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ESTIMATION OF THE LONGITUDE BIAS OF THE NWL9D COORDINATE SYSTEM FROM DEFLECTIONS OF THE VERTICAL, SATELLITE ALTIMETRY AND HIGH DEGREE SPHERICAL HARMONIC EXPANSIONS

Abstract

The NWL9D *(or* NSWC9Z2*) coordinate system is known to have the* x *and* y *coordinates of its origin approximately* 1 *or* 2 *meters from the Earth's gravity center, but its zero meridian plane is supposed to form an angle* ($\Delta\lambda$) *of* $-0''.5$ *to* $-0''.8$ *with the astronomical (Greenwich) zero meridian plane. If biased geodetic longitudes are used for the calculation of an astrogeodetic geoid profile at, e.g., latitude* $\varphi = 30°$ *extending* 1000 km, *then the geoid undulation differences between the two endpoints will differ more than* 2 m *from the true geoid. This error will easily show up by comparison with a geoid calculated from a high-degree spherical harmonic expansion or the approximately observed geoid undulations obtained from satellite altimetry. This gives a possibility for determining* $\Delta\lambda$ *as the correction* $(-\cos\varphi\cdot\Delta\lambda)$ *which gives the best least squares agreement between the deflections of the vertical and other gravity field-dependent quantities.*

An estimation of $\Delta\lambda$ *has been made using least squares collocation, because this method gives result which fulfil a least squares minimum principle. The following datasets were used.*

(A) 301 *deflections of the vertical in an area with latitude between* 29° *and* 33°, *and east longitude between* 264° *and* 279° *along the coastline of the Gulf of Mexico.*

(B) *coefficients to maximal degree* 180 *of two spherical harmonic expansions, and*

(C) *sea-surface heights determined from satellite altimetry along the same coastline treated as if they were biased geoid undulations.*

The deflections were used with the coefficients alone or combined with the sea-surface heights. A value of $\Delta\lambda = -0''.55 \pm 0''.08$ *was obtained for the data combination which gave the smallest standard deviation. This result is not significantly different from the value of* $\Delta\lambda = -0''.50$ *adopted for the transformation between the North American Datum, 1983 and* NWL9D (NSWC9Z2).

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1. Introduction

The origin of the NWL9D coordinate system is known to be very close to the Earth's gravity center. If the coordinates of the gravity center are $(\Delta x, \Delta y, \Delta z)$, then $|\Delta x|$ and $|\Delta y|$ are below one meter, and $|\Delta z|$ is below 5 m; see (West, 1982), (Hothem et al., 1982). However, the zero meridian plane forms a rather large angle, $\Delta\lambda$, with the astronomical (Greenwich) zero meridian plane, estimated to be between $-0''.5$ and $-0''.8$, see e.g. (Hothem et al., 1978) or (White and Huber, 1979). Various results are summarized in (Boucher, 1983, *Table 2*).

This longitude bias is quite disturbing. It results, for example, in an inconsistency between gravimetrically computed prime vertical components of the deflection of the vertical (η_g) and astrogeodetically obtained values (η_a) given in NWL9D,

$$\eta_g = \eta_a - \Delta\lambda \cos\varphi \qquad (1)$$

where φ is the latitude of the point in question. (The minus sign on $\Delta\lambda$ in eq. (1) is due to the fact that the prime vertical component of the deflection of the vertical is the astronomical longitude minus the geodetic longitude, multiplied by $\cos\varphi$.)

A bias $\Delta\lambda = -0''.50$ will at $\varphi = 30°$ result in a bias of the prime vertical deflection component of $0''.43$. This order of magnitude of the bias shows that it may be very difficult to determine $\Delta\lambda$ using eq. (1), because errors in η_g due to the so-called remote zone effect is of the same magnitude.

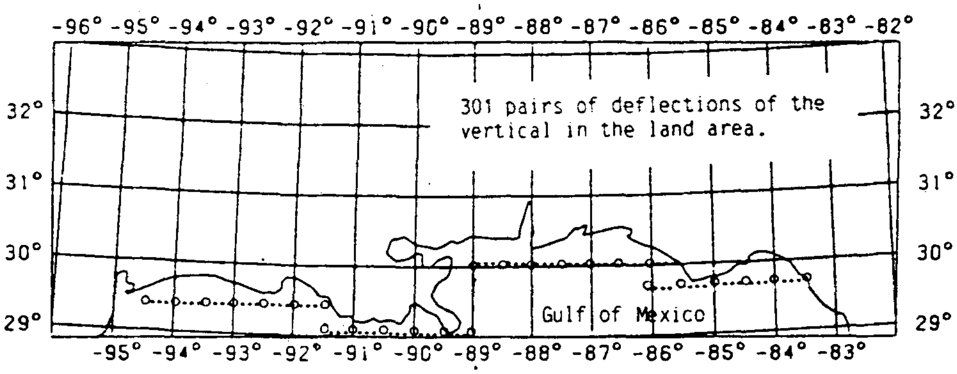
Furthermore, suppose astrogeodetic deflections with the same bias are used for the calculation of an astrogeodetic profile at latitude $\varphi = 30°$, and that this profile is 1000 km long. Then the geoid undulation difference between the two endpoints will differ considerably from the true geoid undulation difference. Since a deflection of the vertical of $1''$ corresponds to a geoid undulation difference of 5 m over 1000 km, then a deflection bias of $0''.43$ corresponds to a difference of approximately 2 m

Geoid undulation differences at sea are, on the other hand, at many places known better than 0.5 m. This knowledge is obtained by regarding sea-surface heights obtained from satellite altimetry as being equal to geoid undulations having a constant bias in a given region. Therefore it seems possible to determine $\Delta\lambda$ with a standard deviation of $\pm 0''.1$, if we are able to construct an astrogeodetic profile along a coastline extending 1000 km in an east-west direction. In fact, such a situation is found along the northern part of the Gulf of Mexico, see ***Figure 1***. In this area it also seems reasonable to treat sea-surface heights obtained from satellite altimetry as geoid undulation differences, see (Maul and Herman, 1985, ***Fig. 4***.)

In the following, the determination of $\Delta\lambda$ from deflections η_a, sea-surface heights and spherical harmonic coefficients will be described. In section 2 we discuss various alternatives for the computation of $\Delta\lambda$, and the data and results obtained using various combinations of subsets of the data are described in section 3. The results and possibilities for further improvements are discussed briefly in section 4.

2. Methods for determining $\Delta\lambda$

The old problem of fixing a geodetic datum did not include the determination of a longitude bias. The application of Laplace conditions in the adjustment of geodetic networks assured (in theory) the parallellity of the various coordinate axes, but the



o Sea-surface heights used for parameter determination.

... Sea-surface heights used for comparison with GEM 10 C, OSU81 and GMP 2.

Fig. 1 – Data collection area.

origin had to be fixed. This was done by minimizing the square-sum of the deflections of the vertical viewed as a function of the deflections of the vertical and geoid height in a fundamental point, see e.g. (Heiskanen and Moritz, 1967, section 5–10).

With coordinate systems like NWL9D, which primarily have been fixed through Doppler satellite observations, we have a slightly different problem. Due to the weak (absolute) directional information, the directions of the axes are not well determined.

One possibility for determining $\Delta\lambda$ would then be to compare (in a least squares adjustment mode) sets of geodetic coordinates given in NWL9D (or a similar satellite-related coordinate system) with the coordinates of geodetic coordinate systems which have been adjusted using the Laplace condition. This has been used by (Hothem et al., 1978) for the transcontinental traverse, east-west across the United States and by (Schlüter et al., 1979) for parts of the European Datum, 1979. Results close to $\Delta\lambda = -0''.50$ have been obtained in both cases, but they have not been considered the final answer because of their disagreement with results obtained, for example, from VLBI , see (Hothem et al., 1982). It is therefore of interest as suggested in the introduction to use an independent type of data like undulation differences in order to determine $\Delta\lambda$. We are then back to methods like the one used to fix a geodetic datum. However, we also know, that the results obtained earlier when fixing the North American Datum, 1927 (NAD 27) and the European Datum, 1950 (ED 50) were not very successful. The centers of the reference ellipsoids are for both systems located more than 100 m from the Earth's gravity center. The reason is that the square-sum of the deflections should have been minimized not just for a set of individual stations in a limited region, but for a globally and regularly distributed set.

The requirement of using a global data set may be dropped, if a best least squares agreement is searched for using deflections computed with respect to a high-degree spherical harmonic expansion of the Earth's gravity potential. And the need of having a regularly distributed dataset may be disregarded, if we take into account the (physical) correlation between the various observations.

The use of a spherical harmonic expansion, presupposes that the errors in the coefficients of low degree are small. They are estimated to be small, but the error estimates are difficult to verify. On the other hand, a comparison with geoid undulation differences, and with the meridian components of deflections of the vertical (ξ) will reveal such errors. Such a comparison has been carried out using the data described in section 3, and did not reveal any systematic errors.

Finally we have the possibility for computing an astrogeodetic geoid and comparing this with geoid undulation differences as discussed in the introduction. This was first tried in Scandinavia (see Tscherning, 1985b) but with a rather unsatisfactory result due to the limited extent (400 km) of the geoid segment and the use of deflections given in the not rigorously adjusted ED 1950 .

The method used was least squares collocation, which here may be regarded as a generalisation of the method using the minimum principle when fixing the origin of a geodetic datum, however now taking into account the physical correlation between the used observations.

Suppose the observations x_i , $i = 1, \ldots, n$ (ξ , η and other quantities) are related to the anomalous gravity potential, T , through linear functionals L_i , that they depend on k parameters X_j , $j = 1, \ldots, k$ and have an error e_i . Then

$$x_i = L_i(T) + \sum_{j=1}^{k} A_{ij} X_j + e_i \ . \qquad (2)$$

The constants A_{ij} will, for example, here be $-\cos\varphi$, if X_j is $\Delta\lambda$, cf. eq. (1). Then an approximation $\tilde{T}$ and estimates $\tilde{X}_j$ of the parameters are obtained (see Moritz, 1980) from

$$\tilde{T}(P) = \sum_{i=1}^{n} a_i C_{Pi} \qquad (3)$$

$$X_j = (A^T \bar{C}^{-1} A)^{-1} A^T \bar{C}^{-1} x \qquad (4)$$

$$\{a_i\} = \bar{C}^{-1} \left\{ x_i - \sum_{\ell=1}^{k} A_{\ell i} X_\ell \right\} \qquad (5)$$

A is the n x k matrix with elements $A_{\ell j}$, C_{Pi} is the covariance between the i'th observation and the value of the anomalous potential in the point P , $\bar{C} = C_{ij} + D_{ij}$ a n x n matrix, where C_{ij} is the covariance between the i th and the j th observation and D_{ij} is the variance covariance matrix of the errors.

The solution has the property that if all C_{ij} are zero, then we are back to the earlier used minimum principle.

Note that the method does not make an "explicit" comparison between geoid undulation differences and computed astrogeodetic values. This is done implicitly when using eq. (4) with both geoid heights and deflections as observations. However, geoid values (N_P) may be predicted using eq. (3) and Bruns' equation

$$N_P = T(P)/\gamma \ ,$$

where γ is the reference gravity in the point P. Also, we are not restricted to using only deflections of the vertical and geoid undulation differences as observations. Any quantity physically correlated with these quantities will contribute to the determination of $\Delta\lambda$.

3. Data used and Results

Deflections of the vertical (ξ_a, η_a) in the so-called Preliminary North American Datum, 1983, (PNAD 83) are available in the National Geodetic Survey database for the United States. They have been obtained from values in NAD 1927 using an empirically determined transformation between NAD 1927 and NWL9D, followed by a rotation $\Delta\lambda = -0''.50$, a shift of the equatorial plane, $\Delta z = 2.0$ m and a differential scale change $\Delta L = -0.50$ ppm (T. Vincenty, private communication, 1984). The longitude shift is applied in order to eliminate the largest part of the longitude bias, and our results given below will confirm this choice.

We used a total of 301 pairs of deflections of the vertical, all within the area shown on *Fig. 1*. (One pair with a 20" deflection component was rejected.) The deflections vary extremely smoothly in the area as shown in *Table 1.* For all the deflections we had given individual error estimates, generally around $\pm 0''.3$.

Professor R.H. Rapp, The Ohio State University, provided us with a set of adjusted and interpolated sea-surface heights based on results from satellite altimetry (see Liang, 1983). These sea-surface heights were treated as geoid undulations with a standard deviation of ± 0.2 m, having a common bias, N_0.

From the available data we selected four east-west profiles close to the coast; see *Fig. 1*. As observations we then used point values spaced $0^\circ.5$ apart, a total of 24 values. However, 94 points, $0^\circ.125$ apart were used for comparison purposes in *Table 1.*

Table 1

Mean values $\bar{X}$ and standard deviations of geoid heights and deflections and of their differences with respect to three sets of potential coefficients.

Coefficient set	Geoid heights $\bar{X}$ (m)	Geoid heights σ (m)	ξ_a $\bar{X}$	ξ_a σ	η_a $\bar{X}$	η_a σ
none	−25.92	±1.48	0".82	±2".56	0".71	±2".30
GEM 10C	0.20	1.73	−0.57	2.48	−0.05	2.25
OSU 81	−0.13	0.30	0.02	1.74	−0.02	1.42
GMP 2	0.50	0.27	−0.01	1.74	0.05	1.40

The use of three sets of spherical harmonic coefficients, GEM 10C, (Lerch et al., 1981) OSU 81 (Rapp, 1981) and GMP 2 (Wenzel, 1985) were considered. However, the GEM 10C set gave a surprisingly bad fit to the data and was not used; see *Table 1*. The sets GEM 10C and OSU 81 are complete to degree and order 180 and

GMP 2 is complete to degree and order 200 , but only coefficients of maximal degree 180 were used.

There exist error estimates for the coefficients. Here the estimates of the errors in the OSU 81 set seemed to be much too large, and the variances were scaled with a factor of 1/9 . Meanwhile new error estimates (in the form of so-called degree variances, σ_i^e) have been received from R.H. Rapp, but they have not been used in this investigation.

It should be noted that the potential coefficients have not been used directly as observations in eq. (2). The contributions from the coefficients may be shown to be included in an equivalent manner by using the associated spherical harmonic expansion as an improved reference potential; see, e.g., (Tscherning, 1974). Hence the contribution from the coefficients has been subtracted from the observations and later added to the predicted quantities. Also the correlation model is affected by this, namely, so that the error degree variances take the place of the so-called empirical degree-variances.

Also gravity data could have been used, but we judged that their possible contribution through their correlation with the other observations would have been small.

Having selected the data, we then had to fix some parameters in the covariance function models. We did not try to estimate empirically the covariance function, since this is primarily of importance when predicting gravity field dependent quantities. However, we wanted to be sure, that at least the variances derived from the covariance function models were in agreement with the empirically estimated values; see *Table 1.*

The following generally acceptable model (see, e.g., Forsberg, 1984) was used

$$cov\,(T(P),\ T(Q)) = \sum_{i=0}^{180} \sigma_i^e \left(\frac{R_E^2}{rr'}\right) P_i\,(cos\,\psi) + \sum_{i=181}^{\infty} \sigma_i \left(\frac{R_B^2}{rr'}\right) P_i\,(cos\,\psi)$$

Here σ_i^e are the error degree-variances , R_E the mean Earth radius (= 6371 km) , R_B = 6369.7 km the radius of the Bjerhammar sphere, ψ the spherical distance between P and Q , r , r′ the radial distances of P and Q from the origin and

$$\sigma_i = \frac{A}{(i-1)(i-2)(i+4)} ,$$

where A was selected so that the derived standard deviation of the (residual) deflections of the vertical became equal to the observed value of ± 1″.5 . The derived standard deviation of the geoid undulations was ± 0.28 m for GMP 2 and ± 0.54 m for OSU 81 . The last value is too pessimistic; cf. *Table 1.*

Eqs. (2) – (4) were then used with the contributions from the two coefficient sets subtracted, and with the following data and parameters :

(1) 301 pairs (ξ_a , η_a) and $\Delta\lambda$

(2) as (1) but in addition 24 sea-surface heights treated as biased point geoid heights and a bias parameter N_0.

GRS 80 reference system parameters were used, however with a = 6378136.0 m. An updated version of the FORTRAN program published in (Tscherning, 1974) see (Tscherning, 1985a) was used. The results are given in *Table 2.* The value for NWL9D is obtained by adding $-0''.50$ to the values given in the table.

Table 2

Result of estimation of $\Delta\lambda$, transformation from PNAD 83 to a "correctly" oriented system, and of N_0.

Data : coefficient set	(ξ, η)	Geoid heights	$\tilde{\Delta\lambda}$	$\sigma(\Delta\lambda)$	$\tilde{N}_0$ m	$\sigma(N_0)$ m
OSU 81	301	none	$-0''.06$	$\pm 0''.12$	–	–
GPM2	301	none	$-0''.05$	$+0.08$	–	–
OSU 81	301	24	$-0''.10$	0 .10	0.18	± 0.25
GPM2	301	24	$-0''.05$	0 .08	0.60	0.07

4. Discussion

Our results confirm in an independent manner results obtained earlier by (Hothem et al., 1978) and (Schlüter et al., 1979). This also confirms the value of $\Delta\lambda = -0''.50$ for the transformation from NWL9D to a correctly oriented system, since our values are not statistically different from this value.

Improved results could be obtained if we had better estimates of the sea-surface topography along the coast. Here a detailed analysis of tide gauges and levelling results might be helpful.

Also the use of GPS derived ellipsoidal height differences and levelling along the coast might be used to improve the result. Improved values of the low-order geopotential coefficients might also contribute a great deal. This is, for example, seen from the formal error estimates associated with the parameters in *Table 2.* Estimates obtained using the probably too pessimistic error degree-variances associated with the OSU 81 set are larger than these obtained using the error degree-variances for GPM2.

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