DETERMINATION OF BIAS PARAMETERS FOR SATELLITE ALTIMETRY BY LEAST-SQUARES COLLOCATION

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Abstract

Satellite altimetry from which biases due to orbit errors or oceanographic phenomena have been removed may be treated as geoid undulations. These may be used for the determination of other gravity field dependent quantities such as gravity anomalies and deflections of the vertical. The bias parameters are most frequently determined by a separate track cross-over adjustment.

Least-squares collocation may be used for a simultaneous determination of bias parameters and gravity field dependent quantities. Cross-over differences are used as a special type of gravity field independent observations, and may be used in combination with other data such as sea-gravimetry. Furthermore, the use of the method has the advantage that bias parameters may be determined without cross-over observations. However, it is shown that in this case the computed gravity field dependent quantities are identical to these obtained using differences between consecutive measurement on the same track.

The use of the method is illustrated using two sets of globally adjusted GEOS-3 and SEASAT data from the New England Sea Mount area and the South Pacific Sea, respectively. Geoid heights, gravity anomalies and deflections of the vertical were computed with and without bias determination. The use of bias determination removed 20 mgal errors clearly related to the track biases, this showing the importance of a bias removal.

In print: Proceedings 1. Hotine-Marussi Symposium, Rome, June 1985.

2. Use of least-squares collocation

The general observation equation is

$$x_i = L_i(T) + \sum_{j=1}^{m} A_{ij}\, X_j + e_i, \qquad i=1,\ldots,n \tag{1}$$

where x_i is the observation, L_i the linear functional associated with the observation, T the anomalous potential, $\{X_j\}$ the (m) parameters, e_i the error and A_{ij} the m-vector relating the parameters and i'th observation. If we only deal with bias parameters, the A_{ij} vector will consist of only zero's and one's.

The solution is

$$\tilde{T}(P) = C_P^T \bar{C}^{-1} (x - A\tilde{X}) \tag{2}$$

$$\tilde{X} = (A^T \bar{C}^{-1} A + R)^{-1} A^T \bar{C}^{-1} x \tag{3}$$

where C_P is the vector of covariances between T(P) and the observations, $\bar{C} = C + D$, with C the covariance matrix of the observations and D the variance-covariance matrix of the noise vector $\{e_i\}$. A is the matrix formed be the n m-vectors A_{ij}, and R is a possible a-priori weight-matrix for the parameters.

In principle, the determination of $\tilde{T}$ and $\tilde{X}$ requires that $\bar{C}$ and $A^T\bar{C}^{-1}A+R$ are matrices of full rank. This will generally be the case in practice, since the addition of D serves as a regularizing factor. However, we may be close to a singular situation, especially if we deal with many observations in a small area. If we have many track cross-overs, i.e. many dublicate measurements, then we are in a numerical unstable situation.

This is generally avoided using a data selection algorithm, e.g. selecting only one value per "cell", (e.g. 15' x 15'). But we may then throw away exactly the information, which permits a good parameter (bias) determination.

The solution is (cf. Tscherning (1978)) to permit observations without a $L_i(T)$-part, i.e. only dependent on the parameters. If we regard cross-over observations, then the difference becomes independent of T.

$$\tilde{X} = (A_2^T D_2^{-1} A_2)^{-1} A_2^T D_2^{-1} x_2, \tag{7}$$

i.e. only a part of the available information is taken into account. In the general eq. (3) the X-vector may be determined even without observations of the type x_2!

This is naturally of interest, if we want to know the bias parameters. However, this is sometimes not the case, and the effect of the parameters have been eliminated by using differences between consecutive measurements on the same track, see. e.g. Sandwell (1983). But this is completely equivalent to the introduction of a bias parameter. This is seen as follows in the case where we have only one parameter X and all observations on the same track. Eq. (1) becomes

$$x_i = L_i(T) + X + e_i \ , \qquad i=1,\dots,n \ , \qquad n > 1 \ . \tag{8}$$

The collocation solution is

$$\tilde{T}_0(P) = \sum_{i=1}^{n} b_i C_{Pi} = b^T C_P. \tag{9}$$

Now A is a n-vector with all elements equal to 1, so

$$\begin{aligned} b &= \bar{C}^{-1}(x - AX) \\ &= \bar{C}^{-1}(x - A(A^T\bar{C}^{-1}A)^{-1}A^T\bar{C}^{-1}x) \\ &= (\bar{C}^{-1} - \bar{C}^{-1}A(A^T\bar{C}^{-1}A)^{-1}A^T\bar{C}^{-1})x \end{aligned} \tag{10}$$

The relation between the original measurements and the differences may be expressed using the following matrix

$$H = \begin{Bmatrix} 1 & -1 & 0 \cdots 0 & 0 \\ 0 & 1 & -1 \quad 0 & 0 \\ \vdots & \vdots & \ddots \quad \ddots & \vdots \\ 0 & 0 & \cdots \quad 1 & -1 \end{Bmatrix} \tag{11}$$

the new observations being Hx. The collocation solution is

$$\tilde{T}_1(P) = c^T H C_P \tag{12}$$

ters to be solved for. Each time a new observation is input, the revolution and sequence number is given. If the revolution number has not been encountered before, then a new parameter is defined. A similar method may be used for sea-gravimetry, where each observation also frequently has associated a ship-track number and a sequence number, see e.g. Andersen (1966).

In certain situations we may, however, want to leave out a parameter or want to regard two parameters as being identical (e.g. two consecutive revolutions). In this case all "active" parameters must be specified before data input, but this is also done fairly simple.

The following parameter cataloque has been selected in order to save storage. The idea is best illustrated by an example. Suppose we have 3 data groups:

observation	1-3	dependent	on parameters 1 and 2
-	4-5	-	on no parameters
-	6-8	-	each on one parameter namely 5, 4 and 5.

The numbers 1,2,5 and 4 are supposed to be codes uniquely identifying a parameter. Since their sequence is important they are supposed to be stored in a dimensioned integer variable e.g. IPTYPE(50), with IPTYPE(1) = 1, IPTYPE(2) = 2, IPTYPE(3) = 5 and IPTYPE(4) = 4.

Then the catalogue is stored in a dimensioned integer variable IPACAT(500) as shown in Fig. 2.

IPACAT	1	2	3	4	5	6	7	8	9	10	11	12
	3	2	1	2	5	0	8	-1	5	4	5	0
Note	(c)	(b)	(a)		(c)	(b)	(c)	(b)		(a)		(d)

(a) Parameter codes

(b) Number of parameters associated with observation group ((b) positive), or each individual observation ((b) negative).

(c) Number (subscript) of last observation associated with the parameters given in the next (a) group.

(d) Sentinel, indicating end of catalogue.

Fig. 2. Sep up of parameter catalogue in GEOCOL.

```
      LEQP = MP .GE. 0
      MP = IABS(MP)
   60 IF (K .NE. IFIRST) GO TO 50
      LSAME = .FALSE.
C
      DO 40 II = 1, MP
   40 LSAME = LSAME .OR. (KT .EQ. IPACAT(IPA+II))
C
      IPA = IPA+MP
      IF (.NOT.LEQP) IFIRST = IFIRST+1
   50 IF (LSAME) A(I,K) = APARM(IKP,KT)
   10 CONTINUE
```

Before using the subroutine PARCAT and this piece of code, a preprocessing of the data is necessary.

In least squares collocation one typically will work with an area of size 200 x 200 km. A preprocessing of the data must identify all tracks passing through the area and thereby, through the track (revolution) number, give a unique identification of the parameter (for each satellite). This will give a list of numbers, which may be stored in IPTYPE for LALLP = true. A list of cross-over observations of type (4) may also be produced simply on the form: tracknumber_1, tracknumber_2, latitude, longitude and difference, and dublicate observations may be removed simultaneously.

Let us finally note that the A_1 and the $A_2^T D_2^{-1} A_2$ matrices will have many zero entries. This makes it suitable to use sparse matrix techniques for the determination of the Cholesky factor of eq. (6).

4. Test of bias determination

Satellite altimetry from GEOS-3 and SEASAT has been adjusted using cross-over differences as described in Liang (1983). This adjustment has removed the main part of the biases and tilts associated with the satellite orbit errors. However, there may exist local biases for example associated with oceanographic or atmospheric phenomena.

We therefore wanted to see

(1) how the bias removal would influence the prediction of gravity field dependent quantities,

(2) whether such local biases could be determined using least-squares collocation,

between the four neighbouring points. 3253 cross-over differences were found and before any adjustment their RMS value was 0.78 m.

Bias -parameters were both estimated separately using eq. (7) (with the supplementary condition that the sum of the biases is zero) and estimated using collocation as discussed above. Also a set of pre-adjusted sea-surface heights (with the separately estimated bias-parameters) were used in collocation without bias-estimation. In these computations were used all the (205) values located within the boundaries $38^{\circ}.25$ and $39^{\circ}.75$ in latitude and $295^{\circ}.25$ and $296^{\circ}.75$ longitude, and the (2377) cross-over differences, located on tracks where at least one sea-surface height was situated in this smaller area. 112 parameters were determined, see Fig. 4a, with standard deviations between 0.03 and 0.06 m. The bias-parameters estimated using the two methods agreed well, within ± 0.02 m except for a constant of 0.22 m. Correcting the cross-over differences using these parameters reduces the RMS value of the differences from 0.77 m to 0.58 m. The distribution of the RMS values for each track and the distribution of all cross-over differences are shown in Fig. 4b before and after adjustment. The geoid heights computed in combination with bias estimation was similar to the values computed using the pre-adjusted sea-surface heights except for the constant of 0.22 m. This displacement of the sea-surface in the combined collocation calculations makes this surface average the reference GPM2 geoid surface. The predicted free-air gravity and the deflections of the vertical in the area are similar. However the picture of these results (see Fig. 6, 8, 10 and 12) has become much smoother was compared to the previous results.

We also left out all cross-overs associated with a tracks passing through the middle of the area. The results of the estimated bias as 0.60 cm as compared to 0.62 cm when cross-overs were used.

In the Pacific area we also carried out the computation with and without cross-over differences. This area had originally been selected because problems had occurred caused by biases in the satellite orbits (R.H. Rapp, personal communication). In both computations we used the same 293 sea-surface heights in the area bounded by $-39^{\circ}.5$ and $-35^{\circ}.5$ in latitude and $205^{\circ}.5$ and $210^{\circ}.5$ in longitude. 85 cross-over differences were used. The free-air gravity anomalies computed in the area bounded by $-36^{\circ}.5$ and $-38^{\circ}.5$ in latitude and 206° and 208° in longitude are show in Fig. (13)

and thereby the task of solving a system of equations (6) as large as the number of observations plus the number of parameters. The determination of other parameters, like orbit tilt, requires data on quite a large part of a track in order to be well determined. However, a comparison of the bias values for the same track, but determined in different areas, should give an indication of a possible tilt, and also give a straightforward possibility for its determination.

Acknowledgement

This paper is a contribution to a project supported by the Danish Space Board.

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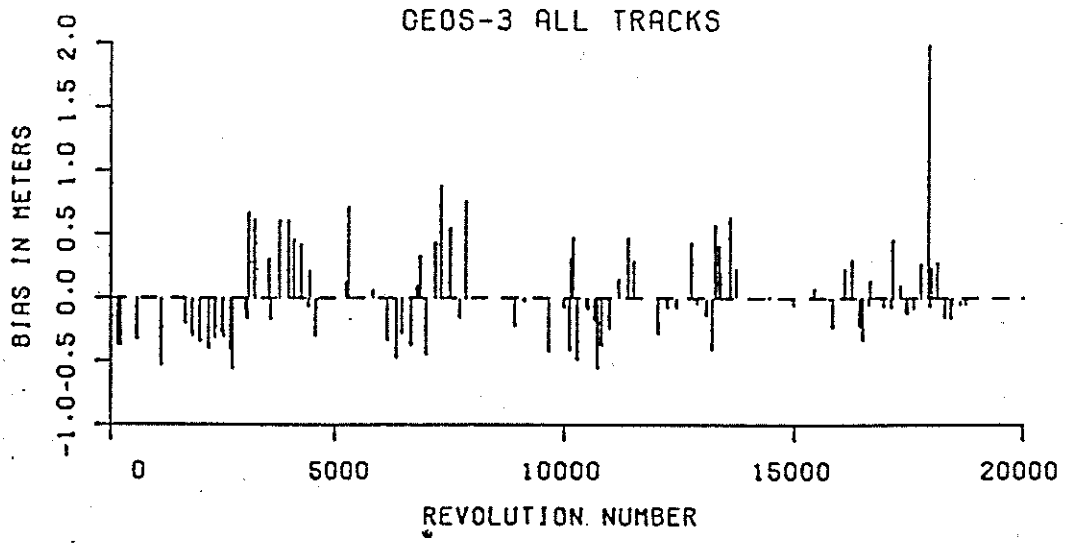

Fig. 4a. Biases for GEOS-3 tracks in New England Sea Mont Area.

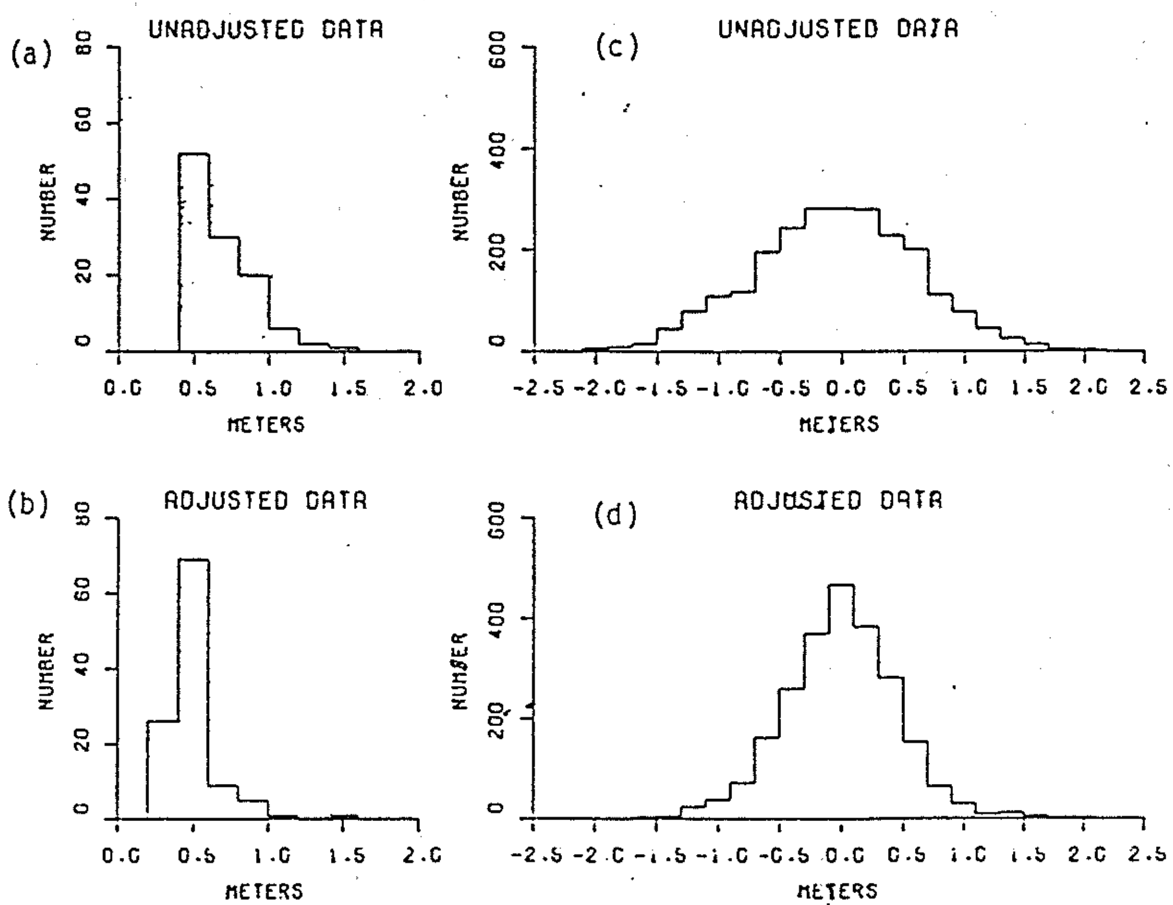

Fig. 4b. Histogram of RMS variations for all tracks before (a) and after adjustment (b), and of Cross-over differences before (c) and after adj.(d).

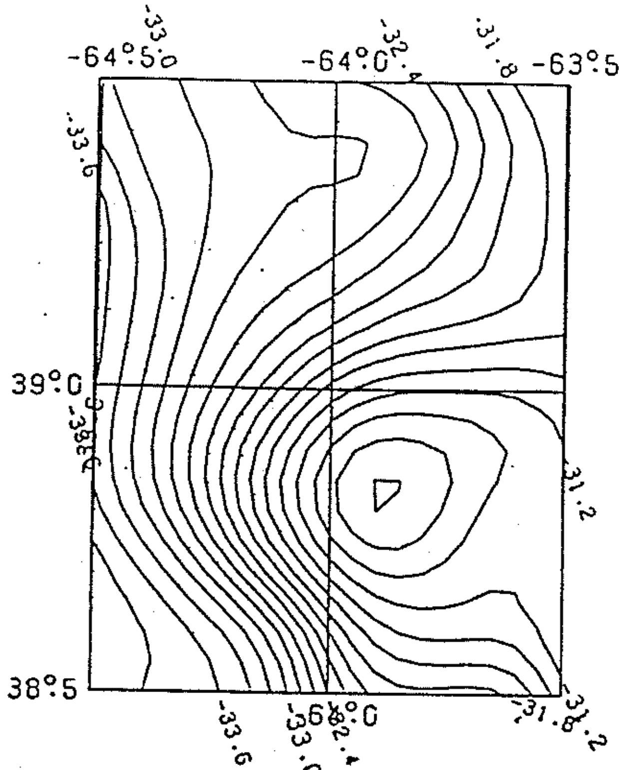

Fig. 9. Geoid heights predicted without bias removal. Contour interval o.2m.

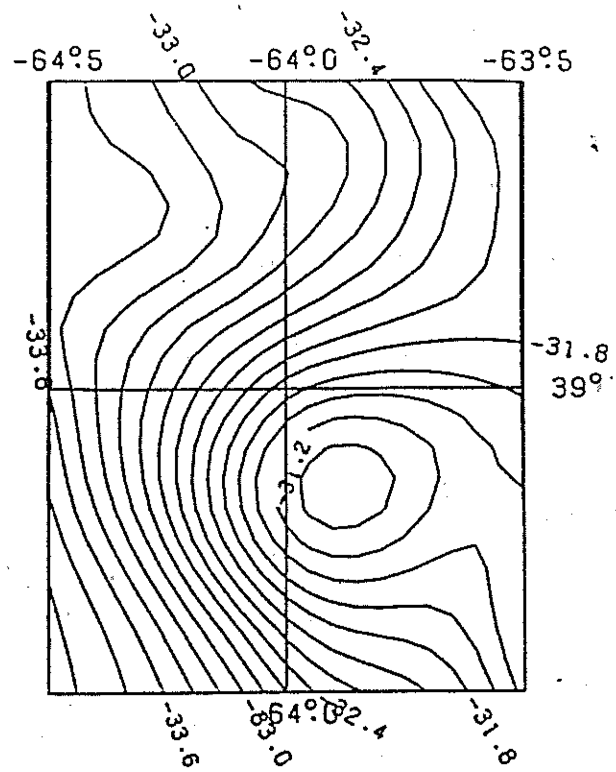

Fig. 10. Geoid heights predicted with bias removal. Contour interval 0.2m.

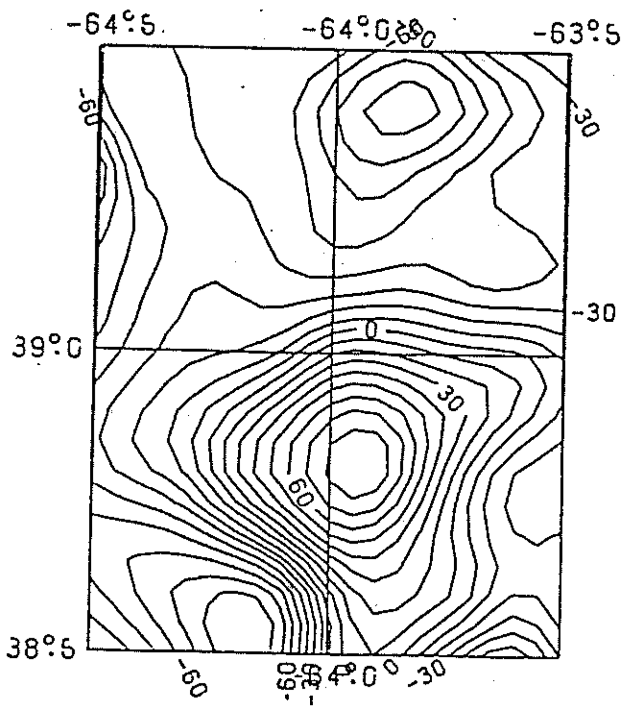

Fig. 11. Free-air gravity anomalies predicted without bias removal. Contour interval 10 mgal.

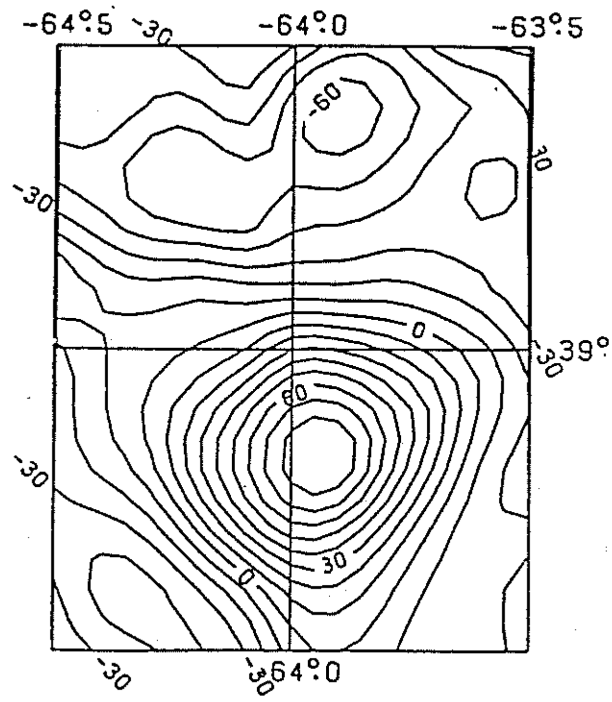

Fig. 12. Free-air gravity anomalies predicted with bias removal. Contour interval 10mgal.

Appendix 1:

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      SUBROUTINE PARCAT(IPACAT,IPTYPE,IPC,IPT,NPARM,IPA,MP,LALLP)
C THE SUBROUTINE INITIALIZES THE PARAMETER CATALOGUES IPTYPE AND
C IPACAT. WHEN THE LOGICAL VARIABLE LALLP IS FALSE, IT IS CHECKED
C WHETHER THE PARAMETERS STORED IN IPACAT(IPA+1),...,IPACAT(IPA+MP)
C ARE NEW PARAMETERS, AND THE CATALOGUE IPTYPE AND THE COUNTER
C NPARM ARE UPDATED. WHEN LALLP IS TRUE, IT IS CHECKED WHETHER
C THE PARAMETER CODES STORED IN IPACAT ARE ACCEPTABLE PARAMETERS,
C I.E. FOUND IN IPTYPE. IF NOT FOUND, THEY ARE PUT EQUAL TO ZERO.
C
C PARAMETERS IN CALL:
C  IPACAT (CALL AND RETURN, INTEGER ARRAY OF DIMENSION IPC) HOLDS
C                PARAMETER CODES TO BE CHECKED IN VARIABLES WITH
C                SUBSCRIPTS IPA+1 TO IPA+MP.
C  IPTYPE (CALL AND RETURN, INTEGER ARRAY OF DIMENSION IPT) HOLDS
C                FOR LALLP TRUE ALL ACCEPTABLE PARAMETER CODES AND
C                FOR LALLP FALSE ALL EARLIER ACCEPTED CODES
C                AND AT RETURN ALL CURRENTLY ACCEPTED CODES.
C  IPC    (CALL, INTEGER) DIMENSION OF IPACAT.
C  IPT    (CALL, INTEGER) DIMENSION OF IPTYPE.
C  IPA    (CALL AND RETURN, INTEGER) IPA+1 POINTS AT CALL AT FIRST NEW
C                PARAMETER CODE IN IPACAT. AT RETURN IT IS IPA+MP.
C  NPARM  (CALL AND RETURN) ACTUAL NUMBER OF ACCEPTED PARAMETERS.
C  MP     (CALL, INTEGER) IPA+MP POINTS AT LAST ACCEPTED PARAMETER.
C
      DIMENSION IPACAT(IPC),IPTYPE(IPT)
      LOGICAL LALLP, LSAME
C
      DO 10 I = 1, MP
      LSAME = .FALSE.
      DO 20 J = 1, NPARM
   20 LSAME = LSAME .OR. (IPACAT(IPA+I).EQ.IPTYPE(J))
      IF (LSAME) GO TO 10
      IF (LALLP) GO TO 50
C
      NPARM = NPARM+1
      IPTYPE(NPARM) = IPACAT(IPA+I)
C THE PARAMETER TYPE CATALOGUE IS UPDATED.
      GO TO 10
C
   50 IPACAT(IPA+I) = 0
C PARAMETER CODE = 0 MEANS INDEPENDENT OF PARAMETERS.
   10 CONTINUE
      RETURN
      END
```