Geoid Determination in the Nordic Countries from Gravity and Height Data (*)

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Summary. – The area covering Denmark, Finland, Norway and Sweden has been divided in blocks of size 2° × 4°. Based on Wenzel's GPM2 spherical harmonic expansion used to maximal degree 180 least squares collocation approximations to the anomalous gravity potential have been constructed for each block. Gravity data spaced 6′ × 12′ within each block, extented 1/2°, was used, with topographic effects calculated from 5′ × 10′ mean heights in all except Norway where 1 km × 1 km mean heights were used.

The heights used were calculated with reference to a 1P moving average mean height surface, thus making it unnecessary to calculate isostatic effects. The terrain-reduced gravity anomalies had variances below 650 mgal^2 and correlation distances larger than 11′, while the unsmoothed values had variances of up to 3000 mgal^2 and correlation distances of down to 3′.

The quality of the result has been evaluated by comparing computed gravity values with values observed, but not used for the construction of the approximation. Root mean square differences for the various blocks ranged between 11 mgal and 2 mgal with most values around 4 mgal. Also deflections of the vertical were calculated giving corresponding agreements between ± 0″.6 and ± 3″, with most blocks around ± 1″. SEASAT sea-surface heights were also used for comparison, most blocks showing standard deviations of differences of ± 0.2 m, however with biases of –0.45 m on Atlantic coastal areas and +0.20 m in the Bay of Bothnia. This bias difference is as yet unexplained.

The computed geoid values differ slightly (–10 cm to 10 cm) along block boundaries, but the general consistency is much improved as compared to earlier solutions. We estimate the error in this preliminary solutions to be 20 cm in areas with moderate topography and 40 cm in mountains for geoid undulations differences within a block. Further improvements are planned by using additional gravity data in the mountains, deflections along lakes and satellite altimetri in the southern part of the Baltic Sea, where very little gravity data is available.

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DETERMINAZIONE DEL GEOIDE NEI PAESI NORDICI UTILIZZANDO DATI DI GRAVITÀ E DI QUOT

Sommario. – L'area comprendente Danimarca, Finlandia, Norvegia e Svezia è stata suddivisa in bloc di dimensioni 2° × 4°. Partendo dallo sviluppo in armoniche sferiche GPM2 di Wenzel fino al grado mass 180, con la collocazione ai minimi quadrati sono state determinate, per ciascun blocco, approssimazioni potenziale anomalo di gravità.

Sono stati impiegati dati di gravità spaziati 6′ × 12′ in ciascun blocco, esteso 1/2°, con effetti topogra calcolati a partire da quote medie spaziate 5′ × 10′ in tutte le aree ad eccezione della Norvegia per la quale s state adottate quote medie con spaziatura 1 km × 1 km.

Le quote utilizzate sono state calcolate con riferimento ad una superficie di questa media con moving average», eliminando pertanto la necessità di determinare gli effetti isostatici.

Le anomalie di gravità ridotte avevano varianze inferiori a 650 $mgal^2$ e distanze di correlazione super a 11′, mentre per i valori grezzi le varianze toccavano i 3000 $mgal^2$ e le distanze di correlazione eran di sotto di 3′.

La qualità del risultato è stata valutata confrontando i valori di gravità calcolati con i valori osservati, non usati per la costruzione dell'approssimazione. Le differenze tra gli scarti quadratici medi dei vari bloc andavano da 10 mgal e 3 mgal, con la maggior parte dei valori intorno a 4 mgal. Anche le deviazioni d verticale furono calcolate fornendo risultati in accordo entro i valori ± 0″.6 e ± 3″, con la maggior parte blocchi intorno a ± 1″. Le quote SEASAT sulla superficie del mare sono state usate per il confronto: la mag parte dei blocchi mostravano deviazioni standard, per le differenze, di ± 0,2 m, con variazioni, comun di – 0,45 m sulle zone costiere dell'Atlantico e di + 0,20 nella Baia di Bothnia. la causa di tali variazio ancora ignota.

I valori calcolati dal Geoide differiscono leggermente (da – 10 cm a 10 cm)lungo i bordi dei blocchi la consistenza generale è molto migliorata rispetto alle prime soluzioni. Riteniamo che l'errore in qu soluzioni preliminari sia 20 cm in aree con topografia non accidentata e 40 cm in aree montagn relativamente alle differenze di ondulazioni geoidiche entro il blocco. Ulteriori miglioramenti sono preved usando un maggior numero di dati di gravità sulle montagne, di deviazioni lungo i laghi e di altimetri satellite nella parte meridionale del Mar Baltico, dove sono disponibili pochissimi dati di gravità.

1 – INTRODUCTION

The Nordic Geodetic Commission has requested the Geodetic Institute o Denmark to execute the computation of a «standard» geoid for the Nordic Area. Th progress of this work has been reported earlier in Tscherning (1982, 1983, 1985a).

From the beginning it was decided to use the method of least squares collocation Reasons for this choice have been given earlier. However, it is appropriate to mentio one of the reasons here, since it in some way created larger difficulties, than it aided th rapid execution of the task. On the other hand, it helped us avoiding various pitfalls, an gave us insight into a number of new problems of which some by now are solved an some still unsolved.

The data distribution (of gravity anomalies, deflections of the vertical etc.) in th Nordic Area is not homogeneous. This is mainly due to the occurrence of the Baltic Sea coastlines with deep fjords and neighbouring countries (USSR, PL, DDR and D) whic do not release any data or only parts of their data. Here least squares collocation woul permit the simultaneous use of data of different kinds, the use of which (at least to a certain extend) would enable the precise computation of for example geoid heigh differences using only data within the Nordic Area. Collocation would work like th astrogravimetric method of integration, where remote zone effects drop out whe forming differences between computed geoid heights.

However, this requires that the data used have reasonable characteristics, i.e. tha there are no large systematic errors, or that such errors if their physical origin is known may be modelled by some parameters.

Biases were expected in parameters connecting the longitude component (η) of th deflection of the vertical (Tscherning, 1982a) and in the datum shift paramete connecting the deflections of the vertical given in ED1950 and the doppler derive geoid heights given in NWL9D with a geocentric system. The uncertainty for th translation vector part of the datum shifts was supposed to be small (1-2 m), but it turne out, that a large rotation in longitude had to be determined, see Tscherning (1985a 1986).

However, even when this was taken into account, we found, in computation carried out in 1983, disturbing differences between (1) doppler derived geoi undulations and computed values (of the order 1-2 m) and (2) observed and compute prime vertical components of deflections of the vertical (of the order of 1″) in Finland These differences were much larger than what should be expected from the erro estimates produced by collocation and/or associated with the datum shift parameters Fortunately, also comparisons by other colleagues (see e.g. Brenneke et al., 1983, tabl 3.1) had shown similar patterns. Also, it was found that the noise in the doppler derive geoid heights could be explained by changing ionospheric conditions (Tscherning and Goad, 1985), while the bias in the Finnish longitudes is not fully explained.

These conditions did not discourage us from using collocation. On the contrar confirmed our decision to use the method, since it was possible to estimate some of t systematic effects using the method. However, we decided to compute first a soluti based on gravity data only, because this kind of data has a very favourable signal to noi ratio, and for which systematic errors are expected to be small. (Systematic errors occ primarily due to inconsistency in the height system (< 0.5 m) used when computi gravity anomalies on land and in sea-gravimetry due to instrument and navigati biases).

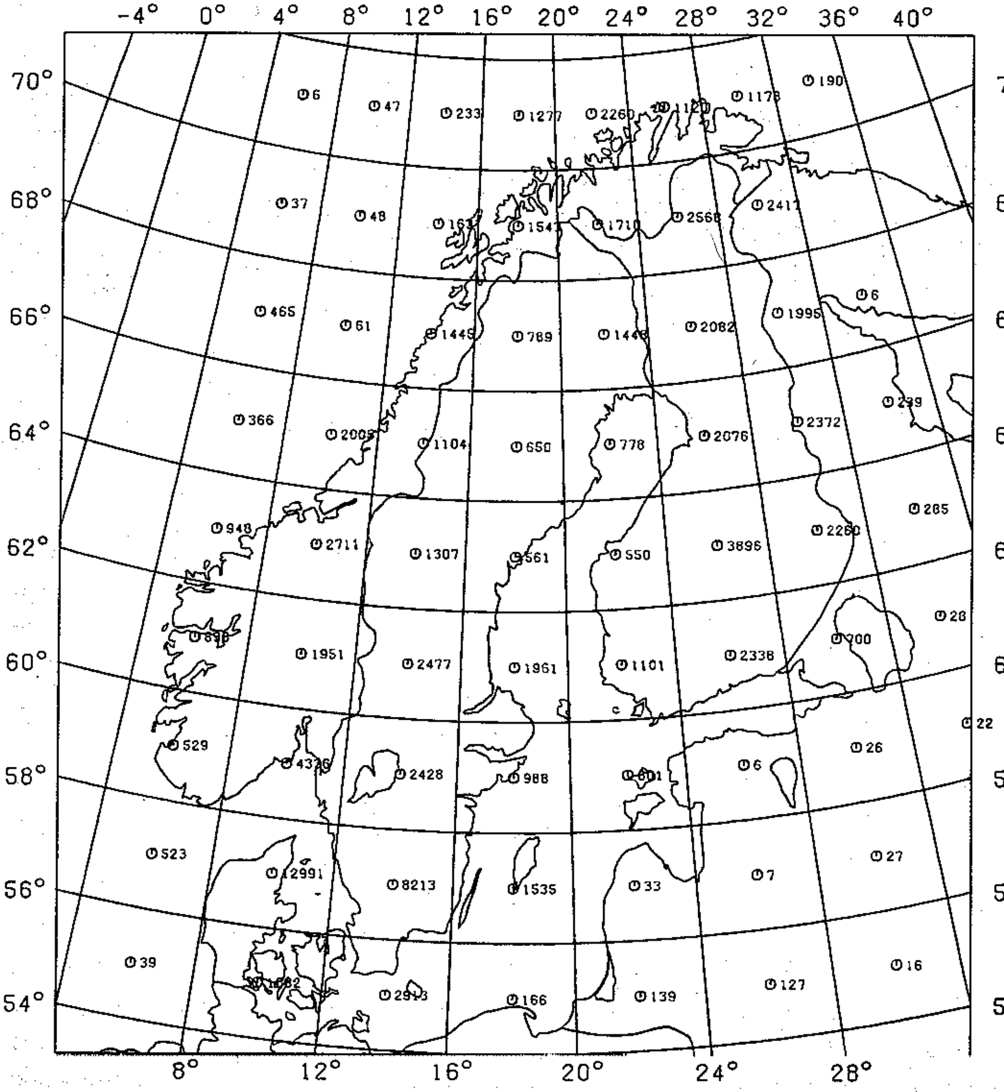

Fig. 1 – Number of gravity observation per 2° × 4° block.

Since the first geoid solution were produced in 1981 requirements to the quality of the geoid have become stricter. The conversion of ellipsoidal height differences obtained using GPS to normal (or orthometric) height differences requires that differences between quasi-geoidal heights (height anomalies) have a standard deviation smaller than 5 cm over 10 km. A reasonable requirement would be 2.5 cm over 10 km. The geoid at sea is also needed with a similar accuracy, however here maybe better expressed as differences between mean values over a certain area size.

The goal may possibly be archieved with the present data distribution (see fig. 1) in areas with moderately varying topography (heights and depths < 500 m), but could not be achieved in the up to 2.5 km high mountains of Norway.

The topography had to be taken into account in a way so that it was consistent with the use of the method of collocation. This means subtracting and subsequently adding the effects of the potential of topographic masses (cf. Tscherning 1979, Forsberg and Tscherning, 1981).

In the following section 2 we will describe how collocation and the so-called residual terrain modelling (RTM) method was used for the computation of approximations to the anomalous gravity potential – and thereby of the quasi-geoid. The area covered by the approximations is not all the Nordic Countries (which also include Iceland, Faroe Islands and Greenland), but Denmark (DK), Norway (N), Sweden (S) and Finland (SF). In section 3 we describe data and data «reduction» procedures, and the result and the evaluation of the result is described in section 4. Future plans are briefly discussed in the final section 5.

2. – THE USE OF COLLOCATION AND RTM

Let T the anomalous gravity potential. Then given data $x_i = L_i(T) + n_i$, $i = 1,\dots,N$, least squares collocation determines an approximation $\tilde{T}$ to T

$$\tilde{T}(P) = \sum_{i=1}^{N} b_i \operatorname{cov}(L_i(T), T(P)), \tag{1}$$

$$\{b_i\} = \{\operatorname{cov}(L_i(T), L_j(T)) + \sigma_{ij}\}^{-1} \{x_j\}. \tag{2}$$

Here P is a point in space, L_i the linear functional associating T with the observation, $\operatorname{cov}(L_i(T), T(P))$, $\operatorname{cov}(L_i(T), L_j(T))$ the covariances between the i'th observation and the value of T in the point P, the j'th observation, respectively, $\{\sigma_{ij}\}$ the variance-covariance matrix of the noise. The height anomaly $\xi(P)$ (or the quasi-geoidal height, when P is on the Earth's surface), is then obtained by evaluating $\tilde{T}$ in the poin and dividing this value by normal gravity, γ_p,

$$\tilde{\xi}\ (P) = \tilde{T}(P)/\gamma_p, \qquad \text{(Bruns formula).}$$

The covariances in eq. (1) and (2) may also be regarded as the inner product the linear functionals in the space dual to a Hilbert space with reproducing Ker $K(P,Q) = \text{cov}(T(P),T(Q))$.

Spherical harmonic coefficients as obtained from analysis of satellite o perturbations combined with information from mean gravity anomalies is today a ma source of gravity field information. They represent a zero order approximation t which we will denote T_0. Used as data in collocation (and with an isotropic covaria function) it is easily seen as being (nearly) equivalent to the subtraction of the value the linear funnctionals applied on the potential T_0. This potential must t subsequently be added to the solution.

The covariances will also be modified, and we will call this for cov_s. Then v $T_S = T - T_0$.

$$\tilde{T}(P) = T_0 + \sum_{i=1}^{N} b_{si}\ \text{cov}_S(L_i(T_S),\ T_S(P)),$$

$$\{b_{si}\} = \{\text{cov}_S(L_i(T_S),L_j(T_S)) + \sigma_{ij}\}^{-1}\ \{x_j - L_j(T_0)\}$$

If we subtract any other (harmonic) function, such as the potential of topogra masses, T_M, we get equations similar to eq. (4) and (5), with $T_S = T - T_0 - T_M$ and T_0 substituted for T_0.

It is important to be aware that T_M could be any harmonic function. Naturally, better a topographic/geological mass model we use, the smoother $T_S = T - T_0 - T_M$. mean square variation of a quantity $L(T_S)$, $\text{Var}(L(T_S))$, or the norm of T_S will be sn This is important because one of the two quantities occur as a scale factor in equat expressing mean square prediction errors, or upper bounds for prediction er respectively. Mean square prediction errors are estimated by

$$\tilde{\sigma}^2(L(T) - L(\tilde{T})) = \text{Var}(L(T_S))$$

$$- \{\text{cov}_S(L(T_S),L_i(T_S))\}^{\text{T}}\{\text{cov}_S(L_i(T_S),L_j(T_S)) + \sigma_{ij}\}^{-1}$$

$$\{\text{cov}_S(L_j(T_S),L(T_S))\},$$

and upper limits by a similar equation.

However, the topography to be considered should be consistent with the fact that long-wavelenght topographic effects already are accounted for by T_0, up to the maximal degree and order minus one (1). As a reference topography one may use spherical harmonic expansion of the topography or similary a smooth mean height surface of corresponding resolution. This has several advantages. One advantage is that topography located further away than 2 times the minimum wavelength has a very small effect. Also the mean value of the observations minus the effect of T_0 will generally be zero if the area has an extend larger than the minimum wavelength represented by T_0. Using the residual topography this zero mean value will not be changed. (This does not mean that zero mean values are required for the use of collocation. But it is one of the factors keeping $\text{Var}(L(T_S))$, and thereby the errors small).

We will here use the *empirical* covariance function. The definition of this function for a local region is discussed in (Goad et al., 1984), see also (Tscherning, 1985b) and (Forsberg, 1984). The estimation of the covariance function in the considered part of the Nordic Area is discussed in (Forsberg, 1986). It was modelled analytically using

$$\text{cov}_S(T_S(P), T_S(Q)) = \sum_{i=2}^{N_0} \sigma_i^0 \left(\frac{R_E^2}{rr'}\right)^{i+1} P_i(\cos\psi)$$

$$+ \sum_{i=N_0+1}^{\infty} \frac{A}{(i-1)(i-2)(i+4)} \left(\frac{R_B^2}{rr'}\right)^{i+1} P_i(\cos\psi). \tag{7}$$

Here ψ is the spherical distance between P and Q, P_i the i'th Legendre polynomial, r, r' the radial distances of P, Q from the origin, respectively, N_0 the maximal degree of T_0, R_B the radius of a (Bjerhammar) sphere totally enclosed in the Earth, R_E the mean Earth radius (= 6371 km) and σ_i^0 «error degree-variances», representing the expected error of T_0 in the considered Area. A is a factor selected so that $\text{Var}(\Delta g_s) = 225$ mgal^2, where Δg_s is a gravity anomaly at height zero, from which has been subtracted the contribution from T_0 and T_m.

In theory one should use for σ_i^0 the square sum per degree (i) of the error estimates of the potential coefficients. For the results described below we have used the coefficients of the spherical harmonic expansion GPM2 (Wenzel, 1985) with N_0=180. Since this solution has included data from the Nordic Area of globally seen high quality we have in eq. (7) used the error degree variances of GPM2 multiplied by the scale factor 1/2.

A value of $R_E - R_B$ = 3.5 km was used in order to obtain a correlation distance $\psi_1 = 12'$, see (Tscherning, 1985a, fig. 6).

From eq. (2), (5) it is seen that as many equations as the number of observations have to be solved. Since for the area of *DK*, *N*, and *SF* we have available more than 100 000 gravity observations, it is obvious that local solutions must be constructed. General considerations concerning the size of the local areas (blocks), and the selection

of a suitable subset of the available data are given in (Ibid, section 4). We decided to u
$2° \times 4°$ (latitude extend × longitude extend) block, with a $1/2° \times 1°$ overlap, i.e. da
within a $3° \times 6°$ block was to be used. The block boundaries are the meridians a
parallels shown in fig. 1, except for the blocks above latitude 68°, which extend up to 7
in some cases. Using gravity data selected as close as possible to the nodes o
$6' \times 12'$ grid would then permit gravity anomalies to be predicted with a standa
deviation of

$$\sigma(\tilde{\Delta g}-\Delta g) = (\mathrm{Var}(\Delta g_s))^{1/2}\left(\frac{d}{\psi_1}\cdot 0.3\right) \text{mgal},$$

cf. (Ibid, eq. (4.6)). This gives with $\mathrm{Var}(\Delta g_s) = 225$ mgal, $\psi_1 = 12'$ and $d = 6'$ an expec
standard deviation of predicted gravity anomalies of 2.25 mgal. (We shall see in secti
4 whether this has been achieved). The estimate correspond to an estimate of predic
deflections of the vertical of 0."3 – 0."4. For geoid undulation differences over 10 km t
again correspond to a standard deviation of 2 cm, or 20 cm over 100 km. Please keep
mind that these figures are «rule of thumb» estimates. Also, naturally, large random
systematic errors are not accounted for.

3. – DATE USED FOR THE GEOID COMPUTATION OR EVALUATION

3.1. - Spherical harmonic coefficients

Several spherical harmonic expansions complete to degree and order 180 a
available: OSU78 (Rapp, 1978), OSU81, (Rapp, 1988), GEM10c (Lerch et al. 1981) a
GPM2 (Wenzel, 1985) complete to 200. A comparison of the solutions with gravity d
in the Nordic Area (Tscherning, 1986a, table 1) showed that the OSU78 and GP
coefficients were equally good. However, the disturbing correlation of geoid heig
computed from the OSU78 set with the height of the point of evaluation (Ibid, figure
made us select GPM2.

The evaluation of the series for points with varying latitude and/or height is qu
time consuming, while values in a grid may be produced very fast (see Tscherning et
1983). We therefore computed geoid (ζ_0) and freeair gravity anomalies (Δg_0) in two g
in altitude 0 and 2 km with spacing $5' \times 10'$, for an area covering all the bloc
Values of Δg_0 and ζ_0 in arbitrary points in the area was the subsequently evaluated
interpolation in one of the three-dimensional grids. Fig. 4 shows the GPM2 contribut
at h=0.

3.2. - Topographic data

Topographic data were available in a number of different formats, as $5' \times 10'$ m
heights for S and SF, and as 1 km × 1 km grids in 4 UTM zones covering Norway ar

border into the neighbouring countries, see fig. 2. All height data was combined into basic $5' \times 10'$ grid, used for geoid terrain effects and remote zone effects on gravity ar deflections. Using a moving average operator, this grid was also used for providing smooth mean height surface of $1^\circ \times 2^\circ$ resolution, used as a reference surface in tl residual terrain model (RTM) reduction. With this kind of terrain reduction tl topographic data set need not be complete. Hence, no depth information was used, a heights were set to zero at sea.

Terrain effects on gravity stations were computed by splitting the effect into terrain correction and a Bouguer plate reduction to the reference level. Due to the lac of detailed height data terrain corrections were only computed for Norway and bord zones. A two-stage FFT technique was used (Forsberg, 1985), yielding terra corrections in the same grid as the heights, subsequently interpolated to actual static locations. Due to computer storage limitations, the detailed topography had to b subdivided into a number of blocks (24), with each block computed individually.

For height anomalies a similar FFT-technique was used. However, sinc topographic effects on the geoid primarily are of long wavelenght character, see fig. 3, i this case the $5' \times 10'$ grid has been sufficient for computations. This is not the case fo deflections of the vertical, where 1-km height grid spacing is essential for good result However, lack of data has again necessitated the use of the $5' \times 6'$ grid outside Norwa but since most of the area is quite flat the effects on the solution are minor. In Norwa the 1-km height grid was used, and computations were done using a space domain pris integration procedure (Forsberg and Tscherning, 1981).

3.3. - GRAVITY DATA

Gravity data were made available by the National Agencies and by Defens Mapping Agency, Aerospace Center (USA). A simple data base management system wa established in order to handle the large amounts of data. 120 000 points are available fo the territories of *DK*, *N*, *S* and *SF* including surrounding waters, cf. fig. 1. 90 000 poin are available for the North Atlantic Area surrounding the Faroe Islands and Icelan

The management system used the index-sequential access method with keys base on the geographical location of the points. This enabled vary fast selection of all data i an area or of all points closest to the nodes of a grid. The system also was designed so tha dublicates would be rejected, when loading the data base. This feature was used in orde to compare the data received from DMAAC with the data received from the Nationa Agencies or found in our own files. Already this comparison exposed a large number o gross errors, such as 1° shifts in position or 100 m height errors.

A simple gridding and plotting of the gravity data also showed many large error Some of the errors have been corrected and other values have been marked as havin

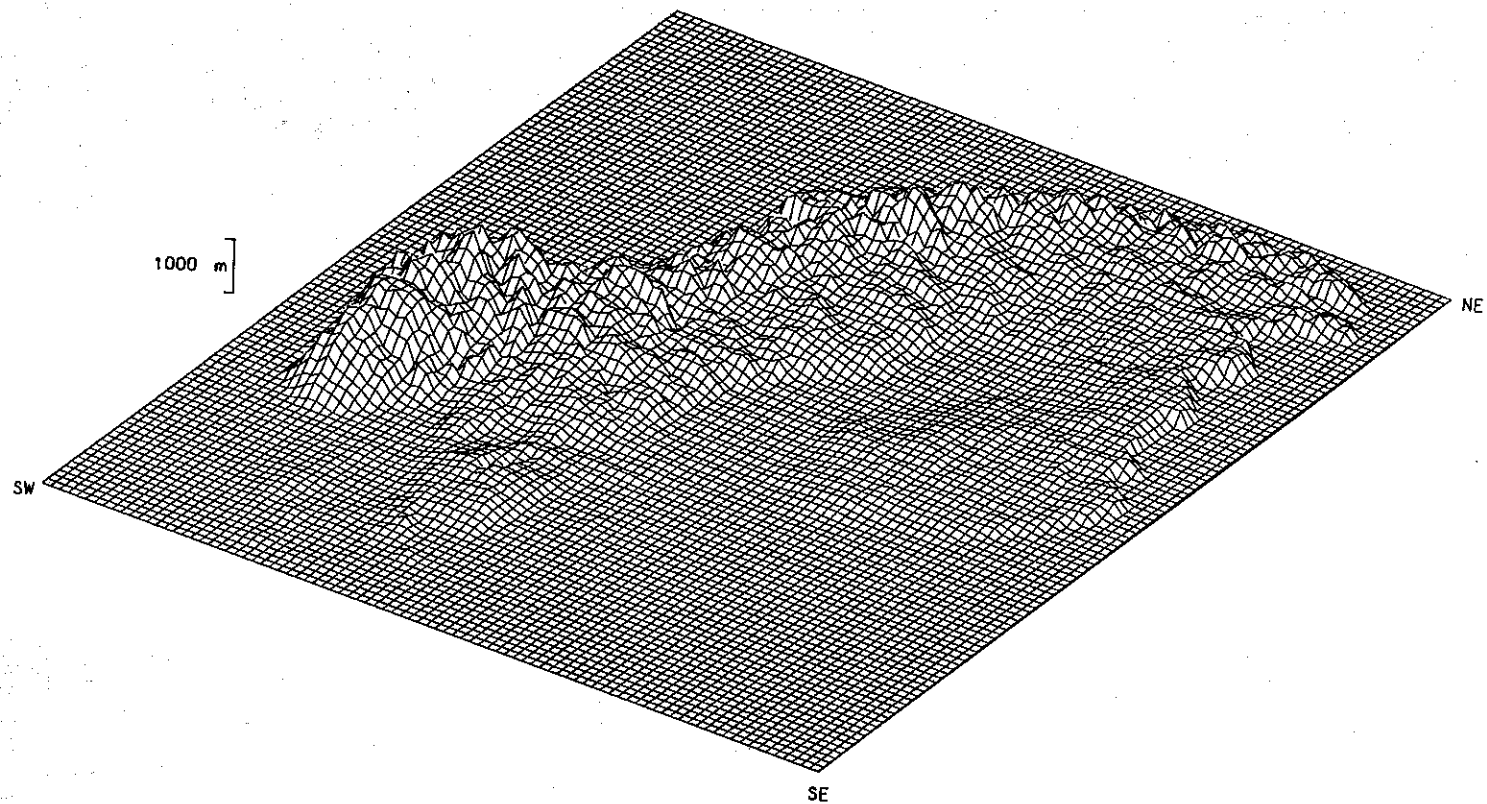

Fig. 2 – Digital terrain model for DK, N, S and SF.

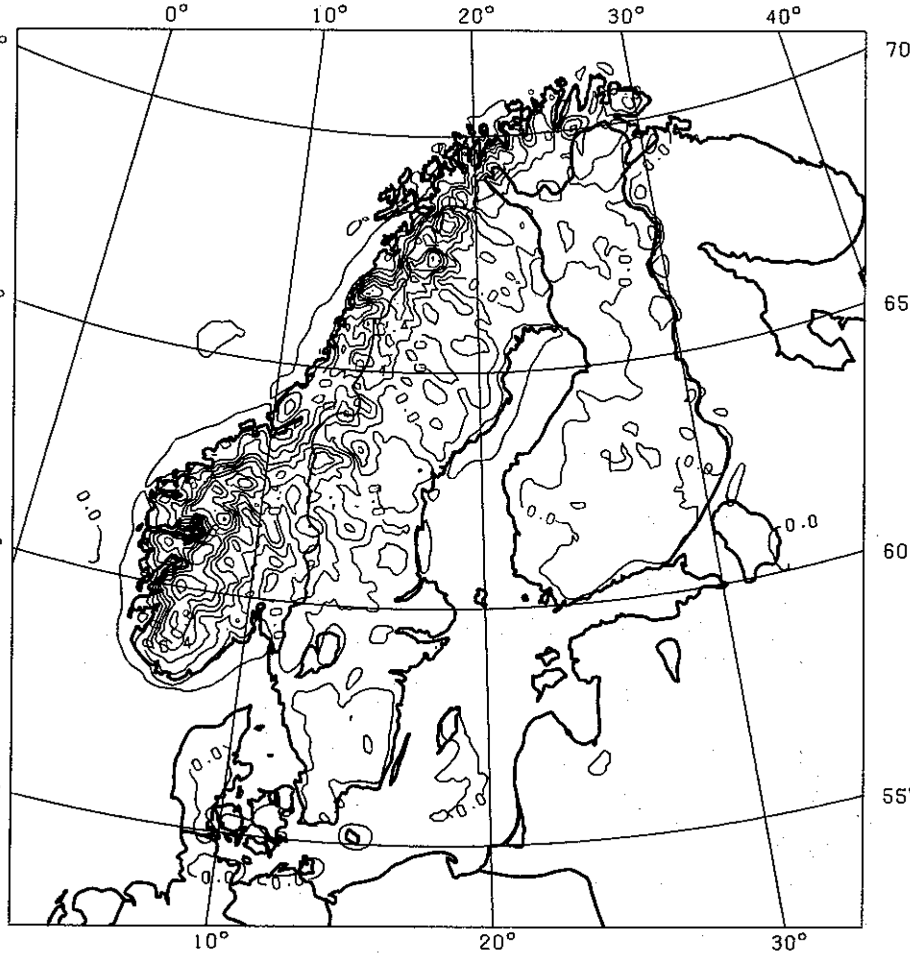

Fig. 3 – RTM-contribution to the geoid, equidistance 0.2 m.

suspected errors, by assigning a large error estimate to the value. Gross errors larger tha
20 mgal are easily detected in area with a smoothly varying gravity field, while we i
mountaneous areas have difficulties in finding gross errors below 30-50 mgal. 1‰ c
the data seems to have gross errors. A revision of the data is still going on, so th
solutions presented in section 4 may include (a few) gross errors.

3.4. - Deflections of the vertical

Deflections of the vertical had earlier been collected from the National Agencies. Usin
earlier solutions a comparison with computed values were made, exposing several gros
errors (up to 6″ - 8″). Some of these errors have been corrected meanwhile.

A comparison with gravimetrically computed values for Finland showed that th η-component of the deflections of the vertical had a large bias. Consequently a speci datum-shift for ED1950 to a geocentric system has been used.

Values in ED1979 have not been used. However the differences between ED195 and ED1979 are within the noise level of the data (< 0.″3), thus with some systemat trend.

3.5. - Doppler derived geoid heights

A number of doppler derived height were available, but not used due to their larg errors.

3.6. - Sea-surface heights

Sea-surface heights determined by SEASAT were made available to us by R.H Rapp. The data has been has been globally adjusted. A regional adjustment of the da gave a further decrease of the standard deviation of the cross-over differences, fro 25 cm to 17 cm.

Several gross errors has been removed from the data. The errors were found b comparing the heights with geoid heights computed from the solutions described i (Tscherning, 1983).

It is known that data in the Baltic Sea and the Bay of Bothnia contain effects due t wind, see (Vermeer, 1983). Corrections computed by the Finnish Geodetic Institute wi be applied in subsequent applications of the data.

4. – PRODUCTION OF THE APPROXIMATIONS TO *T*, AND THE EVALUATIO OF THE RESULT

As a basic coordinate system for the computations we used an ellipsoid wit $a = 6378136$ m and with the flattening of GRS1980. We also used the GM of GRS198

The evaluation of equations (4), (5) and (6) was done using the FORTRAN progra TC (Forsberg, 1984) and Fast Fourier transform programs, cf. section 3.2. The data flo is shown in fig. 5.

Only gravity data was used, situated as close as possible to the nodes o a $6' \times 12'$ grid covering a $3° \times 6°$ area with the $2° \times 4°$ area in the middle. Solutions fo totally 40 blocks were computed. The blocks are identified by the latitude and longituc of the north-west corner of the block, e.g. 6404 for the $2° \times 4°$ block with north-we corner $\varphi = 64°$ and $\lambda = 4°$. An exception is made for the northernmost blocks, which ma

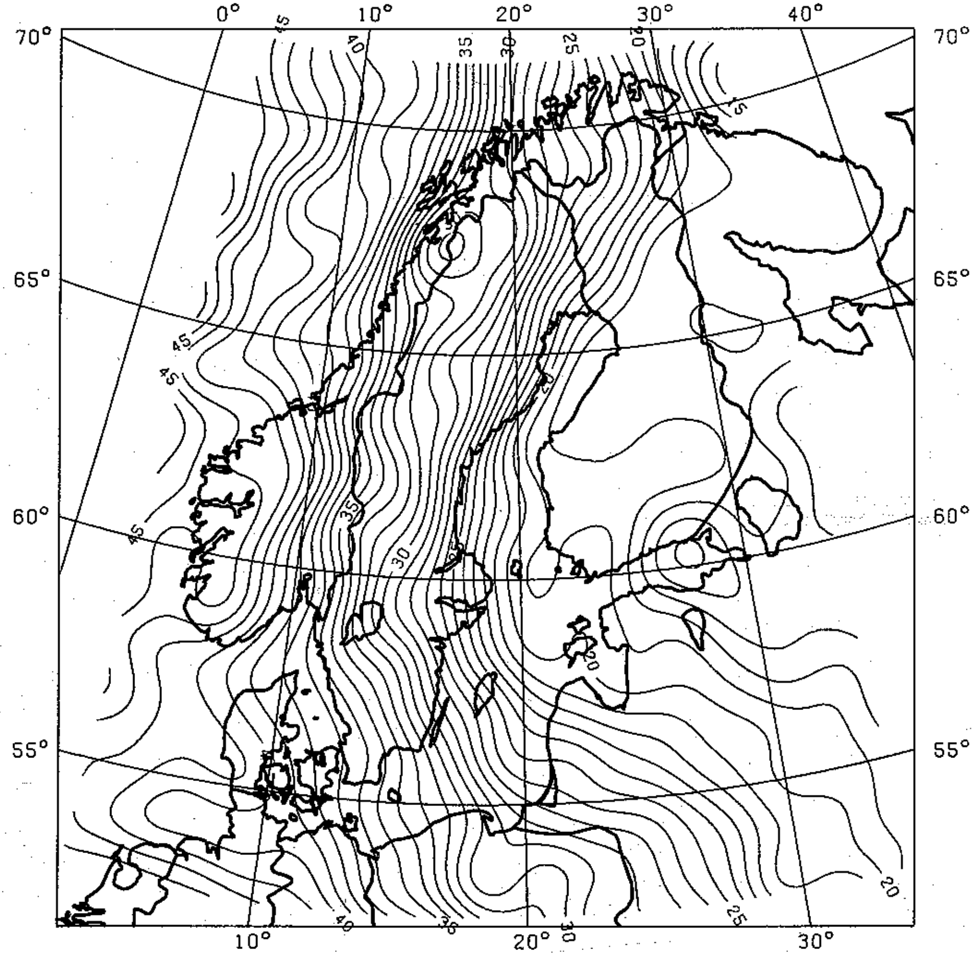

Fig. 4 — Geoid heights derived from GPM2 in GRS1980, equidistance 1 m.

extend up to 71° latitude. The actual number of points used is shown in the secor column of table 1. Fig. 6 shows the contribution to the predicted height anomaly fro GPM2 and RTM reduced gravity anomalies in one of the blocks, and fig. 7 the RT contribution.

The third column of table 1 shows the number of points as close as possible to tl nodes of a $3' \times 6'$ grid covering a $2''\!.8 \times 5''\!.6$ area surrounding the $2^\circ \times 4^\circ$ block. Tl points of this grid, which do not belong to the $6' \times 12'$ grid, will possibly be used in a improved solution, if they contribute significant new information. This is seen l predicting the values using the computed solution, and comparing the observed valu with the predicted.

This comparison is simultaneously an evaluation of the quality of the solution, ar the result, in the form of mean value and standard deviation of the difference, are give as the last two column of table 1. Observations, for which the (absolute) differen

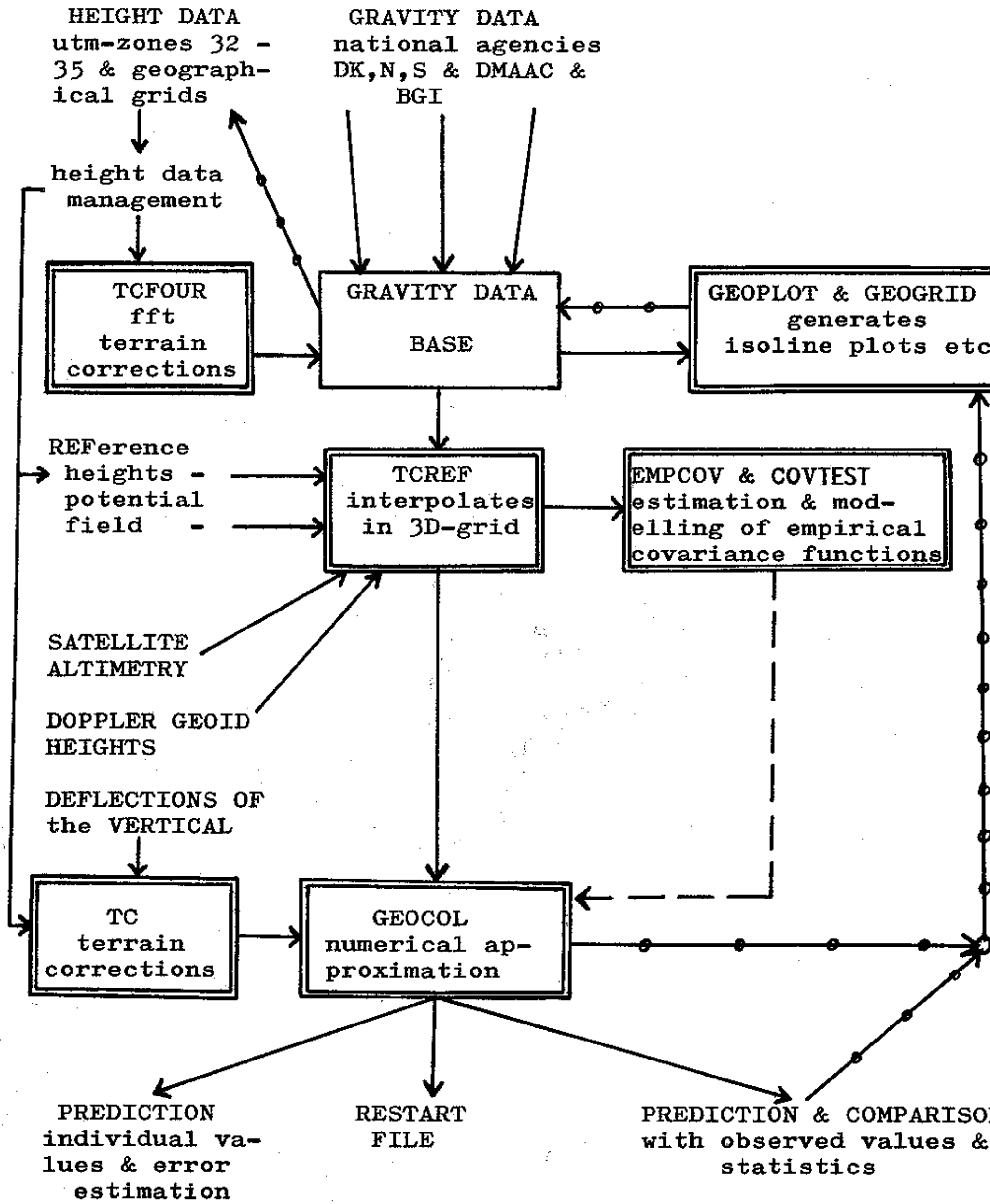

Fig. 5 – Data flow and program modules (with double lines used to form boxes). → means important data flow line for outlier detection. Not all connections are shown in order to preserve clarity.

TABLE 1

STATISTICS CONCERNING GRAVITY ANOMALIES. N = NUMBER OF OBSERVATIONS. STDV = STANDARD DEVIATION, UNITS MGAL

Block	N in grid with masks		Free-air gravity		Gravity-GPM2 - RTM		observed - predicted	
	6'*12'	3'*6'	mean	stdv	mean	stdv	mean	stdv
70 12	409	1008	18.1	51.1	7.7	25.2	2.0	11.0
70 16	770	1626	0.8	43.8	0.6	17.1	0.0	7.0
70 20	959	2113	6.8	32.3	1.8	21.0	-0.2	4.7
70 24	980	2385	7.8	23.9	0.0	14.9	0.1	3.3
70 28	477	1172	6.7	21.1	0.2	12.7	-0.1	2.5
68 12	758	1467	-3.1	52.1	1.0	21.7	0.4	8.9
68 16	739	1460	-1.0	39.7	-6.9	14.8	-3.0	6.3
68 20	838	2265	2.7	20.2	-0.8	14.9	-0.4	4.8
68 24	881	2453	-11.9	14.4	0.0	11.0	0.3	3.9
68 28	391	1146	-13.9	16.4	-1.5	11.1	-0.5	4.2
66 08	517	1024	18.0	23.8	3.7	15.3	0.1	5.7
66 12	634	1009	-0.9	32.3	-3.9	13.0	-0.4	4.0
66 16	608	1150	4.2	18.5	-1.4	11.3	-0.3	4.0
66 20	803	2152	-24.5	20.0	0.5	10.7	-0.2	5.0
66 24	895	2707	-20.4	13.2	-0.8	9.3	0.0	4.3
66 28	470	1409	-4.5	13.6	-1.4	10.6	-0.4	2.9
64 04	704	1587	11.4	41.2	-0.1	17.1	0.7	7.9
64 08	822	1604	17.4	43.5	-1.2	17.1	-0.2	7.6
64 12	637	1141	0.8	23.4	-3.2	13.2	-0.5	4.0
64 16	378	654	-1.7	23.2	0.8	11.2	0.0	4.2
64 20	675	2025	-28.4	15.7	0.3	8.9	0.1	4.0
64 24	897	2874	-7.4	14.5	0.1	7.6	0.0	2.8
64 28	577	1816	3.6	9.8	-0.8	9.1	-0.2	2.2
62 04	771	1558	18.5	53.8	1.6	15.7	1.3	6.9
62 08	858	1795	15.7	48.6	1.4	15.9	0.7	6.9
62 12	684	1334	-8.5	21.1	-2.6	11.7	-0.9	3.5
62 16	427	897	-23.8	18.0	-2.0	8.1	-1.0	4.9
62 20	646	2064	-15.9	17.0	-0.3	10.1	-0.3	4.1
62 24	728	2363	-9.1	18.7	-0.6	9.1	-0.2	3.0
62 28	321	881	-0.7	20.6	1.7	9.0	-0.1	3.0
60 04	628	1395	14.9	36.8	-1.2	15.1	-1.5	5.6
60 08	802	1890	8.5	27.9	0.3	14.0	0.4	4.9
60 12	709	1646	-11.0	15.2	-3.3	10.2	-0.7	2.8
60 16	500	1152	-22.2	17.2	-1.1	9.3	-0.4	4.6
60 20	272	684	-18.4	22.3	0.5	11.2	-0.8	5.0
58 08	799	2281	2.0	14.1	-3.8	11.2	-0.5	3.2
58 12	685	1739	5.0	14.5	-5.1	12.2	1.2	3.0
58 16	248	549	-1.7	15.2	0.8	9.6	-0.4	3.2
56 08	613	1929	7.5	12.5	-3.8	9.8	-1.0	2.8
56 12	455	1239	2.4	14.3	-6.2	12.8	-1.1	3.5
Total		63643	-2.0	29.7	-0.8	13.4	-0.2	4.9

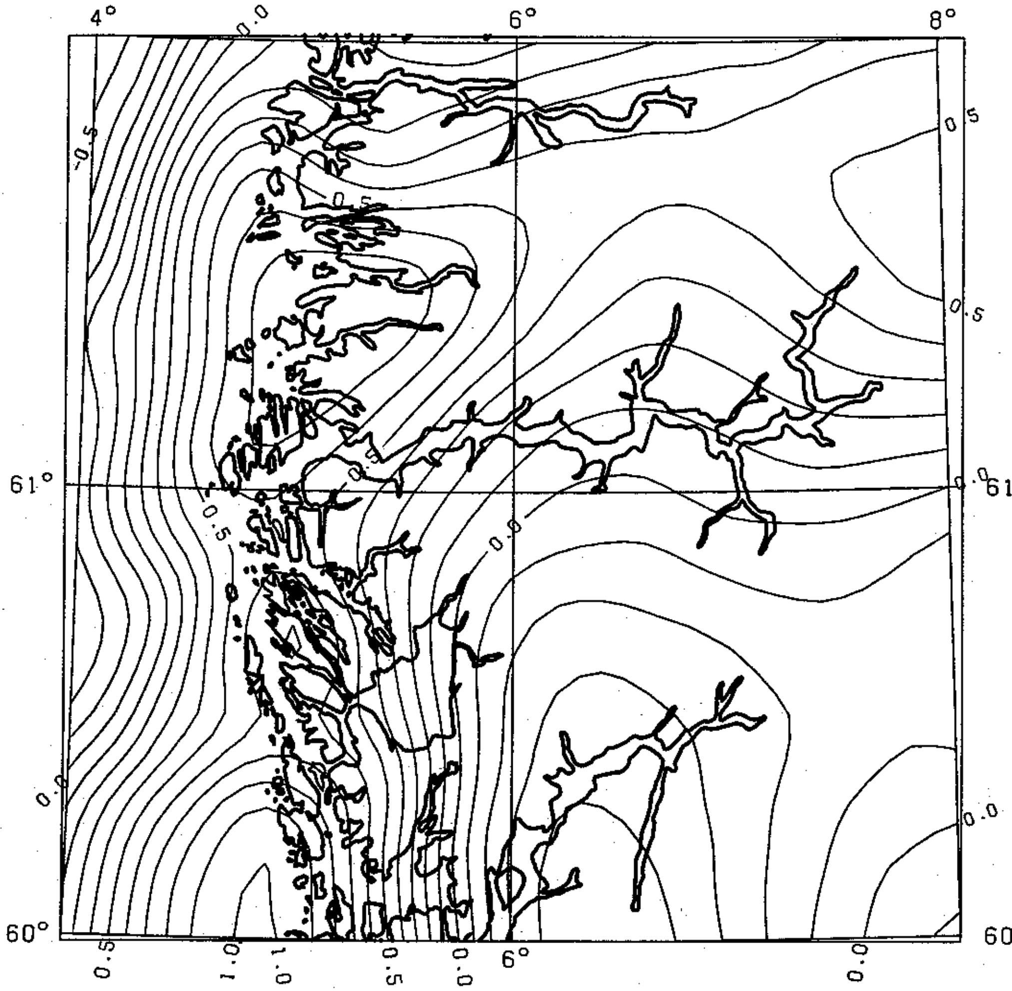

Fig. 6 – Gravity contribution for one block (6204). Equidistance 0.25 m.

between observed and predicted values exceed 10 mgal will be used as new data i updated solution, as long as these observations are not identified as containing gross errors. In the same way large differences may expose observations belonging t the 6′ × 12′ grid, which contain gross errors. Such observations will be eliminated fron the solutions. Note, that if none of the original observations have to be changed, the the reduced normal equations produced when evaluating eq. (5) may be reused. New columns are simply added to the equations, and the reduction continue from the firs new column. The GEOCOL-program is designed so that it may take advantage of this

A further evaluation of the quality of the solutions was made by comparing observed and computed deflections of the vertical and sea-surface heights obtained fron SEASAT, cf. section 3.4 and 3.6. The result of the comparisons are given in table 2 and 3

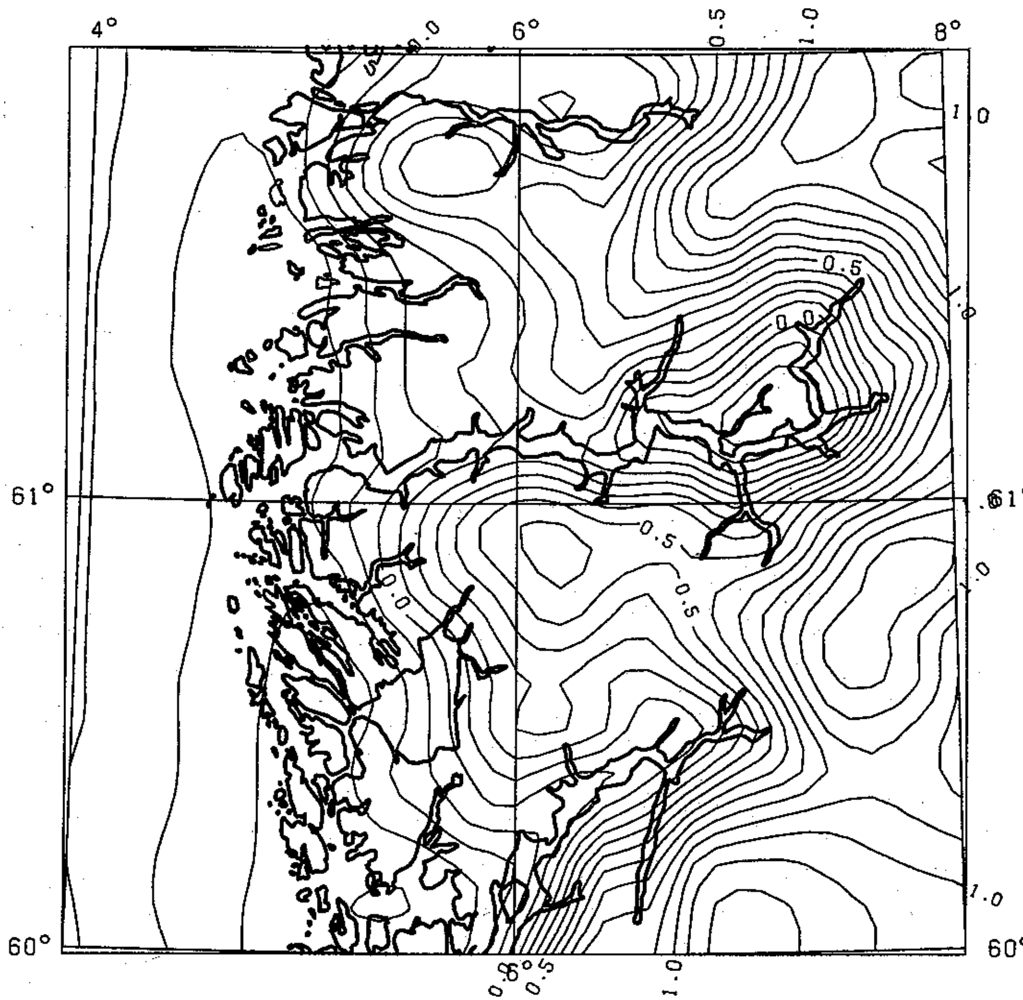

Fig. 7 — RTM-contribution for one block (6204). Equidistance 0.1 m.

respectively. The deflections were transformed to an approximate geocentric system using the parameters recommended by Ordnance Survey (1981), with a further shift of $\Delta Z = 3.0$ m and a rotation of the meridian plan of $-0''.8$ for DK, N and S and $3''.2$ for SF.

Since a main purpose was the computation of a «standard» geoid, we computed quasi-geoidal heights (i.e. height anomalies evaluated at the height of the Earth's surface) in a $6' \times 12'$ grid for each block. The values were transformed to GRS1980, by a datum shift corresponding to the 1 m difference in semi-major axis. A plot of the quasi-geoid is shown in fig. 8.

Formal error estimates were computed using eq. (6). The estimates (standard deviation) varied between 8 cm and 30 cm. The 8 cm occuring in areas with a

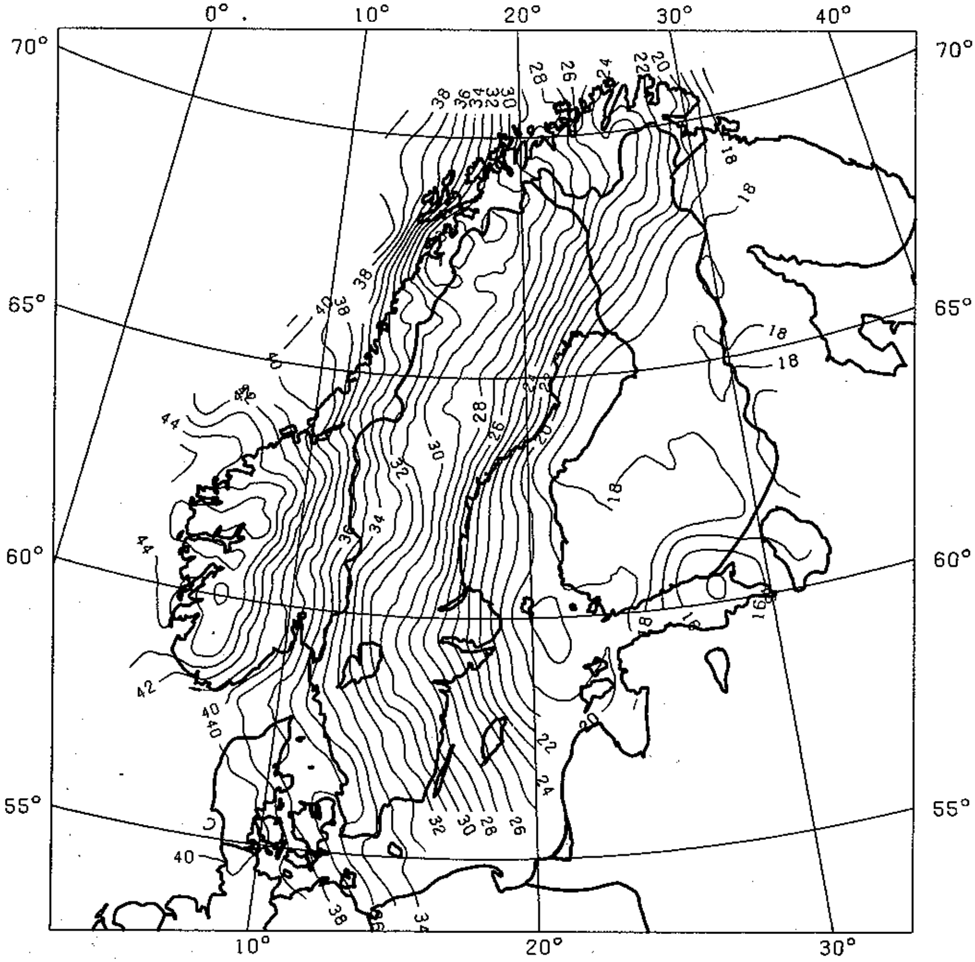

Fig. 8 – Quasi-geoid for DK, N, S and SF, in GRS1980, equidistance 1 m.

dense (6′ × 12′) data coverage, and the 30 cm in area with no data. The error estimat may be scaled using as scale factors the ratio between the actual standard deviation the gravity data, cf. table 1, column 7 and the model standard deviation of 15 mgal. The ratios, which vary between 0.5 and 1.8, correspond quite well to the differences seen between the blocks in the results given in table 2 and 3. However, for the compariso with SEASAT-data, the large differences in biases (mean values of the observed min computed value) is disturbing. A part of the bias is explainable by the uncertainty semi-major axis and GM. Another part is due to sea-surface topographic effects caus by wind effects (M. Vermeer, private communication).

From the various comparisons, one may especially notice the comparisons wi deflections of the vertical. Here the «best» blocks have standard deviations of t differences of typically 0″6, corresponding to blocks with standard deviations

TABLE 2

STATISTICS CONCERNING SEASAT DATA. N = NUMBER OF OBSERVATIONS, STDV = STANDARD DEVIATION, UNITS METERS

Block	N	Observed values		Observed - GPM2 - RTM		Observed - predicted	
		mean	stdv	mean	stdv	mean	stdv
70 12	970	37.65	1.10	-0.15	0.43	-0.11	0.30
70 16	618	32.94	2.07	-0.21	0.46	-0.25	0.31
70 20	234	27.90	1.06	0.51	0.69	-0.19	0.50
70 24	207	22.48	1.29	-0.58	0.41	-0.75	0.42
70 28	286	18.40	0.87	-0.76	0.50	-1.02	0.55
68 12	194	35.47	2.23	0.05	0.61	-0.22	0.32
66 08	432	39.19	0.74	-0.46	0.43	-0.46	0.21
66 20	220	18.71	0.95	0.08	0.30	0.20	0.21
66 24	42	18.18	0.62	0.39	0.34	0.31	0.33
64 04	380	43.61	1.03	-0.16	0.53	-0.20	0.20
64 08	29	40.71	0.50	-0.46	0.46	-0.42	0.44
64 16	114	21.39	1.21	-0.13	0.40	0.16	0.43
64 20	139	19.05	0.85	0.23	0.51	0.19	0.38
62 04	79	44.39	0.54	-0.15	0.37	-0.21	0.21
62 16	280	21.41	1.37	-0.07	0.26	-0.08	0.23
62 20	82	19.55	0.42	0.05	0.30	0.04	0.28
62 24	45	15.88	0.88	0.29	0.54	0.13	0.56
62 28	3	15.66	0.06	0.65	0.05	0.66	0.03
60 04	99	42.56	0.87	-0.33	0.50	-0.46	0.16
60 08	82	37.73	0.69	-0.51	0.47	-0.47	0.21
60 16	258	22.23	2.01	-0.20	0.34	-0.22	0.21
60 20	241	19.14	0.74	-0.39	0.42	-0.32	0.29
58 08	158	37.88	0.88	-0.59	0.27	-0.46	0.17
58 16	254	24.96	2.51	-0.73	0.42	-0.65	0.35
56 08	79	38.52	0.79	-0.87	0.40	-0.69	0.37
56 12	188	34.85	1.36	-0.63	0.34	-0.48	0.25
Total	5713	32.09	1.03	-0.25	0.54	-0.28	0.43

predicted $3' \times 6'$ gravity of 3 mgal. This again correspond to geoid standard deviations of maximally 30 cm over 100 km. The comparisons with SEASAT data indicates that values of 20 cm are realistic. In mountaneous regions the standard deviations of the differences are 7-8 mgal for gravity data. Hence a factor of $8/3 \times 20$ cm ≈ 50 cm is an upper limit for the error of geoid undulation differences over 100 km in these areas. The error is probably smaller, since the gravity prediction error has a high frequency

TABLE 3

STATISTIC FOR DEFLECTIONS OF THE VERTICAL. N = NUMBER OF OBSERVATIONS, DEFLECTIONS FROM DK, N AND S, B: DEFLECTIONS FROM SF. STDV = STANDARD DEVIATIC OBSERVED = VALUES IN ED1950 WITH NO DATUM TRANSFORMATION, OBS. – PRED. OBSERVED MINUS PREDICTED FROM THE SOLUTION IN EACH BLOCK WITH DATUM SHI DEPENDING ON WHETHER DATA BELONGS TO GROUP «A» OR TO GROUP «B». A «*» INDICAT THAT THE BLOCK IS MOUNTANEOUS. ALL UNITS ARCSECONDS

Block		N	Meridian component				Prime vertical compon.			
			Observed		Obs.-pred.		Observed		Obs.-pred.	
			mean	stdv	mean	stdv	mean	stdv	mean	stdv
70 12	a	1	-6.6	0.0	-0.4	0.0	10.9	0.0	1.9	0.0
70 16 *	a	12	-3.3	8.6	-0.8	3.8	3.0	4.5	-1.5	2.4
70 20 *	a	2	1.6	2.8	0.3	1.6	3.6	1.3	-0.6	0.8
70 20 *	b	2	-0.3	1.4	1.7	0.7	8.6	1.8	1.0	0.6
70 24 *	b	13	-3.0	3.0	-0.1	1.1	4.6	3.4	0.2	0.9
70 28 *	a	1	-0.5	0.0	0.2	0.0	3.8	0.0	0.5	0.0
70 28 *	b	5	0.0	1.7	0.9	0.6	6.3	3.0	0.2	1.0
68 12	a	2	-3.3	4.9	0.7	1.1	3.6	0.8	-1.7	0.2
68 16 *	a	1	-3.0	0.0	0.9	0.0	5.2	0.0	0.9	0.0
68 20	a	11	-4.0	2.0	-0.6	1.4	5.2	1.8	0.3	0.7
68 20	b	4	-4.4	0.6	0.3	0.9	3.7	1.3	-0.6	0.7
68 24	b	18	-3.8	1.8	-0.2	1.0	3.7	2.3	0.0	0.9
68 28	b	8	-0.6	2.6	-0.2	1.1	1.6	2.0	-0.6	0.9
66 08	a	2	1.8	2.8	1.2	0.1	-0.2	6.7	-2.6	2.4
66 12 *	a	2	-2.8	1.4	-1.2	1.0	1.5	9.2	-1.7	3.5
66 16	a	9	-3.7	1.6	-0.4	1.2	3.2	3.7	0.1	1.2
66 20	a	7	-5.2	0.8	-0.8	0.5	7.6	1.8	-0.1	0.8
66 24	b	33	-1.4	2.1	-0.4	0.8	-1.0	2.3	-0.5	1.2
66 28	b	17	-0.5	0.9	-0.3	0.6	-1.5	1.7	0.1	0.6
64 04	a	6	-0.2	3.1	1.0	1.4	-1.2	3.1	-1.6	1.4
64 08 *	a	6	1.8	4.4	-0.5	1.0	3.3	5.2	0.2	1.7
64 12 *	a	15	0.9	1.1	-0.7	1.7	5.0	3.5	1.0	2.3
64 16	a	10	-5.0	3.2	-0.2	1.0	9.7	4.6	1.3	1.6
64 20	b	27	0.0	2.1	-0.1	0.8	-0.4	1.6	-0.7	0.7
64 24	b	36	0.0	1.3	0.0	0.6	-1.2	1.1	-0.4	0.6
64 28	b	26	-0.2	1.2	-0.2	0.7	1.0	1.6	-0.3	0.9
62 04 *	a	14	-3.4	6.6	-1.2	3.9	-4.0	5.9	-1.7	2.7
62 08 *	a	13	-1.8	3.6	0.0	1.9	5.8	4.0	-1.1	2.9
62 12	a	18	-4.0	2.1	-0.4	1.4	5.5	3.3	0.8	1.8
62 16	a	10	-1.9	1.0	0.1	0.9	8.8	1.4	1.3	1.4
62 20	b	42	-1.4	1.7	-0.4	0.5	-0.8	1.7	-0.6	0.6
62 24	b	52	-1.4	2.6	-0.1	0.7	1.5	2.6	-0.3	0.7
62 28	b	19	-5.0	2.4	-0.6	0.9	-1.2	1.6	-0.1	0.7
60 04 *	a	22	-7.4	4.1	-0.6	1.2	-3.2	4.7	-0.9	2.2
60 08 *	a	29	-3.4	2.7	-0.7	1.2	2.8	3.5	-0.8	2.3
60 12	a	23	-1.9	2.3	-0.9	1.3	4.7	2.8	0.3	1.6
60 16	a	11	0.3	1.8	-0.1	1.3	5.6	1.8	0.5	1.0
58 08	a	15	-0.4	1.3	-0.7	0.5	0.5	1.9	0.2	0.8
58 12	a	20	-0.4	2.4	-0.6	1.6	2.0	2.5	0.2	1.7
58 16	a	7	0.2	2.4	-0.7	1.1	5.4	1.6	0.0	0.9
56 08	a	19	-2.4	2.3	-0.4	0.6	0.3	2.2	0.3	0.8
56 12	a	10	-3.5	2.9	-0.4	1.2	0.4	2.9	0.2	1.0
Total:										
DK,N,S	a	298	-2.5	2.8	-0.5	1.2	3.0	3.8	-0.2	1.5
SF	b	302	-1.4	2.0	-0.2	0.9	0.5	2.4	-0.3	0.9
All	a+b	600	-1.9	3.3	-0.3	1.3	1.7	4.1	-0.2	1.5

character and the geoid error has a longer wavelength. A value of 40 cm in the mountains is probably realistic.

A disturbing feature of earlier solutions, see e.g. Tscherning (1985, fig. 4) has been the inconsistency of local solutions along the boundary. F. Sansò has suggested (in a private communication) that this was due to the use of different covariance fuctions fo each block. However, since the empirical covariance functions were wildly differen when no height information was used, we had to live with this fact. Now, the statistica characteristics are quite similar from block to block as discussed in (Forsberg, 1986). A least we dared to select the same model for all blocks, knowing that the difference in variance would be accounted for by the data, thus not influencing the prediction bu only the error estimate eq. (6).

While earlier solutions had inconsistencies along block boundaries of up to 40 cm the differences in between the new solutions had decreased to maximally 21 cm (with a standard deviation of 5 cm). This correspond well to the improvement in ability to predict gravity, compare the results in table 1 with table 5 of (Tscherning, 1983).

5. – CONCLUSION

The here presented solution may be somewhat improved. First of all gravity data should be added in areas with a strongly varying gravity field. Detailed heigh information should be used not only in Norway, but also in Sweden and the northernmost part of Finland.

Some coastal regions and the large lakes of Sweden lack gravity data. Deflections o the vertical may aid us in extrapolating the geoid from the coast to the nearby sea o allover the lake. However these data must be carefully checked before use, especially fo systematic errors.

Especially in the Baltic Sea gravity data is missing. Here we may use the SEASAT data as described in (Tscherning and Knudsen, 1986). Also these data must as far a possible be corrected for sea-surface topography.

At sea, it has not been possible to smooth the gravity data (see block 7012 and 6812 in table 1), because a large amount of sea gravity lack bathymetric information. This may probably in the future be obtained from syntetically produced bathymetric maps.

The collocation solutions computed for each block may easily be transferred to users, since they exist on digital form as so-called «restart» files. These files are simply used as input to GEOCOL. However a user must also be able to compute and add the topographic effects. These effects have been pre-computed for gravity and heigh anomalies in grids, which also may be made available to external users. Topographic effects for deflections of the vertical require access to the digital terrain models used, cf fig. 2. These vast models are not easily transferable to other users.

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