C.C. TSCHERNING
Geodaetisk Institut, København.

APPLICATION OF COLLOCATION FOR THE PLANNING OF GRAVITY SURVEYS

Abstract

Least squares collocation can be used to determine the density of a gravity survey, when the object of the survey is :

1. To produce a (free–air) gravity anomaly map, so that point gravity anomaly values can be interpolated with a standard error of $\pm X_1$ mgal ,

2. To interpolate deflections of the vertical with a standard error of $\pm X_2$ arc. sec. between astronomical stations Z km apart,

3. To compute an upward continuation of a point gravity anomaly to a height of Z meters with a standard error of $\pm X_3$ mgal ,

4. To compute mean gravity anomalies of block size Z degrees with a standard error of $\pm X_4$ mgal ,

5. To obtain a (local) gravimetric geoid with a standard error of $\pm X_5$ meters ,

6. To compute density anomalies at a depth of Z km with a standard error of $\pm X_6$ g/cm^3 .

1. Introduction

A gravity survey of a region may be made with the purpose of mapping the gravity variation and/or estimating one or more geophysical quantities from the observed gravity values.

In both cases, the survey must be planned so that the quantities of interest can be estimated with a specified standard deviation. In the first case, where we want to map the gravity variation, we may specify the density of the survey e.g., by requiring that free–air gravity anomalies can be interpolated everywhere within the region with a standard deviation of $\pm X_1$ mgal .

An estimate of the interpolation error may be determined for the linear least squares prediction method, Heiskanen and Moritz (1967, Section 7–6) by a simple equation, Ibid (1967, eq. (7–64)) :

$$m_p^2 = c_{pp} - \{ c_{pi} \}^T \{ c_{ij} \}^{-1} \{ c_{pj} \} , \qquad (1)$$

where m_p^2 is the standard error of least squares prediction, c_{pp} is the mean square variation of the gravity anomalies, $C = \{ c_{ij} \}$ is a $q \times q$ matrix of covariances between the observed gravity anomalies and $C_p = \{ c_{pi} \}$ is the q vector of covariances between the anomalv to be predicted and the observed quantities.

The equation (1) has been used to estimate the standard error of e.g. potential coefficients, height anomalies and deflections of the vertical by simply propagating the error through the equations relating these quantities to the gravity anomalies, namely the equations of Stokes and Vening–Meinesz, see e.g. Groten and Moritz (1964) and Heiskanen and Moritz (1967, sections 7–7 and 7–9).

This type of procedure has some disadvantages. Firstly, several integrals (with singular kernels) will have to be evaluated. Secondly, the procedure cannot be applied directly when data of a different kind (e.g. deflections and gravity anomalies) are used together.

However, the method of least squares collocation will furnish us with estimates of the standard error of prediction. These estimates are computed by an expression similar to (1). The difference is, that the covariances are not between gravity anomalies anymore, but between the quantities used for the estimation (the observations) and the quantity to be estimated.

In Section 2, we will present briefly the theory of least squares collocation as applied in physical geodesy and discuss the interpretation of the estimate of the error of prediction, furnished by this method.

We have applied least squares collocation to a set of point gravity anomalies using different covariance functions, and predicted other anomalies and deflections of the vertical. The computational procedure used is described and a comparison of the actual and the estimated error of prediction is presented in Section 3.

In the following Section 4, the results of some test computations are presented. Eight sets of ficticious point gravity anomalies spaced with variable density in a $1^\circ \times 1^\circ$ block have been used to compute estimates of the error of prediction for point or mean gravity anomalies, height anomalies and deflections of the vertical within the area.

In the final section we explain how the error estimates, obtained by the method of least squares collocation, can be used in the planning of a gravity survey.

2. Least squares collocation

By the method of linear least squares prediction, a gravity anomaly in a point P, Δg_p, is predicted from other gravity anomalies by determining constants a_i', so that

$$\{a_i'\} = C^{-1} C_p \quad \text{and then} \tag{2}$$

$$\widetilde{\Delta g}_p = \{a_i'\}^T \cdot \{\Delta g_i\} = C_p^T \cdot C^{-1} \cdot \{\Delta g_i\} \tag{3}$$

where Δg_i are anomalies observed in points $P_i (i = 1, ..., q)$ and $\widetilde{\Delta g}_p$ is the predicted value.

We could instead determine constants

$$\{a_i\} = C^{-1} \{\Delta g_i\}, \tag{4}$$

in which case the expression

$$\widetilde{\Delta g}_p = C_p^T \cdot \{a_i\} \tag{5}$$

can be regarded as a *functional* representation of the gravity variation for the point P varying, (the single covariances c_{pi} are functions of P).

The determination of a functional expression instead of the estimation of a single quantity is the main difference between least squares prediction and least squares collocation. In fact, the latter method includes the first one, because naturally, the functional expression (5) can be evaluated in a point Q, giving us the estimated value $\widetilde{\Delta g}_Q$.

The statistical model behind the prediction method did originally regard only one kind of random variables, namely the random variables associated with the gravity anomalies. They were supposed to form a stochastic process with the points on the surface of the Earth as an index set.

In least squares collocation, the gravity anomalies and other related quantities such as deflections of the vertical (ξ, η), height anomalies (ζ) or density anomalies (ρ) are all associated with different random variables, Y_s, but associated with the same sample space H, i.e. for the element T of H:

$$Y_{\Delta g_p}(T) = \Delta g_p, \quad Y_{\xi_p}(T) = \xi_p \quad \text{or} \quad Y_{\rho_p}(T) = \rho_p \tag{6}$$

(by ξ_p and ρ_p we mean the value of ξ or ρ in the point P).

The sample space, H, may consiste e.g. of a set of density anomaly distributions and their related potential functions. The covariance between random variables associated to quantities of a different kind can be defined in this statistical model. (See e.g. Grafarend (1973 a, b) and Lauritzen (1973) for a detailed discussion of the statistical models).

We have here a statistical model which enables us to combine different kinds of data. Deflections of the vertical may be used together with gravity anomalies for the prediction of other gravity anomalies, (see Moritz (1972) for a further discussion of this point).

We will denote the covariances between two random variables Y_{s_1} and Y_{s_2} by $cov(s_1, s_2)$. The covariances c_{ij} and c_{ip} of equation (1) are in this notation equal to $cov(\Delta g_i, \Delta g_j)$ and $cov(\Delta g_p, \Delta g_i)$.

Instead of determining the functional representation of the gravity variation (5), a functional representation of either the anomalous potential T or the related density distribution ρ can then be determined (cf' e.g. Krarup (1969), Tscherning (1973a, 1974a, b)).

Using a set of data $x = \{x_i\}$, $i = 1, \dots, q$, the functional representations become

$$\tilde{T}(P) = \sum_{i=1}^{q} cov(T_p, x_i) \cdot b_i \quad \text{or} \tag{7}$$

$$\tilde{\rho}(P) = \sum_{i=1}^{q} cov(\rho_p, x_i) \cdot b_i, \quad \text{with} \tag{8}$$

$$\{b_i\} = \{cov(x_i, x_j)\}^{-1} \cdot \{x_j\}.$$

These functions can then, as mentioned above, be used for the computation (prediction) of an estimate s of a quantity s, (e.g. a gravity anomaly, a deflection of the vertical or a geoid undulation). This is done simply by interchanging, in the equation (7) and (8), the covariance $cov(s, x_i)$ for $cov(T_p, x_i)$ and $cov(\rho_p, x_i)$.

The functions $\tilde{T}(P)$ and $\tilde{\rho}(P)$ will both have the property, that when the value of an observed quantity is predicted, the original value is recovered (contingently with some noise filtered out, in case the observations are not regarded as errorless). This is the reason why the method is denoted collocation.

The error of prediction of s (with related random variable Y_s) becomes

$$Er(s) = s - \tilde{s} = s - Y_s(\tilde{T}) = s - \sum_{i=1}^{q} cov(s, x_i) \cdot b_i . \qquad (9)$$

$Er(s)$ is a random variable, and it will have a variance equal to

$$m_s^2 = cov(s,s) - \{ cov(s, x_i) \}^T \{ cov(x_i, x_j) \}^{-1} \{ cov(s, x_j) \}, \qquad (10)$$

i.e. equal to the expression, eq. (1) for $s = \Delta g_p$ and $x_i = \Delta g_i$.

The usefulness of this quantity depends on our knowledge of the probability measure of the sample space H. In case all the random variables considered had a one dimensional Gaussian distribution, we could interpret m_s^2 in the usual way as the variance of the Gaussian distributed random variable $E(s)$.

Unfortunately, we cannot simply estimate the covariance functions and in this way find the most important probabilistic properties. This is because we only have in hand one member of the sample space, the potential of the Earth. Generally a hypothesis of isotropy is introduced, which enables us to regard the potential of the Earth, rotated around the origin, but not translated, as a new element of the sample space (see Lauritzen (1973)).

The covariance functions of e.g. gravity anomalies or height anomalies will then only depend on the spherical distance and the distance from the origin of the points. The estimation can be performed, e.g. by computing sample means of products of gravity anomalies which are measured in points with the spherical distance ψ between the points within certain sampling intervals.

$$\psi_i < \psi \leq \psi_{i+1}, \; i = 1, \ldots, h$$

These empirical covariance functions may be mathematically represented by Legendre series, see e.g. Tscherning (1972) or Tscherning and Rapp (1974).

There are then two important questions, which must be answered : How sensitive are the predictions and how sensitive are the error estimates (10) to changes in the covariance functions. We shall not try to answer these questions here, but we will in the next section, give an example which illustrates the sensitivity.

3. An example of the dependency of the prediction results and the error estimates on the covariance function

We have chosen to regard an area in the State of Ohio, U.S.A., where the necessary data already had been collected and used in an earlier investigation, cf. Tscherning (1973 b).

Three sets of data formed the set of observations $x = \{ x_1, x_2, x_3 \}$.

x_1 = { a set of potential coefficients of degree less than 21 , cf. R.H. Rapp (1973, table 6) } .

x_2 = { 157 , $1^\circ \times 1^\circ$ mean gravity anomalies covering an approximate circular area of radius 10° and with center in a point with latitude $39^\circ.5$ and longitude 277° } .

x_3 = {117 point gravity anomalies, spaced with approximately 15' differences between the points in latitude and 20' difference in longitude, with the south–west point having latitude $38^\circ.2$ and longitude $275^\circ.0$}.

Two sets of observed quantities were then chosen as "test" values, namely 42 pairs of deflections of the vertical within the same area as covered by the dataset x_3 and 302 point gravity anomalies covering the same area, but spaced with half the distance between the points as was used for the dataset x_3 , cf. Figure 1.

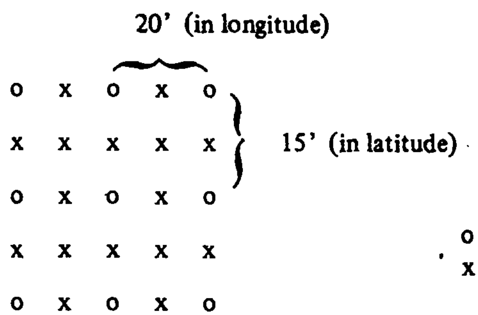

Fig. 1 – Situation of gravity observations and test points.

o Observed gravity anomaly
x Point of error estimation and prediction

This kind of prediction situation may be most efficiently handled using stepwise collocation, cf. Moritz (1973) and Tscherning (1974a). The datasets x_1 and x_2 are used to determine two harmonic functions $\tilde{T}_1$ and $\tilde{T}_2$,

$$\tilde{T}_1(P) = \sum_{i=1}^{q_1} cov(x_{1i}, T_p) \cdot b_{1i} = \{cov(x_{1i}, T_p)\}^T \{cov(x_{1i}, x_{1j})\}^{-1} \{x_{1j}\} \quad (11)$$

and

$$\tilde{T}_2(P) = \sum_{i=1}^{q_2} cov(d_1 x_{2i}, d_1 T_p) \cdot b_{2i} = \{cov(d_1 x_{2i}, d_1 T_p)\}^T \{cov(d_1 x_{2i}, d_1 x_{2j})\}^{-1} \{d_1 x_{2j}\} \quad (12)$$

where

$$d_1 x_{2i} = x_{2i} - \sum_{k=1}^{q_1} cov(x_{1k}, x_{2i}) \cdot b_{1k} \text{ and } d_1 T_p = T_p - \widetilde{T}_1(P) \quad (13)$$

i.e. $U_1 = U + T_1$ is used as a reference field for the dataset x_2, (U is a normal potential). In the same way a function $\widetilde{T}_3$ is determined from the observations x_3, now refering to a reference field $U_2 = U + \widetilde{T}_1 + \widetilde{T}_2$

$$\widetilde{T}_3 = \sum_{i=1}^{q_3} cov(d_2 x_{3i}, d_3 T_p) \cdot b_{3i} = \{ cov(d_2 x_{3i}, d_2 T_p) \}$$

$$\{ cov(d_2 x_{3i}, d_2 T_p) \}^{-1} \{ d_2 x_{3j} \} . \quad (14)$$

with

$$d_2 x_{3i} = x_{3i} - \sum_{k=1}^{q_1} cov(x_{1k}, x_{3i}) b_{1k} - \sum_{k=1}^{q_2} cov(d_1 x_{2k}, d_1 x_{3i}) \cdot b_{2k} \quad (15)$$

and

$$d_2 T_p = T_p - \widetilde{T}_1(P) - \widetilde{T}_2(P)$$

The final estimate of the anomalous potential becomes then

$$\widetilde{T} = \widetilde{T}_1 + \widetilde{T}_2 + \widetilde{T}_3 . \quad (16)$$

The covariance function of the residual observations $d_1 x_{ij}$, where the set of observations x_1 as here are potential coefficients, may easily be derived from a global covariance function, where the coefficients of degree less than 21 in a development of the function in a Legendre series has been set equal to zero (cf. e.g. Tscherning, 1974a).

The covariance of the residual observations $d_2 x_{3i}$ can not be directly derived from a global covariance function. We may instead estimate the covariance function of the residual anomalies as explained in section 2. (This has been done for the dataset x_3, cf. Figure 2). The covariance function will be a local covariance function, i.e. its first zero value ψ_1 will be much smaller than the corresponding value for the global covariance function used for the determination of $\widetilde{T}_1$ and $\widetilde{T}_2$. The empirical covariance function may be analytically represented as explained in Tscherning (1973) and Tscherning and Rapp (1974) by a I'th order local covariance function

$$cov(d_2 x_{3i}, d_2 x_{3j}) = \sum_{\ell=I+1}^{\infty} \sigma_\ell(\Delta g, \Delta g)\left(\frac{R^2}{r_i, r_j}\right)^{\ell+2} P_\ell(cos(\psi_{ij})). \quad (17)$$

where $\sigma_\ell(\Delta g, \Delta g)$ are the so called anomaly degree–variances of degree ℓ, R is the radius of the so called Bjerhammar sphere (i.e. the sphere bordering the set of harmonicity for the functions of the sample space H), r_i and r_j are the distances of the points of observation for x_{3i} and x_{3j} from the center of the sphere, respectively and ψ_{ij} is the spherical distance between the points.

By varying the degree I, it is possible to find an analytic model (17), which has the same first zero–point ψ_1 as the empirically determined function. The radius R and the anomaly degree variances may also be varied, so that it in fact is possible to find functions (17) which approximate the empirically determined function very well.

On the other hand, we may choose different values of I, $\sigma_\ell(\Delta g, \Delta g)$ and R and using these values obtain different estimates $\tilde{T}_3$. This is what we have done in order to illustrate the dependency of the predictions of the covariance function. We have regarded the following situations

$$I = 110, \quad R = 6369.7\text{ km}, \quad \sigma_\ell(\Delta g, \Delta g) = \frac{(\ell-1)}{(\ell-2)(\ell+24)} \cdot 65.9\text{ mgal}^2, \quad (17a)$$

$$I = 150, \quad R = 6369,7\text{ km}, \quad \sigma_\ell(\Delta g, \Delta g) = \frac{(\ell-1)}{(\ell-2)(\ell+24)} \cdot 73.5\text{ mgal}^2, \quad (17b)$$

$$I = 205, \quad R = 6369,7\text{ km}, \quad \sigma_\ell(\Delta g, \Delta g) = \frac{(\ell-1)}{(\ell-2)(\ell+24)} \cdot 83.3\text{ mgal}^2, \quad (17c)$$

$$I = 210, \quad R = 6363,0\text{ km}, \quad \sigma_\ell(\Delta g, \Delta g) = \frac{(\ell-1)}{(\ell-2)} \cdot 0.65\text{ mgal}^2. \quad (17d)$$

We will denote the functions (17), using these constants by (17a), (17b), (17c), and (17d).

The function (17a) gave the best approximation to the empirical covariance function. It is shown in Figures 2a as well. The three other functions are shown in Figure 2b.

The four functions were each used to determined a function $\tilde{T}_3$, and the predictions and corresponding error estimates were then computed for the two above described test sets of gravity anomalies and deflections of the vertical. The error estimates were computed from the dataset x_3 only, the two other datasets were regarded as not giving any contribution to the prediction error. The FORTRAN program described in Tscherning (1974b) was used for the computations.

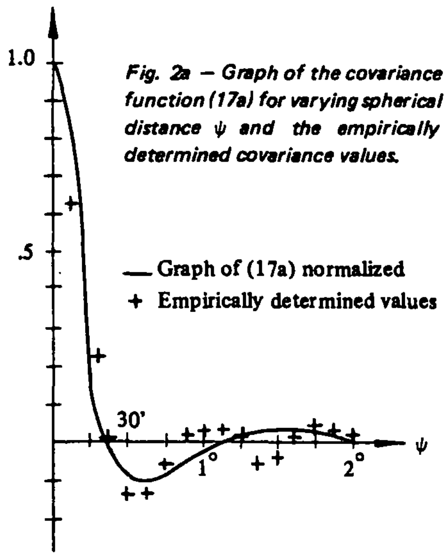

Fig. 2a – Graph of the covariance function (17a) for varying spherical distance ψ and the empirically determined covariance values.

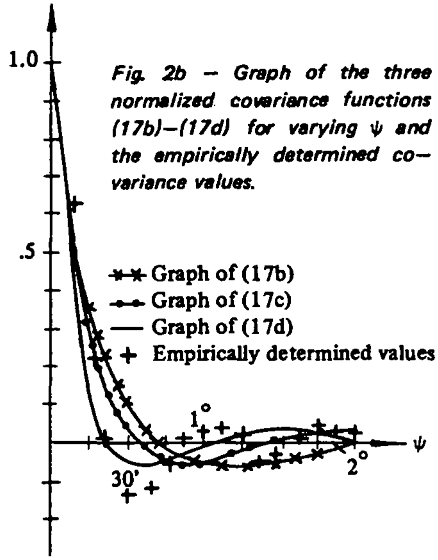

Fig. 2b – Graph of the three normalized covariance functions (17b)–(17d) for varying ψ and the empirically determined co–variance values.

The quality of the predictions can be evaluated e.g. by inspecting the distribution of the difference between the measured and predicted gravity anomalies and deflections and by comparing the means and variances of the differences corresponding to the different used covariance functions. Figures 3a and 3b show the two histograms of the distribution differences of the gravity anomalies and the deflections respectively as obtained using the covariance function (17a).

Histograms showing the distribution of the differences between observed and predicted :

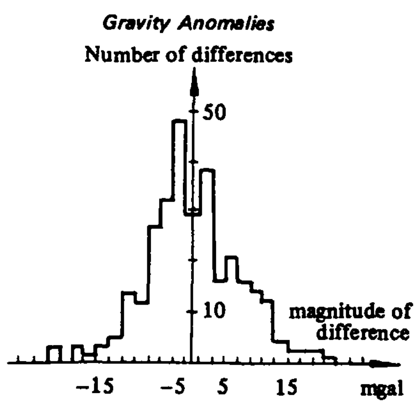

Fig. 3a

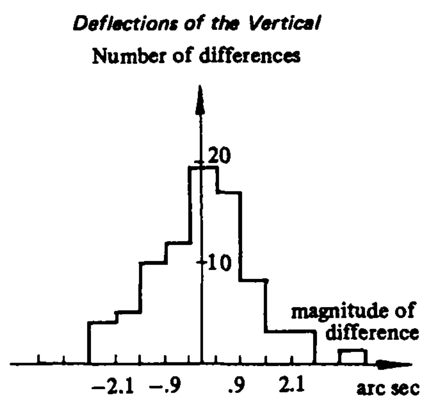

Fig. 3b

The mean and variances are presented in Table 1 corresponding to all four covariance functions. The evaluation of the quality of the estimates of the error of prediction is a more difficult case. We have chosen to regard the distribution of the ratio $Er(s) / \tilde{m}_s$ and the percentage of the differences being less than or equal to the estimated error of prediction. These quantities are presented in Table 1 as well.

Table 1

Comparison of prediction results and error estimates obtained using the covariance functions (17a)–(17d)

	Gravity Anomalies					Deflections of the Vertical						
	Er (s)		$\|Er(s)\|/\tilde{m}_s$			ξ Er (s)		η Er (s)		(ξ,η) $\|Er(s)\|/\tilde{m}_s$		
cov. func	mean mgal	stand. devia mgal	mean	stand. devia	Number of $\|Er(s)\| < \tilde{m}_s$ %	mean arc sec	stand. devia arc sec	mean arc sec	stand. devia arc sec	mean	stand. devia	Number of $\|Er(s)\| < \tilde{m}_s$ %
(17a)	0.5	7.3	0.73	0.71	77	0.0	1.1	−0.1	1.3	0.92	0.51	56
(17b)	0.5	7.2	0.59	0.52	80	0.1	1.2	−0.2	1.3	0.78	0.47	66
(17c)	0.8	7.3	0.57	0.50	82	0.0	1.2	−0.1	1.3	0.73	0.45	73
(17d)	1.0	7.6	0.56	0.49	83	−0.1	1.0	0.0	1.3	0.67	0.42	83

We may from Table 1 conclude, that the covariance function (17a) gives the best error estimates. This covariance function was also the one which approximated the empirical covariance function the best. The prediction results do not differ very much in the four cases.

Let us conclude this section by noting, that it seems to be of value to have a good estimate of the empirical covariance function in case we wish to use least squares collocation to obtain reliable error estimates. This may be a problem in itself, especially, because in an unsurveyed area, there is (naturally) no way of estimating the function. Hence, a preliminary covariance function may be used, which is estimated in an area with similar topographic and geological characteristics. But we will point out, that the covariance function will depend on the quality of the reference field used as well as on topographic and geological features.

4. Variation of the prediction error for varying spacing of the gravity measurements.

One of the main questions which arises in the planning of a gravity survey is how dense is it necessary to space the gravity observations, when we wish to estimate a quantity of a certain kind with a given standard deviation. Another related question is, how much information can then be gained by considering deflections of the vertical or height anomalies observed within the area as well.

The precision will depend on the information available in neighboring areas. This dependency will be expressed through the covariance function, which will have its first zero–point, ψ_1 , nearer to the origin for more available information.

We will here consider the percent–wise gain in precision for a varying density of the points of observation, i.e., we will not consider that more information available also means that the variance of the gravity anomalies decreases.

We have considered a fictitious $1^\circ \times 1^\circ$ square, firstly with only one gravity anomaly measured in each corner and then anomalies spaced more and more densely. We have then regarded points spaced with a distance of $\frac{1^\circ}{3}, \frac{1^\circ}{5}, \frac{1^\circ}{7}$, ..., up to $\frac{1^\circ}{15}$ in latitude and longitude between the points. The variation of the error of prediction of different quantities can then be computed using eq. (10) for the different spacing of the points of observation. We have estimated, (a) the height anomaly, (b) the gravity anomaly at the surface of the Earth, (c) the deflection of the vertical, (d) the gravity anomaly in 100 km's height and (e) the mean gravity over the square. The estimated error will naturally vary with the situation of the point of prediction within the square. We therefore estimated the error in four profiles, two going from the middle and to the boundary, and two going parallel to the boundary, see Figure 4. The error of prediction of a gravity anomaly will be zero, if we predict in a point of measurement. The biggest error will occur in the points situated most distant from the points of observation. The maximal error will occur in points on the boundary. The smallest "maximal" error will occur in the middle of the center subsquare, cf. Figure 4.

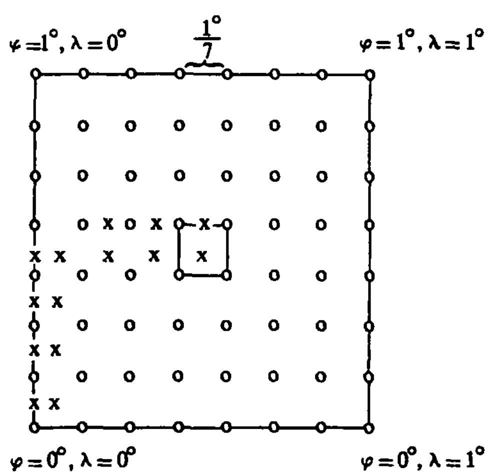

Fig. 4 – Distribution of fictitious observations and points of error estimation for a spacing equal to $\frac{1}{7}$ degree.

o Observed gravity anomaly
x Point of error estimation

(On the other hand, for the deflections, the maximal errors may occur in the points of observation because of the independence of the deflections and the gravity anomaly in the same point).

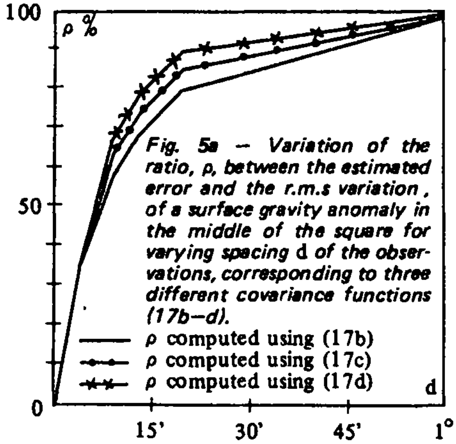

Fig. 5a – Variation of the ratio, ρ, between the estimated error and the r.m.s variation, of a surface gravity anomaly in the middle of the square for varying spacing d *of the observations, corresponding to three different covariance functions (17b–d).*

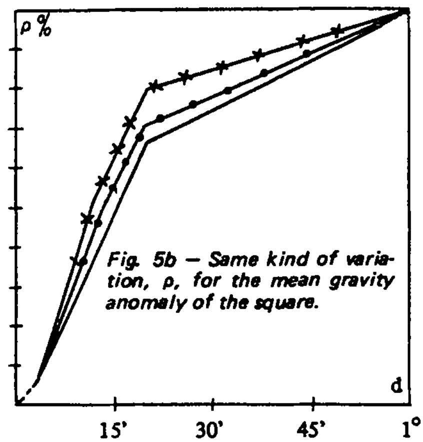

Fig. 5b – Same kind of variation, ρ, for the mean gravity anomaly of the square.

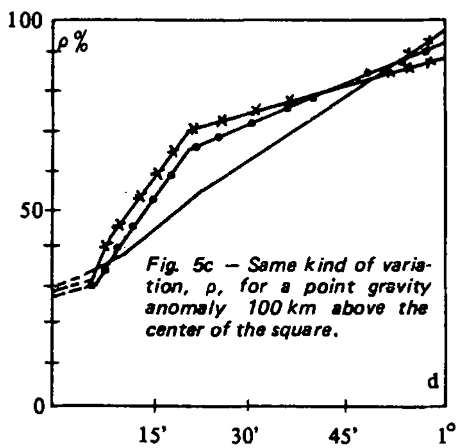

Fig. 5c – Same kind of variation, ρ, for a point gravity anomaly 100 km above the center of the square.

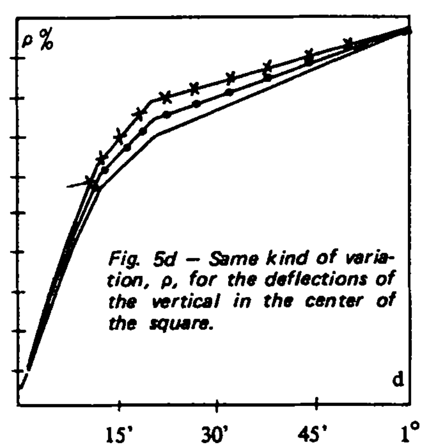

Fig. 5d – Same kind of variation, ρ, for the deflections of the vertical in the center of the square.

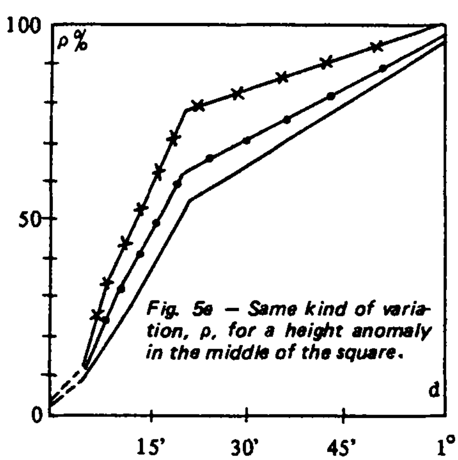

Fig. 5e – Same kind of variation, ρ, for a height anomaly in the middle of the square.

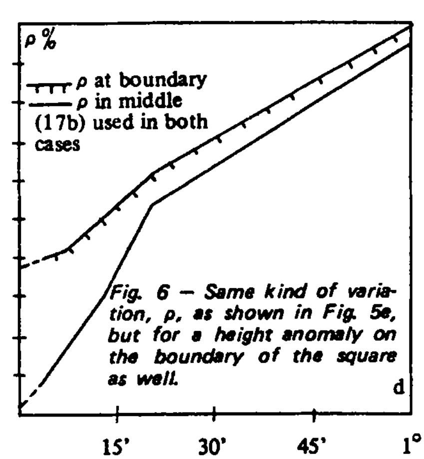

Fig. 6 – Same kind of variation, ρ, as shown in Fig. 5e, but for a height anomaly on the boundary of the square as well.

The variation of the normalized error of prediction, $\rho = (m_p^2 / c_{pp})^{\frac{1}{2}}$, due to varying spacing is shown in the Figures 5a–e for P lying in a favourable position (i.e. in the middle of the center square). The variation of ρ is shown corresponding to the three covariance functions (17b)–(17d) and for the above mentioned 5 different kind of estimated quantities. The variation of ρ is shown in Figure 6 for the height anomaly in the most unfavourable position, i.e. in a point on the boundary, and in a point in the middle of the area.

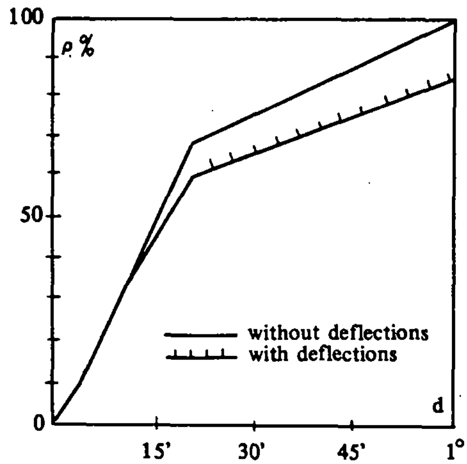

Fig. 7 – Variation of the ratio, ρ, for the mean gravity anomaly of the square for varying spacing d of the fictitious observed Δg with and without a pair of deflections in each corner. The covariance function (17b) was used.

We furthermore added to the observations four pairs of deflections in each corner of the square. The new error estimates of the center square are compared with the original ones in Figure 7 for the mean gravity anomaly.

By analyzing the figures, several conclusions can be drawn : (1) The addition of even a few deflections of the vertical may improve the estimated quantity considerably, when the gravity data are scarce. (2) A real gain in precision is first obtained, when the distance between the points becomes smaller than the spherical distance ψ_1 in which the first zero point of the covariance function occurs, (3) From a certain density it does not help to add new points within the area, in case quantities of a kind different from the observed are estimated. It only helps to add information in the surrounding area.

5. Application to the planning of gravity surveys

Let us suppose, that we are going to plan a gravity survey (i.e. determine the density of the points of observation) in a $1° \times 1°$ square. We will also suppose, that we have a fairly good picture of the height (or depth) variations in the area, and that we know the $1° \times 1°$ mean anomalies in the surrounding area.

It should then be possible to estimate the variance of the point gravity anomalies by considering a set of gravity anomalies in a similar, already surveyed area. This quantity may e.g. be used to scale the covariance function (17a) used in Section 3, and we have hereby obtained a first estimate of the covariance function. (It may be worthwhile to measure gravity values relatively densely spaced in a profile passing through the whole area and from this data get an estimate of the covariance function).

When a covariance function is available, it is possible, as done in Section 4, using fictitious sets of anomalies, to estimate the error of prediction for the quantities we wish to determine. By varying the density of the fictitious observations, it is possible to determine a density, which will result in a satisfactory estimation of the sought quantities. It may very well happen, that the gravity measurements will reveal, that the gravity variation within a part of the area is considerably bigger than the estimated variation. In this case, the new estimated variation can be used to scale the covariance function which is then used to determine a new density of the points of measurements in this particular area. The FORTRAN program described in Tscherning (1974a) may be used for the computation.

The program may, for example, be used to determine the density of the points of measurement, when we want to

(1) produce a (free–air) gravity anomaly map (or a corresponding functional representation) so that gravity anomaly values can be estimated with a standard error of $\pm X_1$ mgal,

(2) determine deflections of the vertical with a standard error of $\pm X_2$ arc. sec., when astronomical stations spaced with a distance Z km already are observed in the area,

(3) compute a point gravity anomaly in a height of Z meters with a standard error of $\pm X_3$ mgal,

(4) compute mean gravity anomalies of a block size Z degrees with a standard error of $\pm X_4$ mgal,

(5) obtain a (local) gravimetric geoid (or astrogravimetric geoid, when deflections are observed as well) with a standard error of $\pm X_5$ meters,

and hopefully, the program may be further developed, so that we also may be able to determine the point density, when we want to

(6) compute density anomalies at a depth of Z km with a standard error of $\pm X_6$ g/cm^3.

Acknowledgment

This work has been carried out while the author was visiting the Department of Geodetic Science of The Ohio State University. The author wishes to express his thanks to Doctors R.H. Rapp and U.A. Uotila for encouraging the work and for helpful advice.

Thanks is also extended to the Instruction and Research Computer Center of the Ohio State University for funds which supported certain computations and program developments described in this study.

o

o o

REFERENCES

E. GRAFAREND : Geodetic Prediction Concepts, Lecture Notes, International Summer School in the Mountains, Ramsau, 1973 (in press).

E. GRAFAREND : Geodetic Stochastic Processes, Lecture Notes, International Summer School in the Mountains, Ramsau, 1973 (in press).

E. GROTEN and H. MORITZ : On the Accuracy of Geoid Heights and Deflections of the Vertical, Institute of Geodesy, Photogrammetry and Cartography, The Ohio State University, Report No. 38, 1964.

W.A. HEISKANEN and H. MORITZ : Physical Geodesy, W.H. Freeman, San Francisco, 1967.

T. KRARUP : A Contribution to the Mathematical Foundation of Physical Geodesy, Danish Geodetic Institute, Meddelelse No. 44, 1969.

S.L. LAURITZEN : The Probabilistic Background of Some Statistical Methods in Physical Geodesy, Danish Geodetic Institute, Meddelelse No. 48, 1973.

H. MORITZ : Advanced Least Squares Methods, Dept. of Geodetic Science, The Ohio State University, Report No. 175, 1972.

H. MORITZ : Stepwise and Sequential Collocation, Dept. of Geodetic Science, The Ohio State University, Report No. 203, 1973.

R.H. RAPP : Numerical Results from the Combination of Gravimetric and Satellite Data using the Principles of Least Squares Collocation, Dept. of Geodetic Science, The Ohio State University, Report No. 200, 1973.

C.C. TSCHERNING and R.H. RAPP : Closed Covariance Expressions for Gravity Anomalies, Geoid Undulations, and Deflections of the Vertical Implied by Anomaly Degree Variance Models, Dept. of Geodetic Science, The Ohio State University, Report No. 208, 1974.

C.C. TSCHERNING : Representation of Covariance Functions Related to the Anomalous Potential of the Earth using Reproducing Kernels, The Danish Geodetic Institute, Internal Report No. 3, Copenhagen, 1972.

C.C. TSCHERNING : Determination of a Local Approximation to the Anomalous Potential of the Earth using "Exact" Astro–Gravimetric Collocation, Lecture Notes, International Summer School in the Mountain, Ramsau, 1973a (in press).

C.C. TSCHERNING : Problems and Results in Least Squares Collocation, Paper Presented at the Annual Fall Meeting of the American Geophysical Union, 1973b.

C.C. TSCHERNING : A Fortran IV Program for the Determination of the Anomalous Potential Using Stepwise Least Squares Collocation, Dept. of Geodetic Science, The Ohio State University, Report No. 212, 1974a.

C.C. TSCHERNING : Some Simple Methods for the Unique Assignment of a Density Distribution to a Harmonic Function, Dept. of Geodetic Science, The Ohio State University, Report No. 213, 1974b.