On the Long-wavelength Correlation between Gravity and Topography

by

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Abstract:

Spherical-harmonic expansions of the topography, the topographic-isostatic reduction potential and the gravity potential of the Earth (OSU78, OSU81, GEM10C) now exist complete to degree (N) and order 180.

A correlation analysis of the various fields by degree has been made. While the general correlation between gravity and topography for the sets OSU78 and 81 is around 50% for N > 15, the correlation with GEM10C is considerably lower for N > 36. This indicates that this set is unreliable above this degree.

The topographic-isostatic reduction potential may be computed either rigorously by integrating the topography and its compensation or by condensating the topography and its compensating masses. In the last case the spherical harmonic coefficients of the isostatic reduction potential are related in a simple linear manner to the spherical harmonic coefficients of the expansion of the topographic heights.

An optimal depth of compensation for each degree has been determined by requiring the reduced field to be as smooth as possible. Depths between 35 and 15 km were found for N > 20, which are much lower than the values found earlier by Rapp using another optimal depth principle.

It was found that the correlation between the primitive condensated topographic-isostatic potential coefficients showed a higher correlation with the OSU78 and 81 sets than did the rigorously computed coefficients derived at Technische Universität Graz. Since this is opposite to what should be expected, the quality of these coefficients must be in doubt.

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1. Introduction

Spherical-harmonic expansions of the topography, the rock-equivalent topography, the topographic-isostatic reduction potential and the gravity potential of the Earth now exist complete to degree (N) and order 180.

The coefficients are of varying quality. But it is difficult to know how good or how bad the sets are. "Good" or "bad" also depends an for which purpose one wants to use the coefficients.

An important application is in the area of gravity field modelling, where the contribution from either the potential of the (isostatically compensated) topography or from the expansion of the gravity potential in subtracted from the observations. In both cases a considerable smoothing is expected, which if achieved should facilitate the use of various approximation or prediction techniques.

A necessary, but not sufficient, condition for a large smoothing to be achieved is the occurrence of a strong correlation between the various spherical harmonic coefficients. If the correlation is below 50% no smoothing is achieved. A small correlation also indicates inconsistencies between coefficient sets, which in principle should represent the same information. This is used as an indicated for the quality of the various sets.

Since a strong correlation may exist even in cases where two sets differ by a large scalefactor, so-called smoothing coefficients are introduced. These quantities are used to describe quantitatively the smoothing per degree achieved. Furthermore, the quantities are used to determine optimal depths of compensation, defined as the depths where the largest smoothing is achieved.

2. Correlation and smoothing

Let us regard the spherical harmonic expansions of two functions with fully normalized coefficients $(\bar{C}_{nm}, \bar{S}_{nm})$ and $(\bar{A}_{nm}, \bar{B}_{nm})$, respectively.

The correlation by degree is then

$$\rho_n = \frac{\sum_{m=0}^{n} (\bar{C}_{nm}\ \bar{A}_{nm} + \bar{S}_{nm}\ \bar{B}_{nm})}{(\sigma_n^2(A, B)\ \sigma_n^2(C, S))^{\frac{1}{2}}} \tag{1}$$

with the degree-variances

$$\sigma_n^2(C, S) = \sum_{m=0}^{n}(\bar{C}^2_{nm}+\bar{S}^2_{nm}) \tag{2}$$

$$\sigma_n^2(A, B) = \sum_{m=0}^{n}(\bar{A}^2_{nm} + \bar{B}^2_{nm}) \tag{3}$$

It is obvious, that the correlation may be high, even if the two sets differ by a scala factor, so the correlation is not necessarily a good measure for an agreement or disagreement between two sets. In fact, it is of more importance in physical geodesy to know which degree of smoothing we obtain, if we subtract one set from the other.

A measure for the smoothing per degree is

$$S_n = \frac{\sum_{m=0}^{n} ((A_{nm} - C_{nm})^2 + (B_{nm} - S_{nm})^2)}{\sigma_n^2(A, B)} \tag{4}$$

The correlation between various potential coefficient sets are shown in Table 1. Here OSU78 is described in Rapp (1978), OSU81 in Rapp (1981), GEM10C in Lerch et.al.(1981), "rock eq" in Rapp (1982) and "topiso" in Grasegger and Wotruba (1983).

We should naturally expect a very strong correlation between the coefficient sets for the spherical harmonic representation of the gravity potential, W, since the expansions have be computed using very much the same data. But this is not the case. OSU78 and 81 seems to be in agreement, but GEM10C shows little correlation with the two OSU sets for $n > 40$.

We would also expect that a good gravity potential coefficient set should show a strong correlation with the expansion of the potential of the topography. This is clearly the case for the two OSU sets, but the topography and GEM10C shows very little correlation by degree. From this one may conclude, that the intermediate wavelength ($40 < n < 120$) information in the two OSU sets is the most reliable. For the very long wavelength coefficients, we know that GEM10C is identical to GEM10B, which has given excellent results for all types of orbit computations (S. Klosko, private communication)).

Furthermore, it is interesting to see that the in principle rigorously computed coefficients of the isostatic reduction potential show less correlation with the OSU78 and 81 sets, than the coefficients computed based on the attraction of a condensated topography. It may therefore be suspected that the former set of coefficients contain numerical errors. That this is possible has been admitted by our Austrian colleagues (Sünkel, 1984, private communication).

Similar phenomena, exposing less reliable coefficients, can be seen when regarding the smoothing coefficients, Fig. 1 - 9. However, the level of smoothing obtained when subtracting the potential of the isostatically compensated topography depends on the adopted depth of isostatic compensation, D.

The degree-variances obtained from the potential of a topographic expansion with coefficients $\bar{A}_{nm}$ and $\bar{B}_{nm}$ and its isostatic compensation are (Lambeck, 1978, p. 592)

$$\sigma_n^2(D) = \left(\frac{3\rho_c}{\bar{\rho}}\right)^2 \frac{1}{(2n+1)^2} \left[1 - \left(\frac{R-D}{R}\right)^n\right]^2 \sigma_n^2(A,B) \qquad (5)$$

where ρ_c is the average crustal density, $\bar{\rho}$ the average Earth density and R the mean Earth radius. The square-root of this equation gives basically the relation between the individual coefficients.

This has been used by Rapp (1982) in order to find compensation depths, so that

$$\sigma_n^2(D) \approx \sigma_n^2\,(C, S),$$

(where the C_{nm}, S_{nm} set was the OSU81 set).

The result was rather large depths of 50 km. However, if we instead suppose that the optimal depth is attained where the smoothing is largest, then the results in Fig. 10 and 11 are obtained. Here more realistic depths of between 15 and 30 km are obtained for $N > 15$. In general the estimated values will be too small, due to the large noise in both coefficient sets. (The depths were found by linearising eq. (5) and solving for D in a least-squares adjustment with one unknown).

In general it is surprising to see that the smoothing coefficients are close to or larger than 1, indicating no smoothing. This is reflected in the results given in Table 2, which show the variation of the geoid undulations and gravity anomalies before and after the subtraction of the potential of the isostatically compensated, but condensed, topography.

3. Conclusion

The analysis of the correlation between various sets of potential coefficients or expansions of the potential of the topography shows unexpected low values and also large variations in the values as a function of the degree. This points at some sets as being of lesser quality than others.

The variation of the smoothness coefficients as a function of adopted depth of compensation shows that it may be useful to use varying depths of compensation for varying degree. However, the smoothing achieved by subtracting the effect of the topography (to degree and order 180) is very small for the geoid and also rather small for gravity anomalies. This is in contrast to results obtained in local areas, where a 25% smoothing generally is obtained by subtracting local topographic effects.

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Table 1. Correlations between sets of potential coefficients for varying degree, n, in %

Coeff. set n	OSU 78 OSU 81	OSU 78 GEM 10 C	OSU 78 topiso	OSU 81 topiso	GEM 10 C topiso	OSU 78 rockeq	OSU 81 rockeq	topiso rockeq
2	100	100	- 44	-43	-43	-44	-43	99
4	100	100	45	46	30	47	47	99
6	100	100	34	33	32	46	45	97
8	99	98	43	42	37	36	34	96
10	97	94	65	68	68	66	68	97
12	97	92	-2	4	8	2	8	94
14	91	79	55	56	52	54	60	94
16	91	84	47	52	45	50	60	96
18	92	78	65	66	59	55	59	93
20	90	75	53	56	48	57	62	91
22	90	78	51	46	42	49	49	91
24	91	81	62	53	39	61	52	93
26	89	67	64	68	59	61	67	95
28	89	75	61	60	50	70	67	92
30	86	55	59	59	28	59	62	95
35	91	65	60	61	38	57	56	91
40	92	72	59	62	47	58	64	91
45	93	77	60	63	47	61	63	90
50	95	85	58	64	52	54	59	88
55	95	78	58	57	49	58	57	88
60	95	75	63	64	42	66	67	89
65	94	70	57	59	46	61	63	86
70	95	70	60	62	44	58	61	85
75	93	62	44	49	29	56	56	79
80	94	65	56	56	44	57	57	78
85	91	65	44	46	34	50	50	78
90	91	53	35	37	28	47	50	75
95	91	52	51	59	38	52	58	74
100	90	49	50	48	31	50	54	66
105	92	49	56	57	39	60	62	73
110	91	50	53	53	31	60	60	73
115	91	55	43	43	24	50	50	60
120	91	45	46	50	24	50	57	72
125	90	39	56	54	20	58	57	66
130	88	36	41	42	20	49	54	58
135	86	30	33	42	2	47	49	57
140	87	27	47	49	9	55	56	62
145	86	19	35	36	6	49	47	57
150	82	30	36	42	7	40	52	49
155	83	19	34	33	6	41	41	55
160	80	10	31	32	6	46	50	51
165	86	6	37	38	3	56	52	57
170	83	12	26	24	0	40	44	48
175	84	6	31	33	-3	47	52	43
176	80	5	28	30	10	39	42	52
178	80	9	43	43	1	45	48	56
179	81	5	43	45	0	48	54	56
180	82	7	32	41	8	40	48	55

Table 2. Mean sq. variation of geoid heights and gravity anomalies from various sets of potential coefficients complete to degree and order 180, or computed from differences between such sets.

Coefficient set (1)	(2)	Mean square variation derived from set (2) Geoid m**2	set (2) Gravity mgal**2	set (1) - set (2) Geoid m**2	set (1) - set (2) Gravity mgal**2
OSU1978	none			915.9	551.6
	top.-iso., D=30	15.4	125.4	933.9	441.1
	rock eq., D=20	10.5	115.4	925.2	431.8
	rock eq., D=25	16.3	167.6	930.8	434.8
	rock eq., D=30	23.2	226.8	937.7	446.7
	rock eq., D optimal	110.9	143.6	805.0	408.0
OSU1981	none			921.3	585.8
	OSU1978	915.9	551.6	3.5	82.0
	top.-iso., D=30	15.4	125.4	938.7	460.7
	rock eq., D=20	10.5	114.4	930.0	446.2
	rock eq., D optimal	113.1	166.1	808.3	419.7
GEM10C	none			920.9	467.6
	OSU1978	915.9	551.6	7.7	307.2
	top.-iso., D=30	15.4	125.4	940.0	445.7
top.iso.	rock eq., D=30			1.9	92.3

The set rock eq.. is the coefficients of the potential of the rock-equivalent topography, condensed, and with its isostatic compensation at the depth D. D optimal means that the different compensation depths have been used for different degrees, so that the best agreement with the coefficient set (1) was obtained.

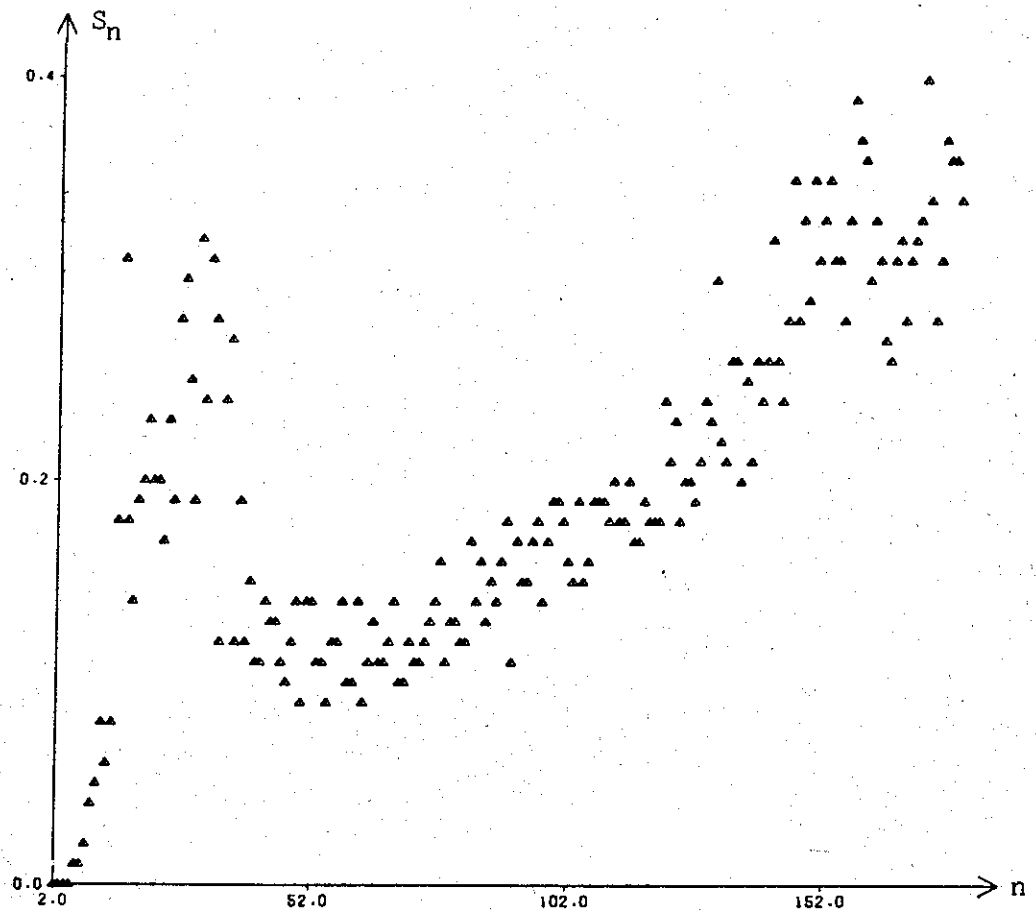

Fig 1. Smoothness coefficients S_n based on coefficients $\{A_{nm}, B_{nm}\}$ = {OSU 81} and $\{C_{nm}, S_{nm}\}$ = {OSU 78}. Note the jump at degree 30

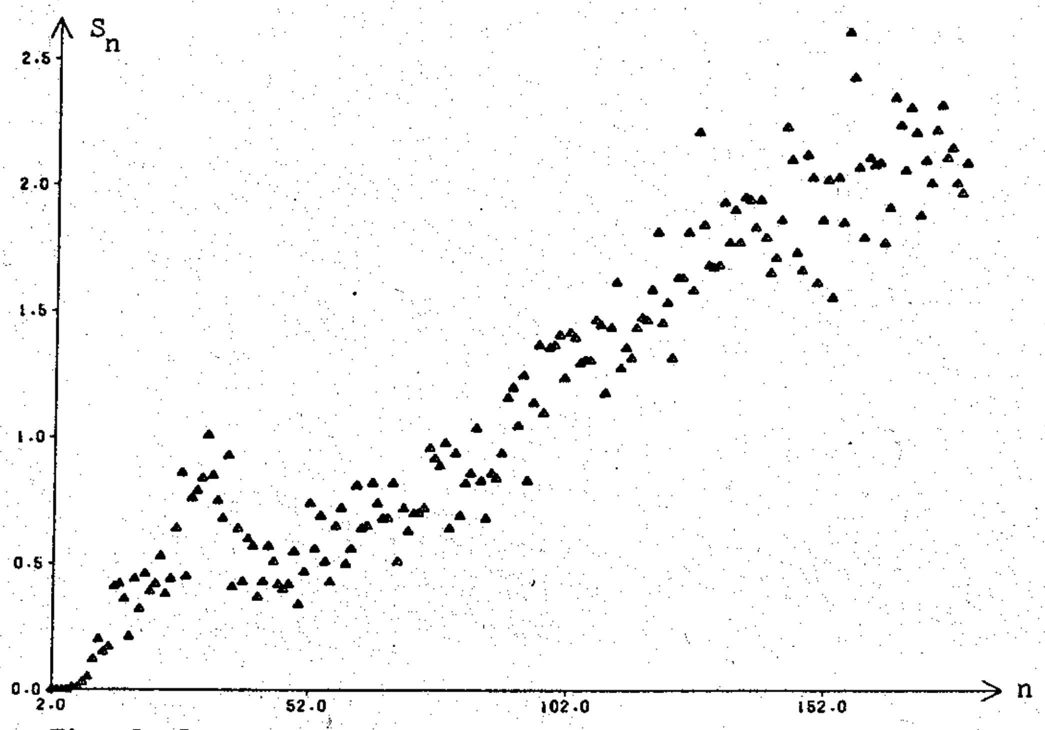

Fig. 2. Smoothness coefficients S_n based on coefficients $\{A_{nm}, B_{nm}\}$ = {OSU 78} set and $\{C_{nm}, S_{nm}\}$ = {GEM 10 C} set.

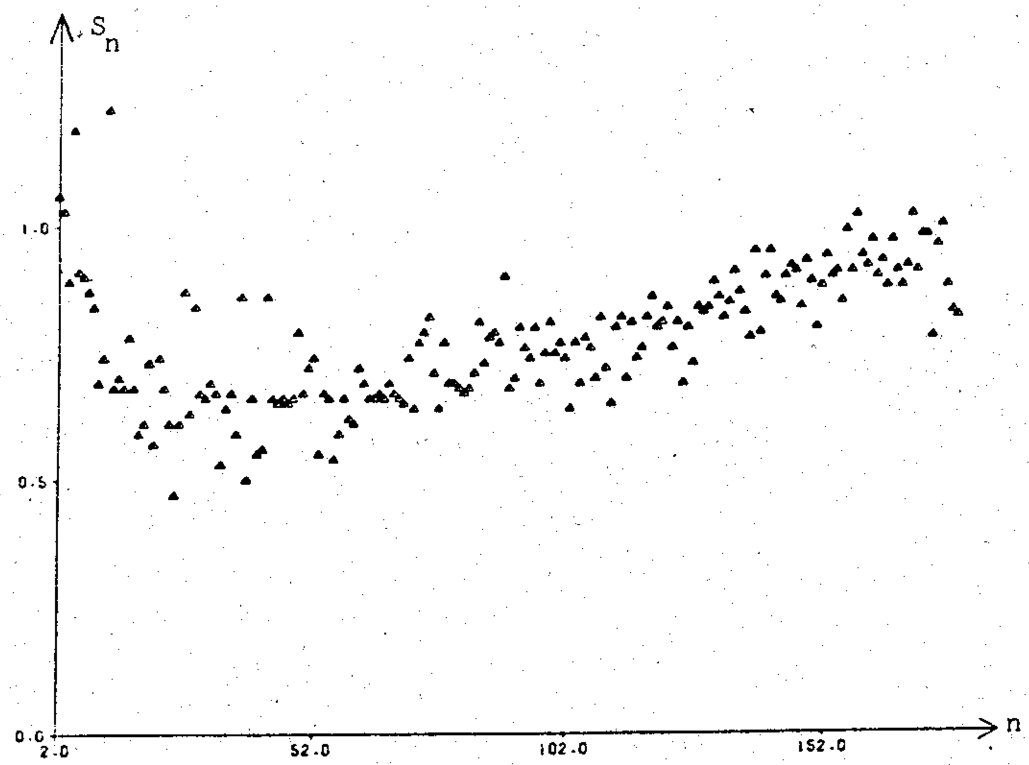

Fig 3. Smoothnes coefficients S_n based on coefficients $\{A_{nm}, B_{nm}\}$ = {OSU 78}set and $\{C_{nm}, S_{nm}\}$ = coefficients of the topographic-isostatic reduction potential. Note the linear trend for n > 30.

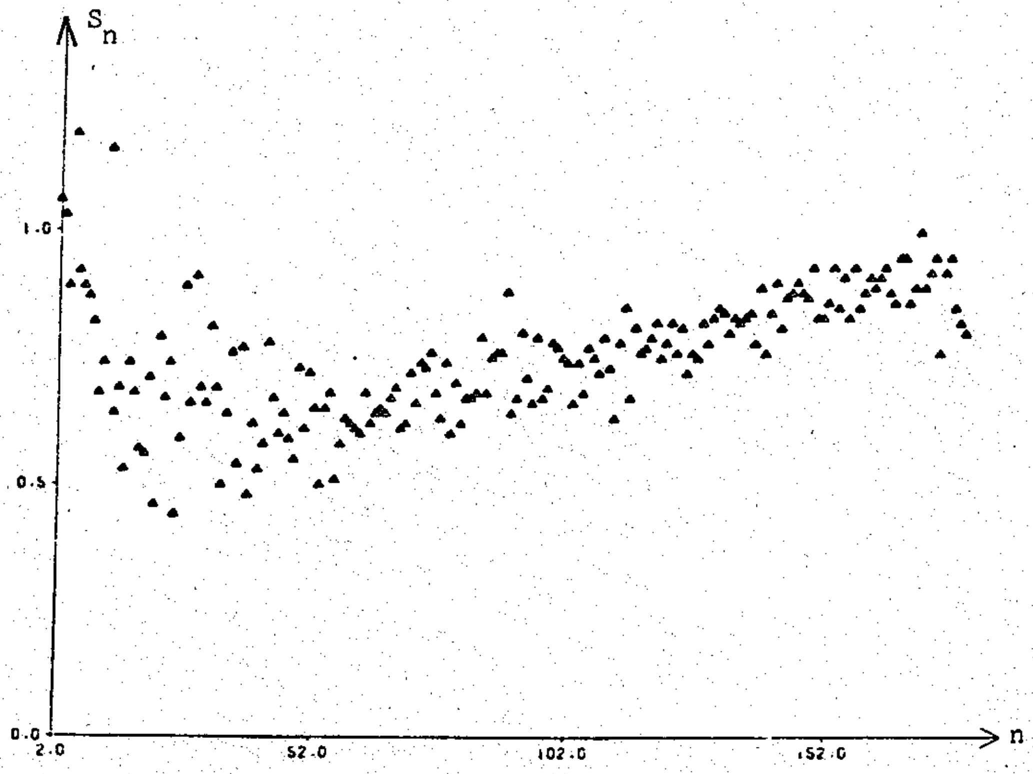

Fig. 4. Smoothness coefficients S_n based on coefficients$\{A_{nm}, B_{nm}\}$= {OSU 81} set and $\{C_{nm}, S_{nm}\}$ = coefficients of the topographic-isostatic reduction potential. Note the linear trend for degree > 30.

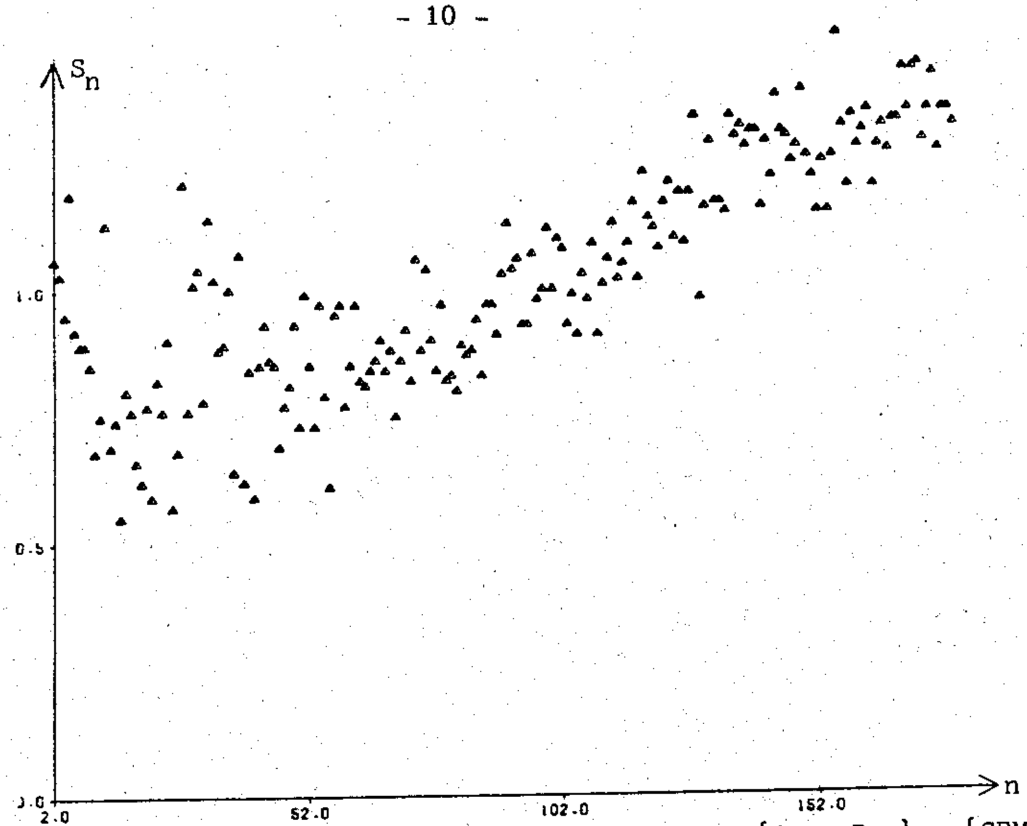

Fig. 5. Smoothness coefficients S_n based on $\{A_{nm}, B_{nm}\}$ = {GEM 10 C} and $\{C_{nm}, S_{nm}\}$ = coefficients of the topographic-isostatic reduction potential. Note, that $S_n > 1$ for the majority of the values of n.

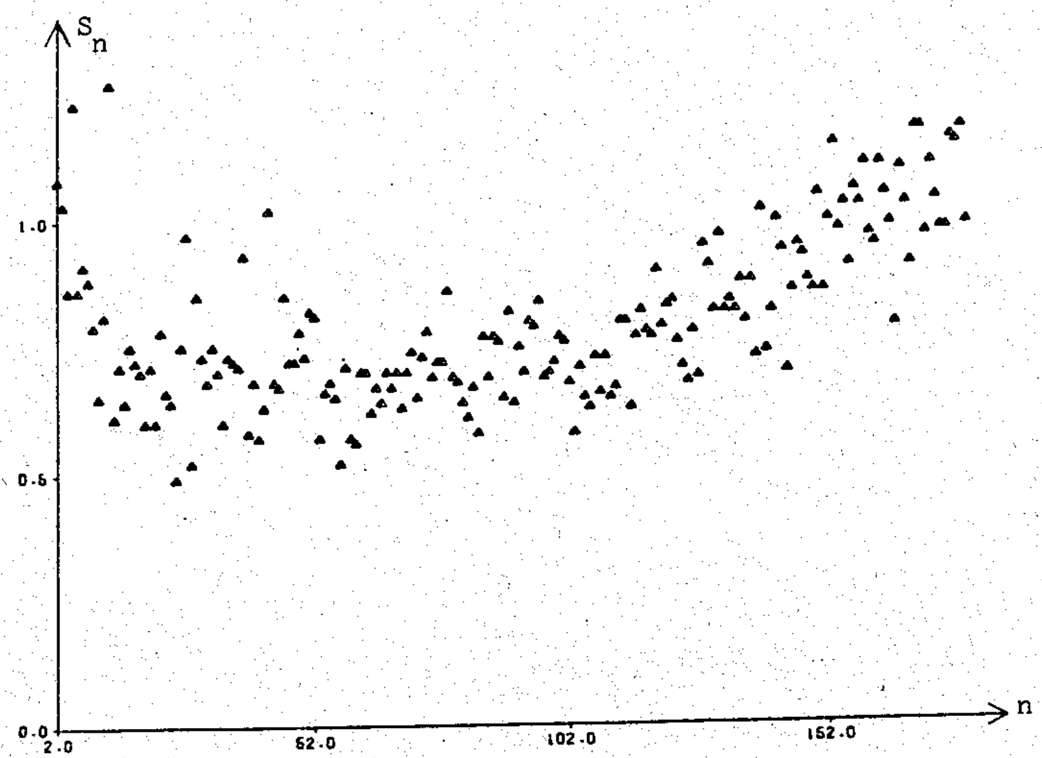

Fig. 6. Smoothnes coefficients S_n based on coefficients $\{A_{nm}, B_{nm}\}$ = {OSU 78}set and the coefficients $\{C_{nm}, S_{nm}\}$ of the potential of the condensed rock-equivalent topography, compensated at D = 30 km.

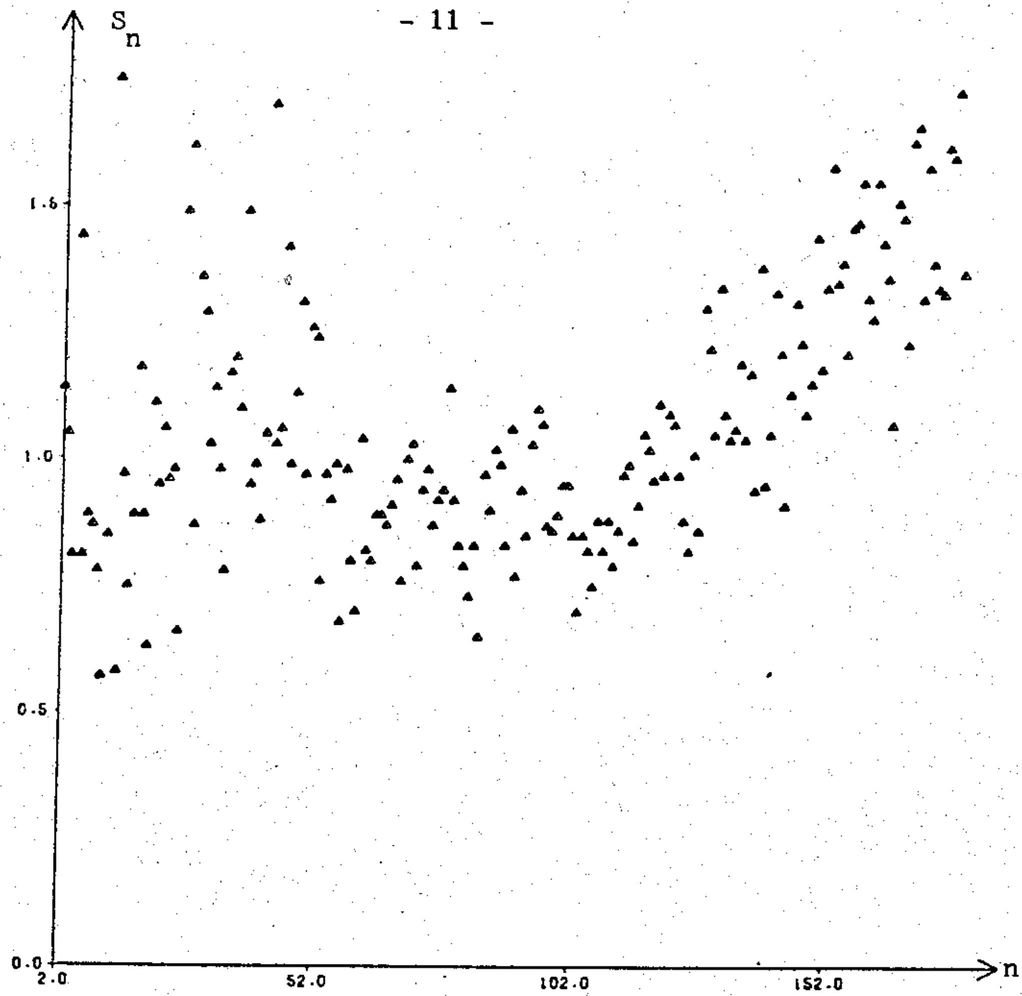

Fig. 7. Smoothness coefficients S_n based on coefficients $\{A_{nm}, B_{nm}\}$ = {OSU 78} set and $\{C_{nm}, S_{nm}\}$ of the potential of the condensed rock-equivalent topography, compensated at D = 50 km.

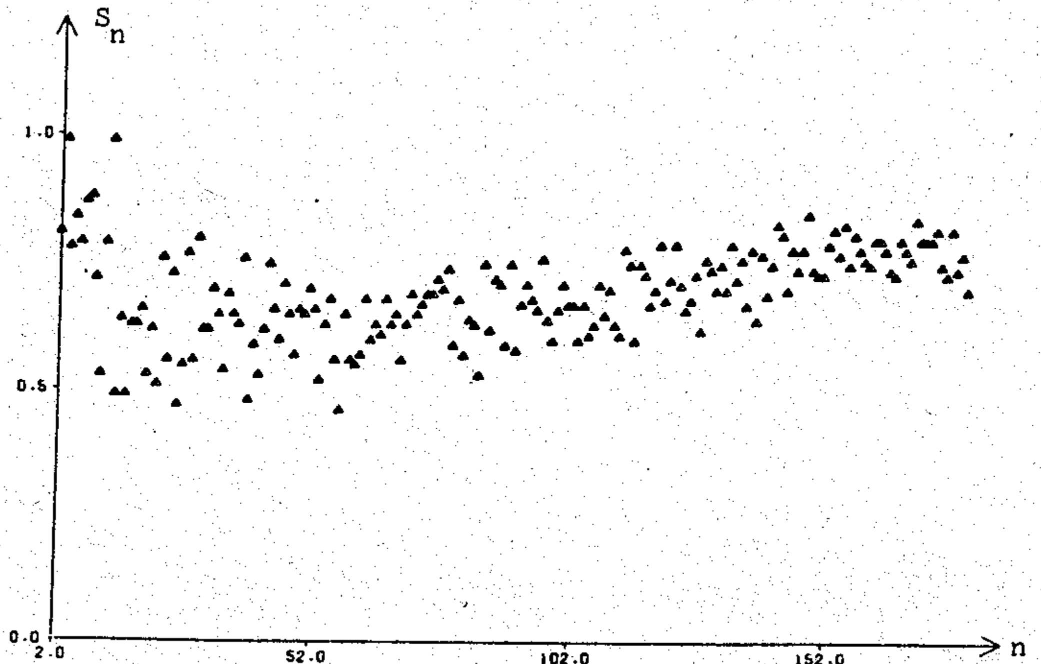

Fig. 8. Smoothness coefficients S_n based on coefficients $\{A_{nm}, B_{nm}\}$ = {OSU 78} and $\{C_{nm}, S_{nm}\}$ derived from rock-equivalent topography using optimized depths, cf. Fig. 10.

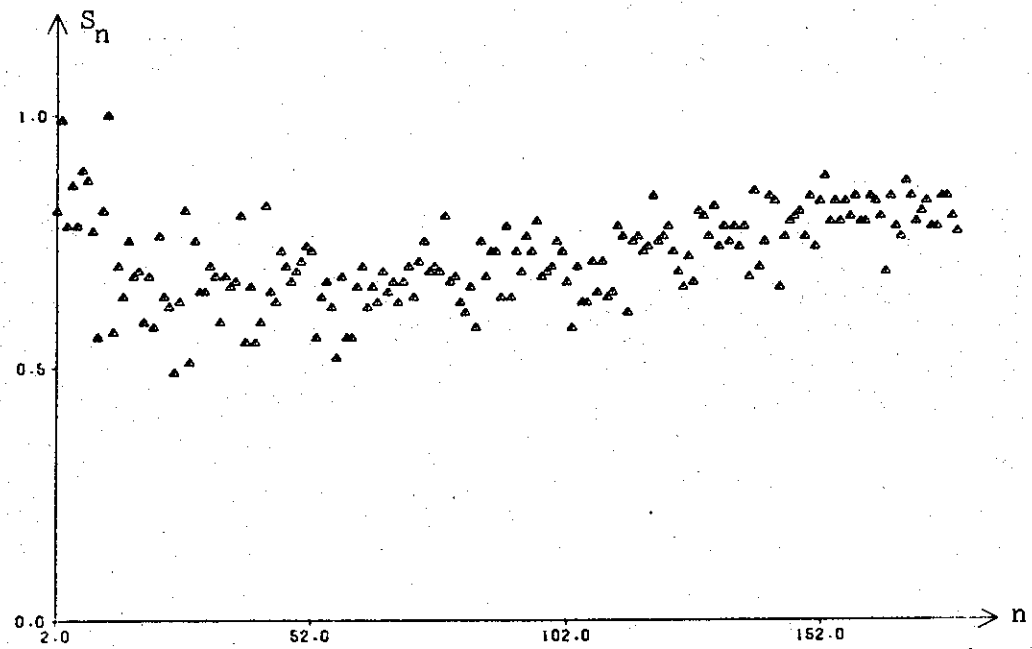

Fig. 9. Smoothness coefficients S_n based on coefficients $\{A_{nm}, B_{nm}\}$ = {OSU 81}set and $\{C_{nm}, S_{nm}\}$ derived from coefficients of rock-equivalent topography using optimized depths, cf. Fig. 11.

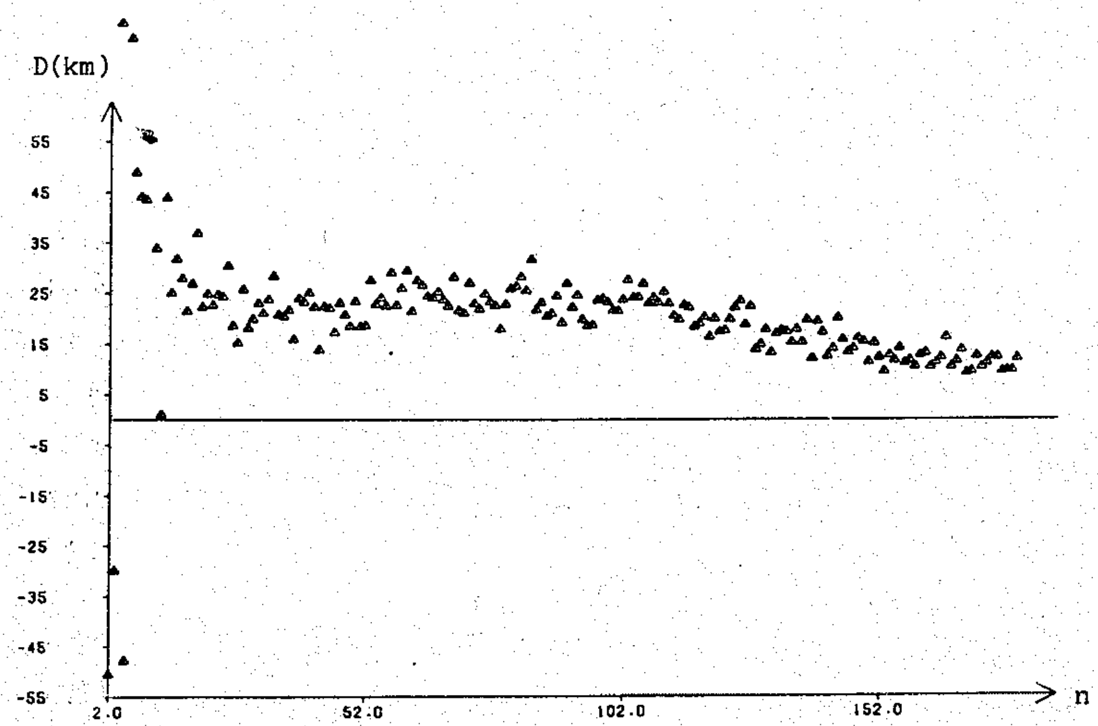

Fig. 10. Optimal depths of compensation, D, as a function of degree, computed using {OSU 78} set and the coefficient of the rock-equivalent topography. Note, that some values are negative. However, they are very uncertain.

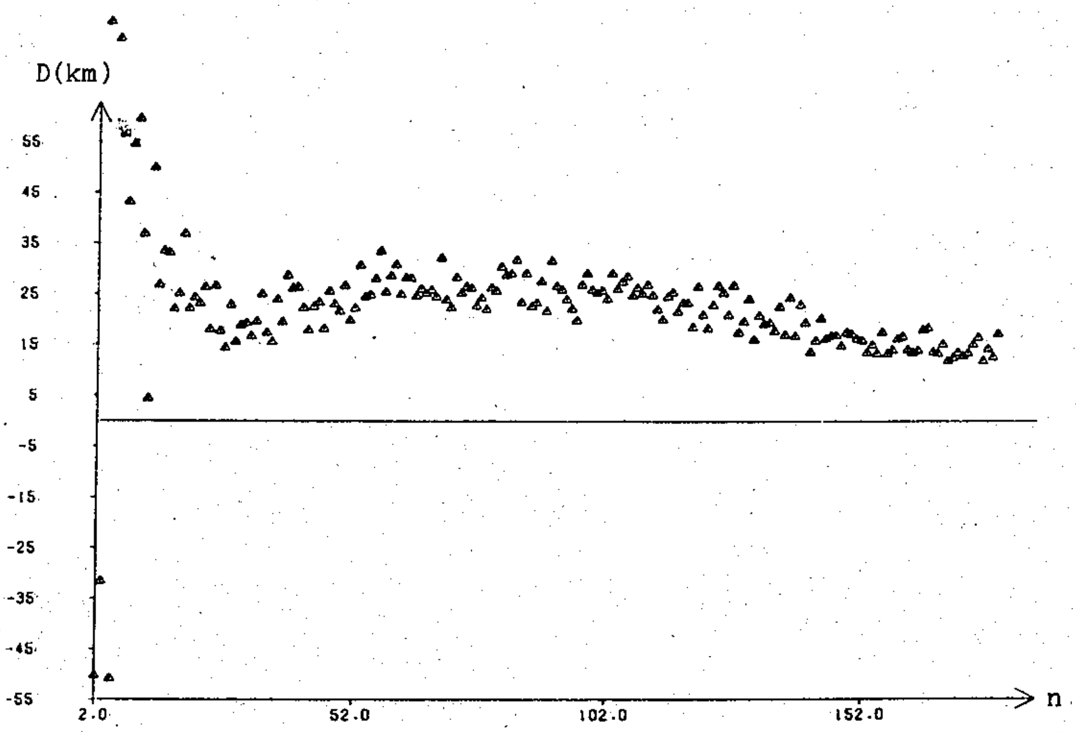

Fig. 11. Optimal depth's of compensation, D, as a function of degree, computed using {OSU 81} set and the coefficients of the rock-equivalent topography.