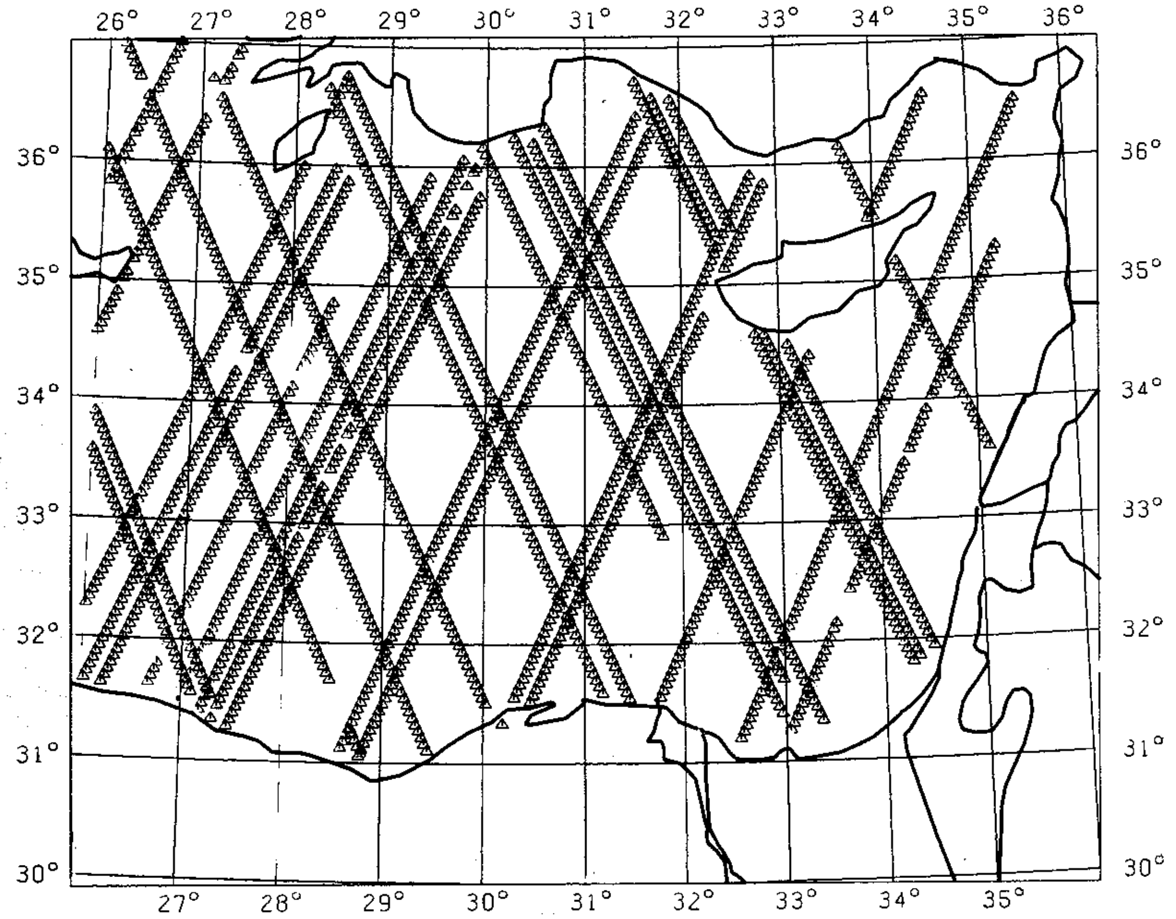

Figure 2. SEASAT altimetry in Eastern Mediterranean. Repeat tracks shown only once.

for the area well, except possibly for a bias common for the whole Mediterranean sea. We shall come back to this point later.

3.3 Potential coefficients

Several sets of geopotential coefficients are available, complete to or above degree 180. We decided to investigate the OSU81 set (Rapp, 1981) and the GPM2 set (Wenzel, 1985). Statistics concerning the mean and s.d. of the differences Δg^a and ζ^a are found in Table 1. We decided to use the OSU81 set, since the bias in ζ^s was the smallest. However, the s.d. of the differences are extremely large. Generally, mean differences and s.d. below 1 m arc found (see Tscherning & Forsberg (1986) Table 2; Knudsen 1986).

3.4 Bathymetry and heights

For the Mediterranean sea, mean $5' \times 7.5'$ depths have been digitized using bathymetric maps of scale 1:750.000, having an equidistance of the isolines equal to 200 m (Morelli *et al.* 1975), see Fig. 3. For the surrounding continental area, mean $5' \times 7.5'$ elevations have been interpolated from Morelli *et al.* (1975) and from a $10' \times 10'$ global data set available from US National Geophysical Data Center, at Bounder, Colorado.

For the computation of the effects of the topography a detailed grid is used close to the computation point and a coarse grid further away. Here the detailed grid of $5' \times 7.5'$ mean values were used up to 90 km distance, while a $10' \times 15'$ grid was used at larger distances. The reference surface needed for the rtm-reduction was formed as $20' \times 30'$ means of the $5' \times 7.5'$ blocks. This grid is smoothed further by taking moving averages over the 3×3 adjacent blocks. Consequently, the topography having wavelengths between 1° and $\frac{1}{3}$° is not accounted for. This will not affect our solution, since we subtract and add back the effect of the same masses.

4 COVARIANCE AND TILT ESTIMATION

The starting point for the use of LSC in a local area is the computation and analytic modelling of the empirical covariance function. Its estimation is discussed in detail in Goad *et al.* (1984). Numerically the estimation is simply done as the computation of the mean value of products of quantities lying within the same interval of spherical distance. The quality of the estimation depends on the size

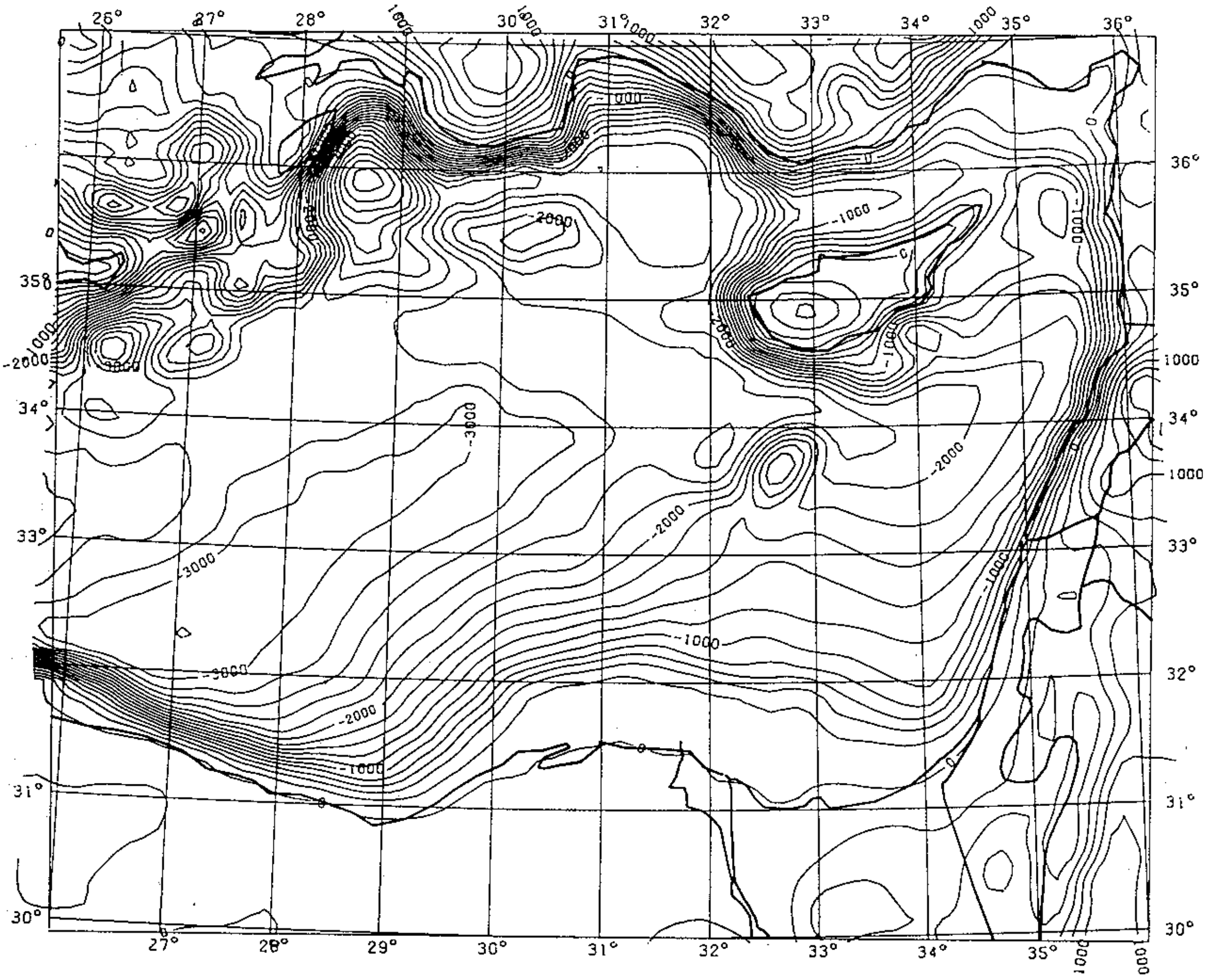

Figure 3. Bathymetry and heights in Eastern Mediterranean. Contour interval 200 m. Note that both the coast line and the bathymetric data contain errors.

of the sampling error and the regularity of the data distribution. The size of the area should also be so large that the mean value of the quantities (from which the contribution from the OSU81 set has been subtracted), y_i^s, is zero. For a spherical harmonic expansion to degree 180 this generally will be achieved for an area of the size we are considering. However, as seen from Table 1 this is not the case for the satellite altimeter data, while the hypothesis that the gravity anomalies Δg^s have mean value zero cannot be rejected.

Despite the non-fulfilment of the requirements for a reasonable covariance estimation, we nevertheless carried out the estimation of auto- and cross-covariance functions for Δg^s and ζ^s. We then tried to model the empirical values using an analytic model for the gravity anomaly covariance function of the form

$$C(P,Q) = a\sum_{i=2}^{N} \bar{\sigma}_i \left(\frac{R_E^2}{rr'}\right)^{i+2} P_i(\cos\phi_{PQ}) + \sum_{i=N+1}^{\infty} \frac{A(i-1)}{(i-2)(i+4)} \left(\frac{R_B^2}{rr'}\right)^{i+2} P_i(\cos\phi_{PQ}). \quad (6)$$

Here r' is the distance of the point Q from the Earth's centre, R_E the mean earth radius (=6371 km), $\bar{\sigma}_i$ error anomaly degree-variances (in units of $mGal^2$) associated with the OSU81 or GPM2 coefficients, R_B the radius of the so-called Bjerhammar-sphere, P_i the Legendre polynomial of degree i, ψ_{PQ} the spherical distance between P and Q, a is a unitless free parameter, while A is a second free parameter in units of $mGal^2$. Also the summation limit N is a free parameter. Its value reflects the degree to which the spherical harmonic coefficient information is considered reliable for the area, where the covariance function is estimated. Values of $a = 2$, $A = 722.3$, $R_E - R_B = 3.5$ km and $N = 90$ were determined, using GPM2 as a reference field. The agreement with the empirical values were far from satisfactory. In fact, it looked as if the auto-covariance of the geoid heights had too much low degree-power.

With these preliminary covariance functions we then used gravity data for the prediction of altimeter data and altimeter data to predict the gravity data. Topographic information was used to smooth the field. While the prediction of gravity data gave very discouraging results, the prediction of the altimeter values from gravity gave us some important information. Inspecting the residuals (the differences between observed and predicted values), it became apparent that they had a clear linear trend, and that north-going and south-going tracks behaved differently. In

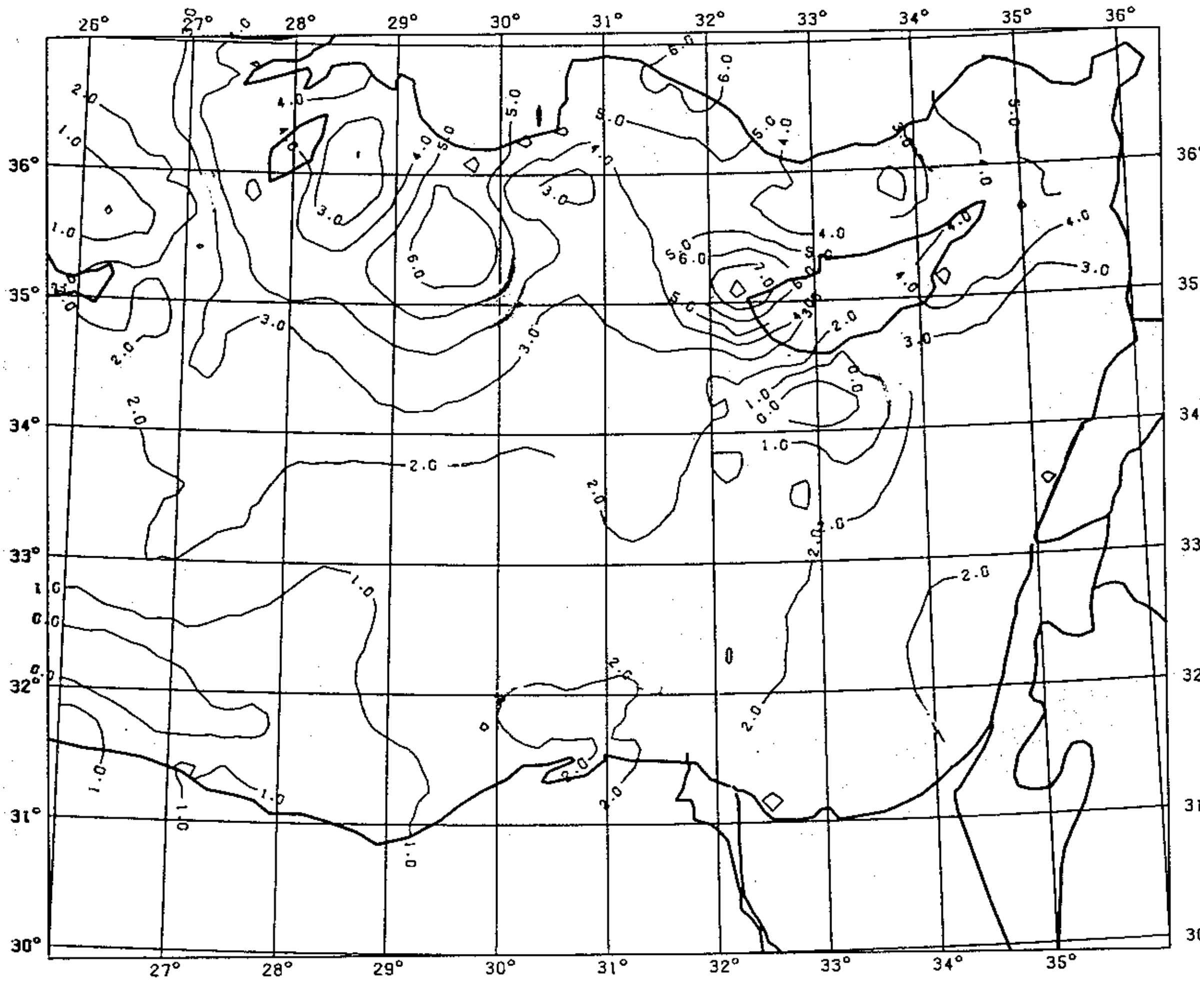

Figure 4. SEASAT altimetry–OSU81, contoured by interpolation at 0.5 m interval.

fact now knowing what to look for, we could even see this trend in the ζ^s-values (see Fig. 4). The values at the Anatolian coast are 5 m higher that at the coast of Egypt.

A trend analysis of the ζ^s-values showed a tilt of $0.0026\ \mathrm{m\ km^{-1}}$ in an eastern direction and $0.0067\ \mathrm{m\ km^{-1}}$ in a northern direction. After the correction for the tilt, the standard deviation of the ζ^s-values decreased from ± 1.53 m to ± 1.07 m. We also estimated tilts separately for each track, but found it unreasonable considering the probable physical reason for the tilt.

The reason is orbit errors, which again are due to errors in the low-degree spherical harmonic coefficients, i.e. being approximately constant for tracks shorter than 1000 km. They are strongly correlated (similar in magnitude) for a specific geographical location like the Eastern Mediterranean (see Engelis 1986). Also the use of separate tilts for each track would have made the s.d. of the cross-over differences increase. Now, using two tilts (and not one per track), this s.d. would remain the same.

Having removed—at least partly—this large error, we made a new covariance function estimation. The modelling of the empirical values by an analytic expression of the form (6) now caused no problems; see Figs 5–7 for ζ^s and Δg^s, and Figs 9–11 for ζ^t and Δg^t, where the rtm method had been used for the calculation of topographic effects. We also

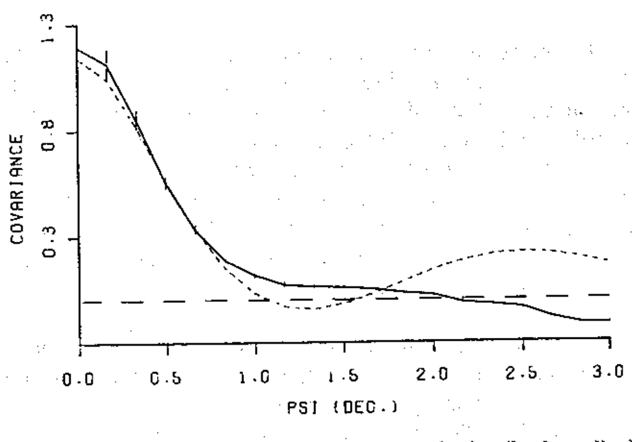

Figure 5. Empirical (solid line) and synthetic (broken line) covariance functions of SEASAT data. OSU81 and tilt removed from the original values. Units m^2 (1112 altimetry points and 1272 gravity points used for the estimation of the empirical covariance functions in this and the following cases).

calculated the covariance function for topographic–isostatic reduced values, but this is not shown here. It is similar to the one obtained by rtm, but with a slightly larger correlation length. The constants a, A, N and $R_B - R_E$ are

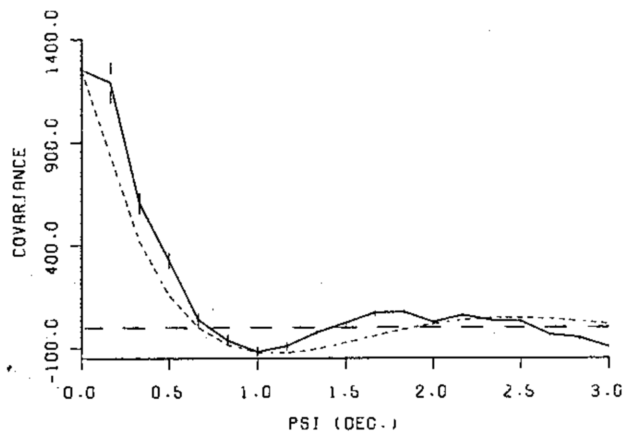

Figure 6. Empirical and synthetic covariance functions of gravity anomalies. OSU81 removed from the original data. Units mGal^2.

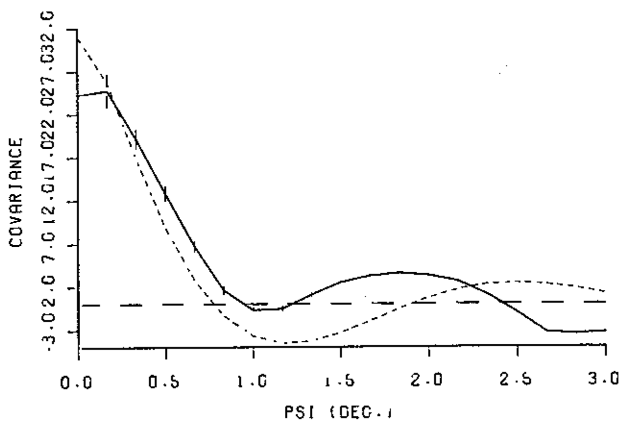

Figure 7. Empirical and synthetic cross-covariance functions between gravity and SEASAT altimetry data. Units mGal$*$m.

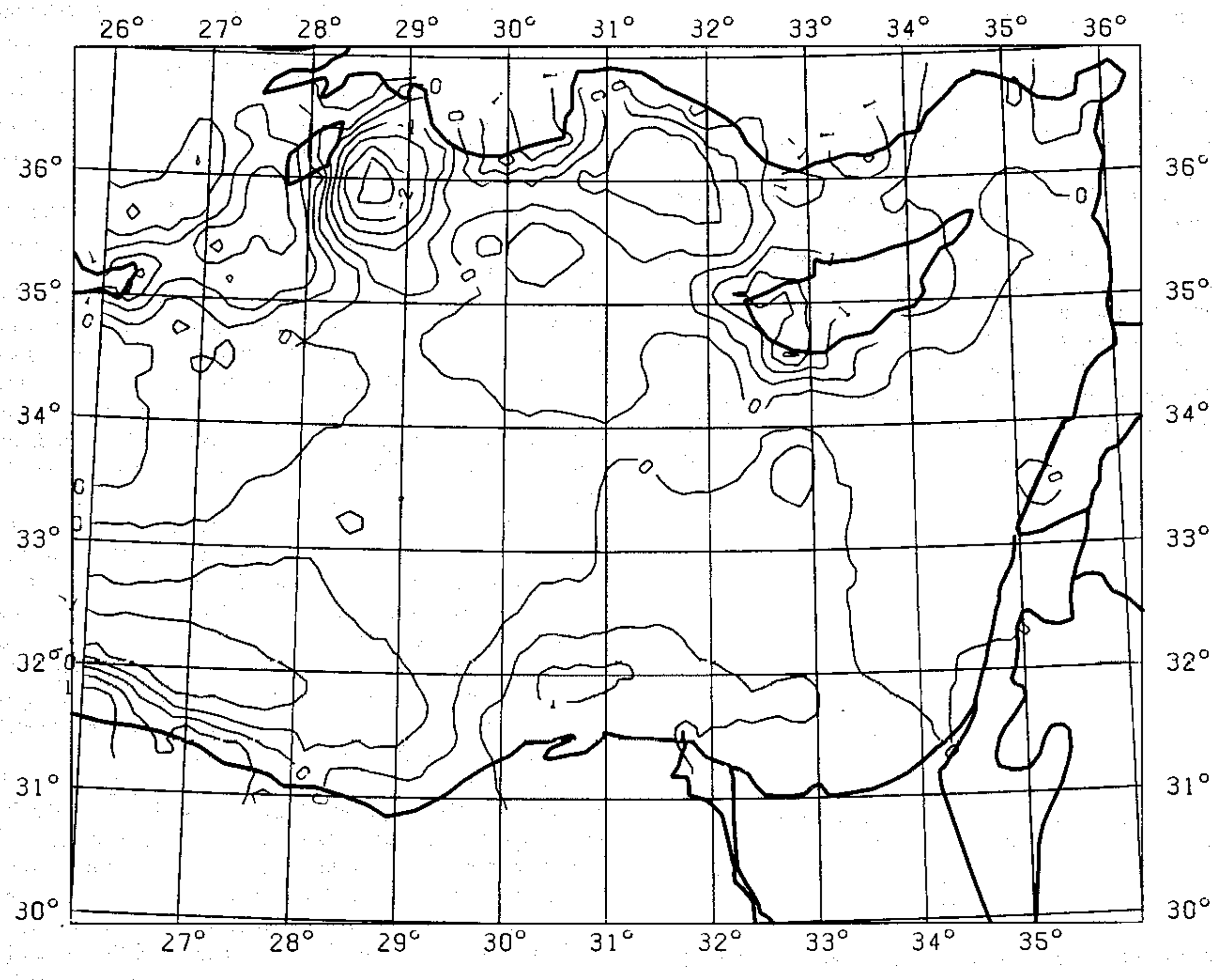

Figure 8. RTM effect on altimetry. Contoured by interpolation at 0.5 m interval.

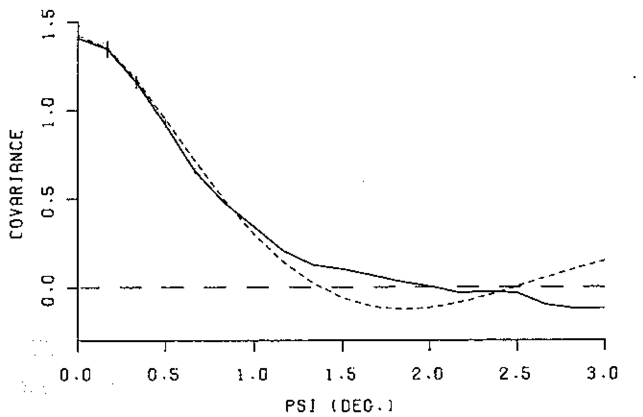

Figure 9. Empirical and synthetic covariance functions of SEASAT altimeter data. OSU81, tilt and RTM effect removed from the original values. Units m^2.

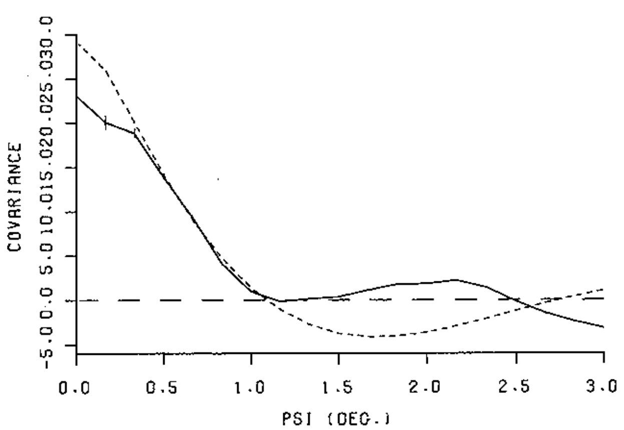

Figure 11. Empirical and synthetic cross-covariance functions between gravity and SEASAT data. Units mGal * m.

given in Table 2 for all three cases. Note that the value of the constant a is larger for the topographic–isostatic values than for the rtm values. Correspondingly the value of N is smaller in the first case. This is caused by the biases introduced due to the incomplete topographic isostatic reduction.

5 PREDICTION RESULTS

Having developed analytic models for the covariance function, it is now an easy task using the FORTRAN-program GEOCOL (Tscherning 1985a) to predict geoid heights, gravity anomalies, deflections of the vertical or gravity gradients, if needed. Also any combination of data of this kind may be used as input data. However, the execution of the program, and the calculation of error estimates, becomes very time-consuming if many more than 1000 values are used at a time. Also we needed some data as control values, in order to check the quality of the prediction and the error estimates.

We then selected every third altimeter value in the area $31.5° < \phi < 36°$, $26° < \lambda < 35°$, to be used as input data. This resulted in 603 points. Selecting every third point gives the same regular data distribution as the whole data set. In fact, a regular data distribution is important for getting a good solution. For the gravity data we then selected all points in the area as close as possible to the knots of a $10' \times 10'$ grid. This gave 487 points.

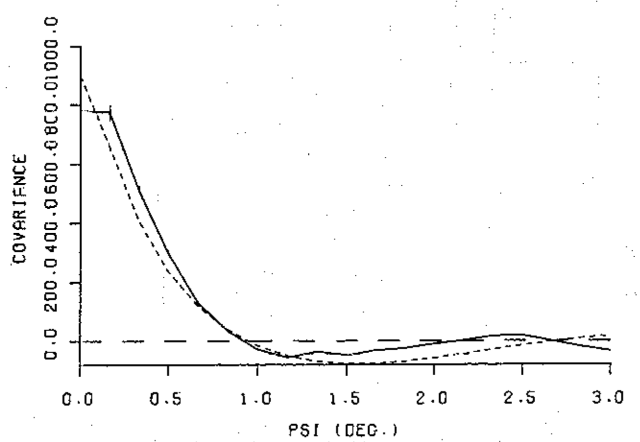

Figure 10. Empirical and synthetic covariance functions of gravity data. OSU81 and RTM effect removed from the original data. Units $mGal^2$.

As control values we used 222 regularly distributed altimeter points and 339 regularly distributed gravity values. We then carried out prediction tests using (a) gravity alone; (b) altimetry alone; (c) gravity and altimetry simultaneously.

In all cases (a)–(c) the contribution from the OSU81 field had been removed. Also we made two further tests using data from which topographic effects had been removed, using either rtm or topographic–isostatic modelling. The results are summarized in Table 3.

The geoid contributions from the topography using rtm and topographic–isostatic modelling are shown in Figs 8 and 13, respectively.

Note that altimeter data predicted from altimeter data always have a very small s.d. of the differences, equal to or just below the *a priori* s.d. of the noise of 0.1 m. It drops to 0.08 m when all data are used and topographic–isostatic effects have been removed. This is also the data combination which gives the best results when predicting gravity anomalies, namely ±6.28 mGal, just above the expected noise level of the data.

The relative improvements in predicting gravity achieved when combining the two data types is largest when using data which have not been reduced topographically. This is to be expected, since the topographic data well represent the same high-frequency part as the gravity data. However, some improvement is achieved in all cases. This may not in itself be statistically significant, but, for example, gross errors are much more easily detected from topographically reduced data than from unreduced data.

Finally, we have computed a geoid for the area with estimates of its error, see Figs 13 and 14. The absolute magnitude of these error estimates may not be correct, but the relative error seems very reasonable. A comparison of the error estimates for the different data combinations showed that a relative improvement of the estimate of the geoid for points at a maximal distance from the tracks but located in between the tracks improved between 55 and 65 per cent, using a combination of all data types. This

Table 2. Parameters used for the computation of the covariance functions.

Type of reduction	Variance of point grav. anomalies (mGal²)	A (MGal²)	a	$R_E - R_B$ (km)	N
none	1253	1118.2	0.2	5.5	130
RTM	900	636.6	0.2	5.5	90
topographic–isostatic	1000	614.2	0.5	5.5	70

corresponds quite well to the results presented in Knudsen (1986).

6 CONCLUSIONS

We have seen how least-squares collocation may be used to combine gravity and satellite altimetry and that an improved estimation of both types of quantities is achieved thereby. The use of topographic information also improves the result, especially when no gravity is available to represent the high frequency part of the spectrum.

Satellite altimetry in a closed sea, like the Mediterranean, must be carefully checked for local tilts. Such tilts could have been estimated directly using LSC, but may also be estimated separately. This is to be recommended, since the tilt-corrected altimeter data must be used for the estimation of the empirical covariance function before applying LSC. The error due to the tilts will be directly reflected in the computed error estimates if the estimation of the tilts is done within the LSC process.

The tilts we have found may also probably be seen in other parts of the Mediterranean. The comparison between geoid heights along the coast of Italy with altimeter data

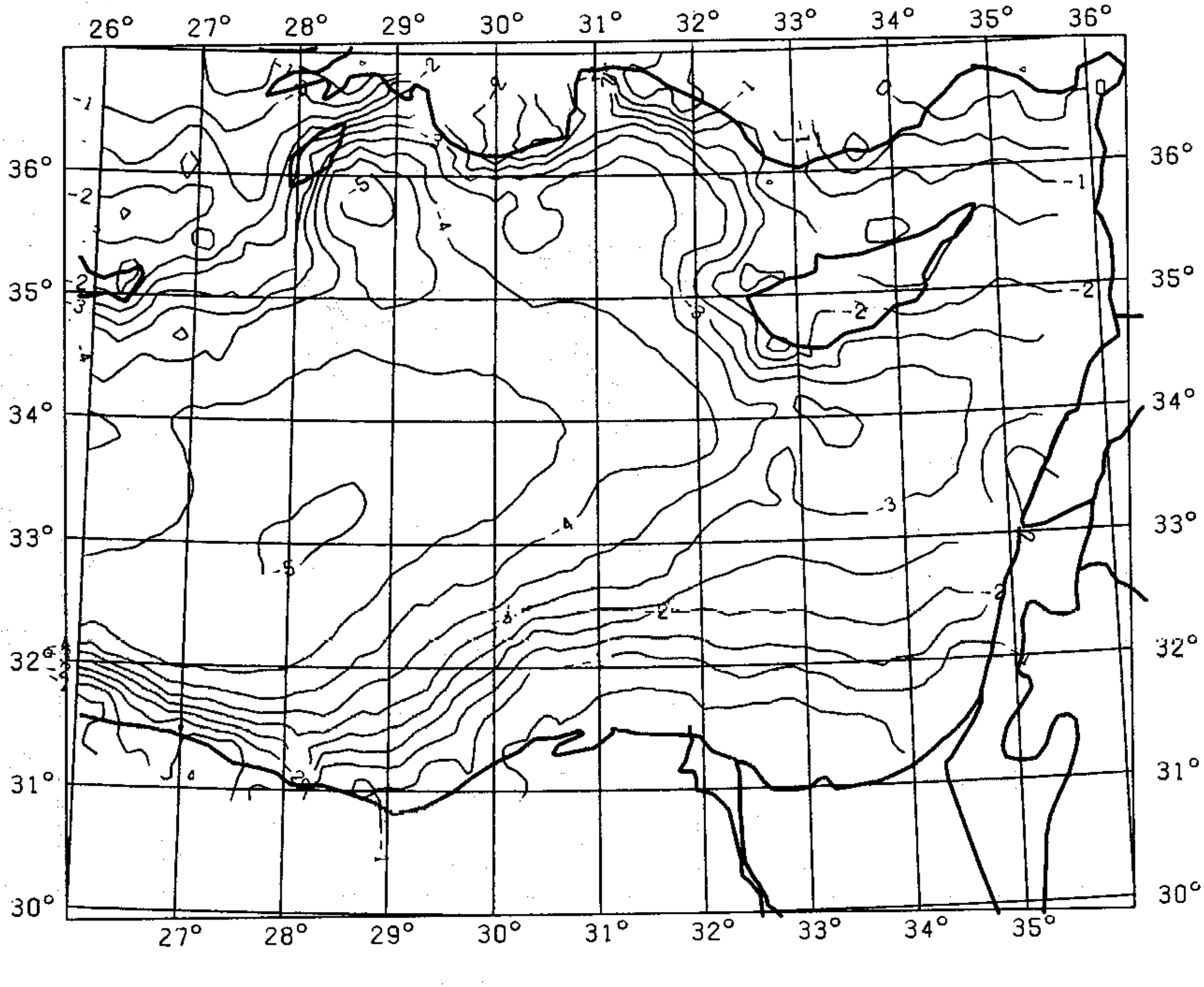

Figure 12. Isostatic effect on altimetry data. Contoured by interpolation at 0.5 m interval.

Table 3. Prediction results of tests using altimetry, gravity and both in combination.

Input data	Number of points used	Altimetry obs.–pred. (222 points) Mean (m)	Standard deviation (m)	Gravity obs.–pred. (339 points) Mean (mGal)	Standard deviation (mGal)	Remarks
altimetry	603	−0.01	0.10	−5.81	16.58	*
gravity	487	0.66	0.42	−1.61	10.06	—
combination	1090	−0.01	0.10	−1.56	7.37	—
altimetry	603	−0.01	0.10	−2.87	11.86	†
gravity	487	0.28	0.42	−0.46	6.90	—
combination	1090	0.01	0.09	−1.09	6.68	—
altimetry	603	0.01	0.09	1.47	10.88	‡
gravity	487	1.09	0.44	1.25	7.35	—
combination	1090	0.00	0.08	0.00	6.28	—

* Without taking into account topographic masses.
† rtm reduction up to 600 km.
‡ Isostatic reduction up to 600 km.

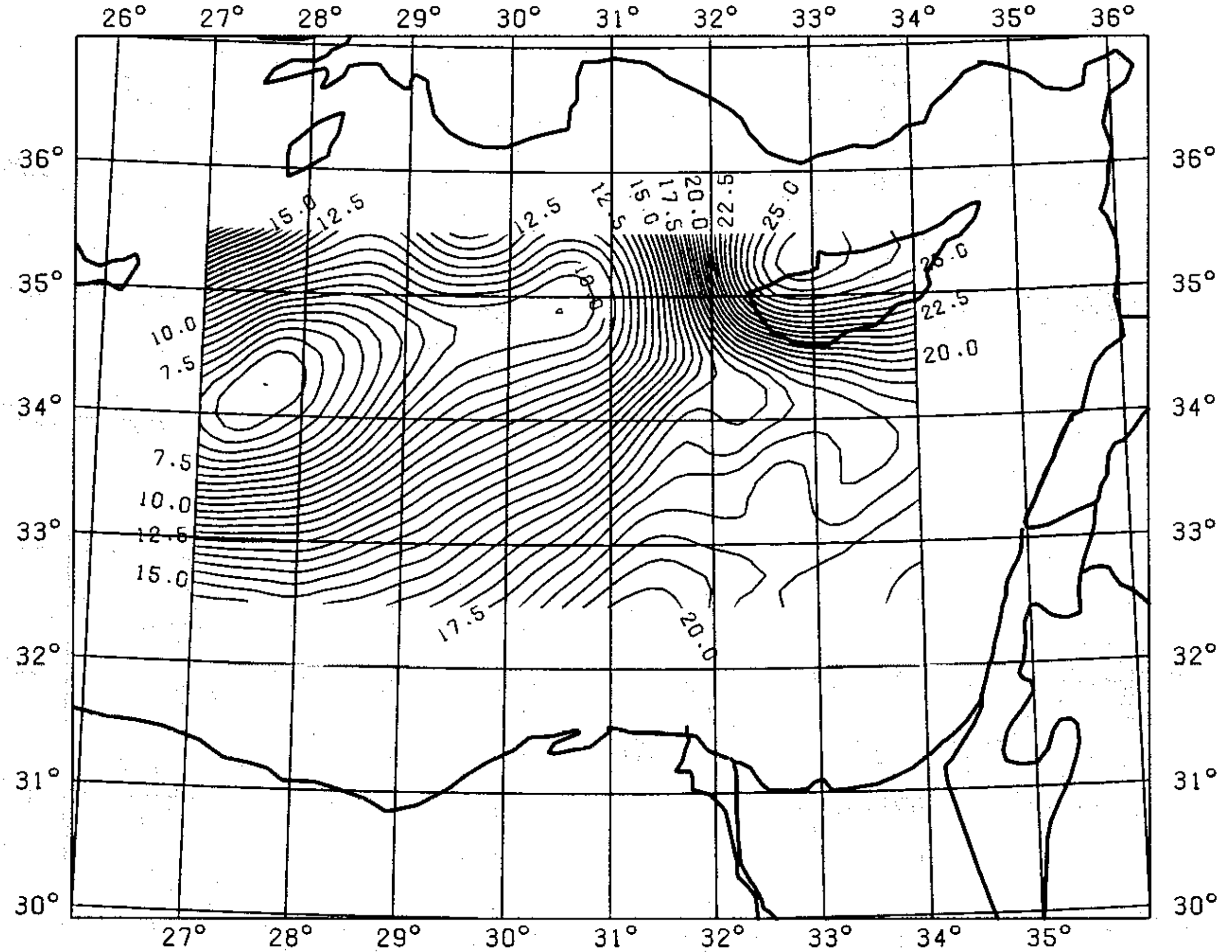

Figure 13. Geoid computed from gravity, altimetry, OSU81 coefficients and topographic information. Reference ellipsoid with a = 6378136 and GRS1980 flattening used. Contour interval 0.5 m.

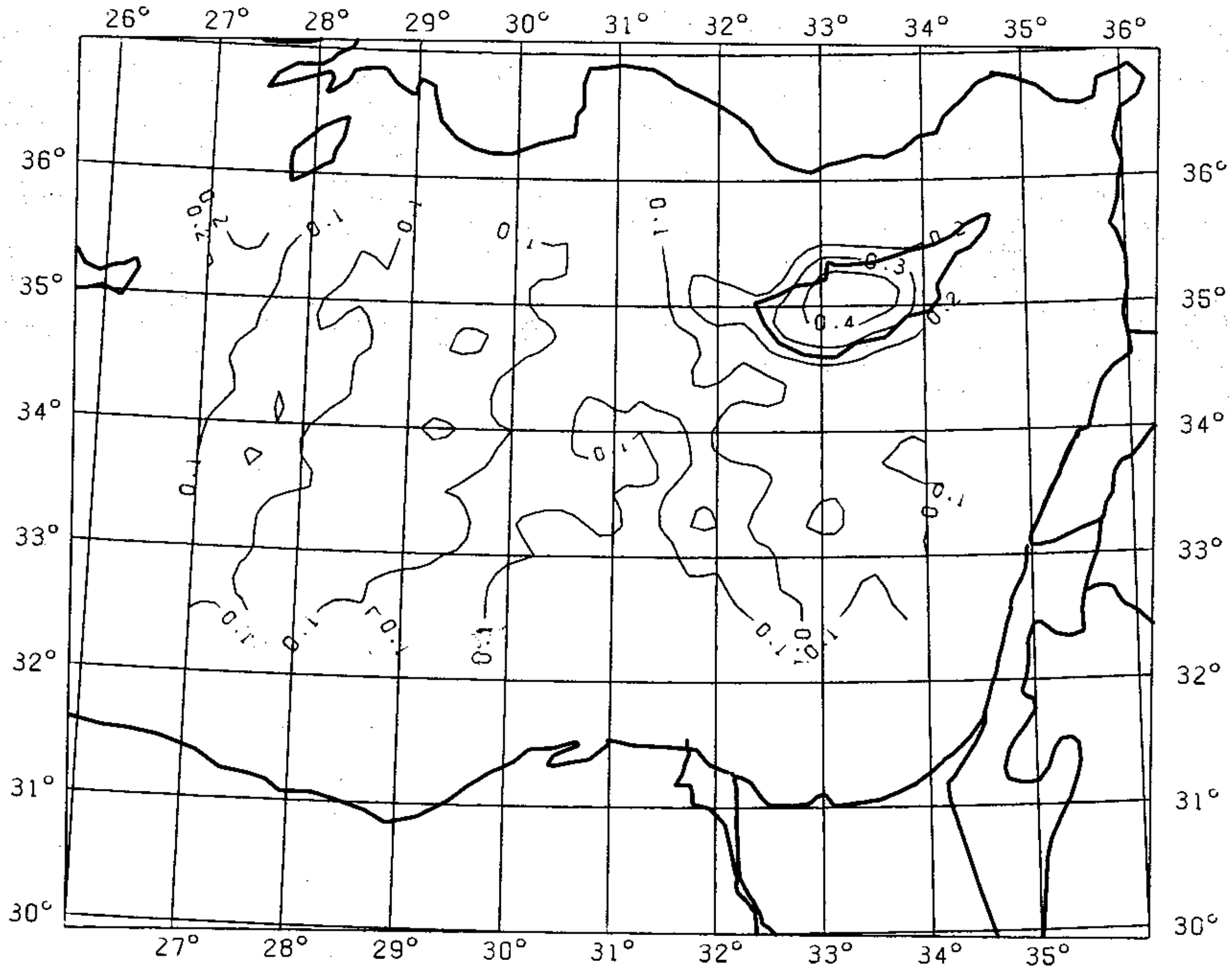

Figure 14. Error estimation of the combined solution. Contour interval 0.1 m.

(Benciolini *et al.* 1984) have large mean differences, which could be explained by a large N–S tilt.

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