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September 1983

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Modifications of COVAX for the direct computation of covariances of torsion-balance observations

by

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Abstract:

The subroutine COVAX permits the computation of covariances of certain second order derivatives of the anomalous gravity potential, but not directly these which are associated with torsion balance observations. However, as it is shown in the paper, covariances of these quantities may be easily computed after a few modifications of COVAX.

1. Introduction

The FORTRAN subroutine COVAX described in (Tscherning, 1976) permits the computation of covariances of certain second order derivatives of the anomalous gravity potential, such as

$$\frac{1}{r^2}\frac{\partial^2 T}{\partial\varphi^2},\ \frac{1}{r^2\cos^2\varphi}\frac{\partial^2 T}{\partial\lambda^2},\ \frac{1}{r^2\cos\varphi}\frac{\partial^2 T}{\partial\lambda\partial\varphi},\ \frac{1}{r\cos\varphi}\frac{\partial^2 T}{\partial\lambda\,\partial r},\ \frac{1}{r}\frac{\partial^2 T}{\partial r\partial\varphi},\ \frac{\partial^2 T}{\partial r^2}\ .$$

Combined with covariances of certain first order derivatives, all covariances of second order derivatives may be computed, included the covariances of torsion balance observations

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial T}{\partial \varphi}\right) = \frac{\partial^2 T}{\partial x_3\, \partial x_2} \tag{10}$$

$$\frac{\partial}{\partial r}\left(\frac{1}{r\cos\varphi}\,\frac{\partial T}{\partial \lambda}\right) = \frac{\partial^2 T}{\partial x_3\, \partial x_1} \tag{11}$$

$$2\,\frac{1}{r}\,\frac{\partial}{\partial \varphi}\left(\frac{1}{r\cos\varphi}\,\frac{\partial T}{\partial \lambda}\right) = 2\frac{\partial^2 T}{\partial x_1\, \partial x_2} \tag{13}$$

$$\frac{1}{r^2}\left(\frac{1}{\cos^2\varphi}\,\frac{\partial^2 T}{\partial \lambda^2} - \frac{1}{\cos\varphi}\left(\frac{\partial}{\partial \varphi}\cos\varphi\,\frac{\partial T}{\partial \varphi}\right)\right) = \frac{\partial^2 T}{\partial x_1^2} - \frac{\partial^2 T}{\partial x_2^2} \tag{15}$$

Here r is the radial distance, φ the latitude, λ the longitude, (x_1, x_2, x_3) the coordinates in a local cartesian coordinate system with axes East, North and "up", see Krarup and Tscherning, (1983).

In the following we will see, that it would have been simpler to base COVAX on the functionals (10) - (15) and $\frac{\partial^2 T}{\partial r^2}$, together with the Laplace equation. We will also explain how COVAX has been modified in order to include the possibility for having covariances of these quantities computed directly. Also the possibility for having the 2 x 2 matrices of covariances between the quantities (10)-(11) and (13) - (15), see Krarup and Tscherning, (1983, section 4) has been included.

2. Evaluation of the new covariances.

First note that the functional in eq. (15) may be written

$$\frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} = \left(\frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2}\right)\circ\left(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2}\right) = \frac{\partial^2}{\partial x_6\, \partial x_4},$$

where we have introduced two new variables. We may then with these old and new variables again use (Tscherning, 1976 eq. (43) - (46)), where the application of the functionals on a kernel $K(t, r, r')$ are expressed as derivatives of a function F with respect to $t = \cos\psi$ and the derivatives of t with respect to the variables.

ψ is the spherical distance between two points of evaluation, P, Q, and r, r' are the radial distances from the origin of P, Q, respectively. The function F may be K itself or the functions D and E, see (Krarup and Tscherning, 1983, eq. (7) - (9)).

The reason why we get more simple equations using the torsion functionals is that

$$\frac{\partial^2 t}{\partial x_2 \partial x_1} = \frac{\partial^2 t}{\partial x_6 \partial x_4} = 0$$

We may then like in (Tscherning, 1976) arrange the derivatives of t in a matrix, with entries corresponding to the evaluation functional, and the derivatives with respect to x_2, x_1, (x_1, x_2) or (x_4, x_6), x_4 and x_6, see Table 1

Table 1. Derivatives d(i, j) of t with respect to the variables, x_1, x_2, x_4, x_6 i P and y_1, y_2, y_4 and y_6 in Q.
We use cp = $\cos\varphi$, cq = $\cos\varphi'$, sp = $\sin\varphi$, sq = $\sin\varphi'$, sd = $\sin(\lambda'-\lambda)$, cd = $\cos(\lambda'-\lambda)$

i	entry: j	1	2	3	4	5	6
		ev_q	$r'\frac{\partial}{\partial y_2}$	$r'\frac{\partial}{\partial y_1}$	$r'\frac{\partial}{\partial y_4}$	$(r')^2\frac{\partial^2}{\partial y_1 \partial y_2}$ $(r')^2\frac{\partial^2}{\partial y_4 \partial y_6}$	$r'\frac{\partial}{\partial y_6}$
1	ev_p	t	sp·cq −cp·sq·cd	−cp·sd	d(1,3) +d(1,2)	0	d(1,3) −d(1,2)
2	$r\frac{\partial}{\partial x_2}$	cp·sq −sp·cq cd	cp·cq +sp·sq cd	sp·sd	d(2,3) +d(2,2)	0	d(2,3) −d(2,2)
3	$r\frac{\partial}{\partial x_1}$	cq·sd	−sq·sd	cd	d(3,3) +d(3,2)	0	d(3,3) −d(3,2)
4	$r\frac{\partial}{\partial x_4}$	d(3,1) +d(2,1)	d(3,2) +d(2,2)	d(3,3) +d(3,2)	d(4,3) +d(4,2)	0	d(4,3) −d(4,2)
5	$r^2\frac{\partial^2}{\partial x_1 \partial x_2}$ $r^2\frac{\partial}{\partial x_4 \partial x_6}$	0	0	0	0	0	0
6	$r\frac{\partial}{\partial x_6}$	d(3,1) −d(2,1)	d(3,2) −d(2,2)	d(3,3) −d(3,2)	d(5,3) +d(5,2)	0	d(5,3) −d(5,2)

The derivatives given in Table 1 may also be combined with the functionals 4 - 6 used in (Tscherning, 1976, Table 3). In this manner we may combine the old and new functionals and use the algorithm in (Ibid, 1975, page 17) if we for the sum of functional numbers larger than 3 always substitute the sum of the numbers by the integer 5. (The sum of the functional numbers is used to get the subscripts of the element in the matrix d(i,j), where the mixed derivative is found. This derivative will here be identical zero). Let us then with the derivatives in P associate the integers i, j and with the derivatives in Q the integer k, m. They may take on the values 0, 1, 2, 3, 4 and 6. We also need auxilliary variables j1 and m1, which are equal to 1 when j and m are equal to zero,(indicating no second differentiation in P, Q, respectively), and otherwise equal to j or m. Let x, y, z and v be any of the variables x_1, x_2, x_4, x_6, y_1 y_2, y_4 or y_6. However never more than two are associated with one of the points. Then

$$r^p(r')^q \frac{\partial^2 F}{\partial x \partial y} = d(i,1)\,d(j1,1)\,d(1,k)\,d(1,m1)F_2 + d(i+j,k+m)F_1 ,$$

$$r^p(r')^q \frac{\partial^3 F}{\partial x \partial y \partial z} = (d(i,k)\,d(j1,m1) + d(i,m1)\,d(j1,k))\,F_2$$

$$+ d(i,1)\,d(j1,1)\,d(1,k)\,d(1,m1)\,F_3$$

$$(r\,r')^2 \frac{\partial^4 F}{\partial x \partial y \partial z \partial v} = (d(i,k)\,d(j,m) + d(i,m)\,d(j,k))\,F_2$$

$$((d(i,k)\,d(j,1) + dlj,k)\,d(i,1)d(1,m)$$

$$+(d(i,m)\,d(j,1) + d(j,m)\,d(i,1))\,d(1,k))\,F_3$$

$$+d(i,1)\,d(j,1)\,d(1,k)\,d(1,m)\,F_4 ,$$

with $F_i = \frac{\partial^i F}{\partial t^i}$, and p, q equal to 0,1 or 2 depending on the values of i, j, k and m.

3. Programming the changes

Most of the required changes are easily done, because space was left in the old version for a new variable, here the one associated with eq. (15). The /DATA/ statements are updated, and the assignment of values to the array d was made following Table 1. In this manner the separate division with $\cos\varphi$ and $\cos\varphi'$ which is associated with

$$\frac{\partial}{\partial x_1} = \frac{1}{r\cos\varphi}\,\frac{\partial}{\partial\lambda},\quad \frac{\partial}{\partial y_1} = \frac{1}{r\cos\varphi'}\,\frac{\partial}{\partial\lambda'},$$

was avoided. This permit that most of the covariances now can be evaluated at the poles!

A modification is also needed, because we here used the functionals (10), and (11) instead of $\frac{1}{r\cos\varphi}\frac{\partial^2}{\partial r\,\partial\lambda}$, $\frac{1}{r}\frac{\partial^2}{\partial r\partial\varphi}$. The factors by which the degree-variances are multiplied becomes $l+2$ instead of $l+1$ see (Tscherning, 1975, Table 1) and (Krarup and Tscherning, 1983 eq, (7) - (9)).

It is often so that the variances are needed in pairs or quadruples, e.g. covariances between two pairs of deflections of the vertical. These 2×2 matrices or 1×2, 2×1 vectors may easily be computed simultaneously. A new common /CW/CV(2,2) has been introduced to facilitate the transfer of these quantities to the calling program.

This permit a check of the algorithm, by "rotating" the matrices as discussed in (Ibid, 1983, section 4). With nd the maximal order of differentiation the result must be

$$nd = 1:\quad \begin{Bmatrix} sF_1 & 0 \\ 0 & 0 \end{Bmatrix} \frac{1}{r} \text{ or } \frac{1}{r'}$$

$$nd = 2:\quad \begin{Bmatrix} s^2F_2 - tF_1 & 0 \\ 0 & -F_1 \end{Bmatrix} \frac{1}{rr'} \qquad \text{(differentiation in P and Q)}$$

$$\begin{Bmatrix} s^2F_2 & 0 \\ 0 & 0 \end{Bmatrix} \frac{1}{r^2} \text{ or } \frac{1}{(r')^2} \qquad \text{(differentiation in either P or Q)}$$

$$nd = 3: \begin{Bmatrix} s^3 F_3 - 2ts F_2 & 0 \\ 0 & 2sF_2 \end{Bmatrix} \frac{1}{rr'} * \frac{1}{r} \text{ or } \frac{1}{r'}$$

$$nd = 4: \begin{Bmatrix} s^4 F_4 - 4ts^2 F_3 + 2(t^2+1)F_2 & 0 \\ 0 & 4(tF_2 - s^2 F_3) \end{Bmatrix} \frac{1}{(rr')^2}$$

where $s = \sin(\psi)$.

References:

Krarup, T. and C.C. Tscherning: Evaluation of isotropic covariance functions of torsion balance observations. Submitted Bulletin Geodesique, 1983.

Tscherning, C.C.: Covariance expressions for second and lower order derivatives of the anomalous potential. Reports of the Department of Geodetic Science, No. 225, The Ohio State University, Columbus, 1976.

Appendix 1. Modified version of COVAX, and of a calling program with a set of test data. Changes are marked in the margin.

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C TEST OF COVARIANCE FUNCTION SUBROUTINES, PROGRAMMED BY C.C.TSCHERNING,
C DEP.GEODETIC SCIENCE, OSU AND GEODAETISK INSTITUT,KOEBENHAVN JUNE 75.
C MODIFIED SEP 1975 ACCORDING TO REF.(C).
C REFERENCES:
C (A) TSCHERNING,C.C.: COVARIANCE EXPRESSIONS FOR SECOND AND LOWER ORDER
C      DERIVATIVES OF THE ANOMALOUS POTENTIAL, REPORTS OF THE DEP. OF
C      GEODETIC SCIENCE NO. 225,1975.
C (C) KRARUP, T. AND C.C.TSCHERNING: EVALUATION OF ISOTROPIC COVARIANCE
C      FUNCTIONS OF TORSION BALANCE OBSERVATIONS. SUBMITTED BULLETIN GEO-
C      DESIQUE, 1983.
C
      IMPLICIT REAL *8(A-H,O-Z),LOGICAL (L)
      COMMON /CW/ CV(2,2)
      COMMON /CMCOV/CI(12),CR(51),SIGMA0(300),SIGMA(300),HMAX,KI(25),N1,
     *N2,LOCAL,LSUM
C THE COMMON AREA IS USED FOR THE TRANSFER OF DATA TO AND FROM THE SUB-
C ROUTINE COVAX.
      DIMENSION COV(181,7),KP(7),KQ(7),HP(7),HQ(7)
     *,KX(3),KY(3),IP(3),IQ(3),LA(3),SM(2001)
      DATA GM,RE/3.98D14,6371.0D3/
     *,D0,D1,D2/0.0D0,1.0D0,2.0D0/,PI/3.1415926535D0/
C RE IS THE MEAN RADIUS OF THE EARTH AND GM IS THE PRODUCT OF THE GRAVI-
C TATIONAL CONSTANT AND THE MASS OF THE EARTH.
C
      WRITE(6,10)
   10 FORMAT('0TEST OF COVARIANCE FUNCTION SUBROUTINES, VERS. SEP 83.',
     *//,' COVARIANCES BETWEEN QUANTITIES OF KIND KP,KQ ARE COMPUTED. ',
     */,' THE KINDS AND CORRESPONDING UNITS ARE AS FOLLOWS: (E=EOTVOS'
     *,'):',/,
     *' (1) THE HEIGHT ANOMALY (METERS), (2) THE NEGATIVE RADIAL DER-',/
     *,' IVATIVE DIVIDED BY THE RADIAL DISTANCE (E), (3) THE GRAVITY',/,
     *' ANOMALY (MGAL), (4) THE RADIAL DERIVATIVE OF (3) (E), (5) THE',/
     *' SECOND ORDER RADIAL DERIVATIVE (E), (6),(7) THE LATITUDE AND',/,
     *' THE LONGITUDE COMPONENTS OF THE DEFLECTIONS OF THE VERTICAL',/,
     *' (ARCSECONDS), (8),(9) THE DERIVATIVES OF (3) IN NORTHERN AND',/,
     *' EASTERN DIRECTION, RESPECTIVELY (E), (10),(11) THE DERIVATIVE',/
     *,' OG (2) IN THE SAME DIRECTIONS (E), (12) - (15) THE SECOND',/
     *,' ORDER DERIVATIVES WITH RESPECT TO LATITUDE, IN NORTHERN',/,
     *' AND EASTERN DIRECTION * 2, WITH RESPECT TO LONGITUDE, AND',/,
     *' IN EASTERN MINUS NORTHERN DIRECTIONS, RESPECTIVELY (E).',//)
C
C INPUT OF THE VALUE OF A LOGICAL VARIABLE, LTEST, TRUE WHEN TEST OUT-
C PUT IS NEEDED.
      READ(5,11)LTEST
   11 FORMAT(L2)
C INPUT OF QUANTITIES SPECIFYING THE DEGREE-VARIANCE MODEL TO BE USED IN
C THE FOLLOWING SEQUENCE: THE RATIO BETWEEN AN ADOPTED BJERHAMMAR-SPHERE
C RADIUS (RB) AND RE, SQUARED, THE QUANTITY A(I) IN REF(A),EQ.(17), DI-
C VIDED BY RB**2 IN UNITS OF MGAL**2, THE INTEGERS K(2),K(3) OF EQ.
C (17), WHEN APPLICABLE, OTHERWISE A ZERO, THE VALUE OF A LOGICAL VARI-
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C ABLE, LOCAL, TRUE,WHEN THE DEGREE-VARIANCES UP TO-AND INCLUSIVE DEGREE
C N ARE ZERO AND FALSE WHEN EMPIRICAL ANOMALY DEGREE-VARIANCES UP TO
C ORDER N WILL BE INPUT, THE INTEGER N, THE INTEGER KT EQUAL TO THE
C DEGREE-VARIANCE MODEL NUMBER (1,2 OR 3), THE VALUE OF THE LOGICAL
C VARIABLE LSUM, WHICH IS TRUE WHEN A FINITE LEGENDRE SERIES (MAXIMAL
C DEGREE 2000) MUST BE USED FOR THE EVALUATION OF COVARIANCES IN ALTI-
C TUDES GREATHER THAN HMAX AND OTHERWISE EQUAL TO FALSE, THE VALUE OF
C N2 (WHICH MUST BE LESS THAN OR EQUAL TO 2000), THE VALUE OF THE HEIGHT
C HMAX IN METERS, AND FINALLY THE VALUE OF LAST1, TRUE, WHEN THE LIST
C OF INPUT PARAMETERS IS THE LAST ONE.
C
  100 READ(5,9)S,A,KI(3),KI(4),LOCAL,N,KT,LSUM,N2,HMAX,LAST1
    9 FORMAT(2D14.7,2I5,L2,2I5,L2,I5,D14.7,L2)
      IF (N2.LT.2) N2 = 2
      IF (N2.GT.2001) N2 = 2000
      N2 = N2+1
      KI(5) = KT
      WRITE(6,12)S,A,KI(3),KI(4),N,KT
   12 FORMAT('0PARAMETERS SPECIFYING THE MODEL DEGREE-VARIANCES:',/,
     *' S,A =',2D14.7,/,' K0-K3,N,KT=   -2   -1',4I5/)
      IF (LSUM) WRITE(6,6)HMAX,N2
    6 FORMAT(' WHEN THE HEIGHT OF ONE OF THE POINTS OF EVALUATION IS ABO
     *VE ',D14.7,' METERS',/,' WILL THE COVARIANCES BE EVALUATED BY MEAN
     *S OF A LEGENDRE SERIES HAVING ',I5,' TERMS.')
      RB2 = RE*RE*S
      LMODEL = N.EQ.0
C
      CI(8) = A*RB2*1.0D-10
      CI(10) = S
C CONVERTING THE CONSTANT A INTO UNITS OF (M/SEC)**4 AND SUBSEQUENT
C STORING OF A AND S IN THE ARRAY CI ACCORDING TO THE SPECIFICATIONS
C GIVEN IN COVAX.
C
      IF (N.NE.0) GO TO 101
C N EQUAL TO ZERO IMPLIES, THAT ALL DEGREE-VARIANCES ARE EQUAL TO THE
C MODEL DEGREE-VARIANCES. THIS AGAIN IS EQUIVALENT TO HAVING THE DEGREE-
C VARIANCES OF DEGREE 0,1,2 EQUAL TO ZERO, I.E. THE COVARIANCE FUNCTION
C USED IS A 2'-ORDER LOCAL COVARIANCE FUNCTION.
      LOCAL = .TRUE.
      N = 2
C
  101 N1 = N+1
C INPUT OF EMPIRICAL ANOMALY DEGREE-VARIANCES IN UNITS OF MGAL**2.
      IF(.NOT.LOCAL)READ(5,13)(SIGMA0(I), I = 1, N1)
   13 FORMAT(12F6.2)
      IF (.NOT.LOCAL)WRITE(6,7)(SIGMA0(I), I = 1, N1)
    7 FORMAT(' EMPIRICAL ANOMALY DEGREE-VARIANCES IN UNITS OF MGAL**2:',
     */,25(12F6.2/))
C
      N2 = 2001
      CALL COVAX(SM)
C THE ARRAY SM OCCURRING IN THE CALL OF COVAX IS USED TO STORE THE
C DEGREE-VARIANCES WHEN LSUM IS TRUE. THE DIMENSION OF THE ARRAY IS
C TRANSFERRED TO THE SUBROUTINE BY MEANS OF THE VARIABLE N2 OCCURRING
C IN THE COMMON AREA /CMCOV/.
C
C CONVERSION OF ANOMALY DEGREE-VARIANCES TO POTENTIAL-DEGREE-VARIANCES.
      WRITE(6,8)(SIGMA0(I), I = 1, N1)
    8 FORMAT('0MODEL DEGREE-VARIANCE CORRECTIONS:',/,50(6(1X,D11.4),/))
```

```
C
      KPP=0
      KQQ=0
C INPUT OF QUANTITIES SPECIFYING THE OUTPUT TABLE. THE TABLE WILL CON-
C TAIN MT COLUMNS OF COVARIANCES OF KINDS KP, KQ (TO BE INPUT SUBSEQUE-
C NTLY) COMPUTED IN POINTS P AND Q. EACH COLUMN WILL CONTAIN NCOV VALUES
C CORRESPONDING TO Q MOVING IN AN AZIMUTH (ALFA) IN STEPS OF LENGTH DT
C (MINUTES). THE SPECIFICATION CONSIST OF NCOV, DT, ALFA IN DEG.,MIN.,
C SEC AND A LOGICAL, LAST2, WHICH IS TRUE WHEN THIS IS THE LAST SPECI-
C FICATION FOR THE DEGREE-VARIANCE MODEL UNDER CONSIDERATION.
  102 READ(5,14)NCOV,DT,IDEG,MIN,SEC,LAST2
   14 FORMAT(I5,F7.2,I5,I3,F6.2,L2)
      CALL RAD(ALFA,IDEG,MIN,SEC,1)
      RT = 60.0*DT/206264.806
      LPOLE = DABS(ALFA).LT.1.OD-6.OR.DABS(ALFA-PI).LT.1.OD-6
      IF (LPOLE) GO TO 103
C
      CA = DCOS(ALFA)
      SA = DSIN(ALFA)
  103 IF (LTEST) WRITE(6,15)NCOV,DT,IDEG,MIN,SEC,ALFA
   15 FORMAT('ONCOV,DT,DEG,MIN,SEC,ALFA=',I4,F6.2,I5,I3,F6.2,D14.6)
C
      MV = 0
      MT = 1
C
      IF (NCOV.LE.181) GO TO 104
      NCOV = 181
      WRITE(6,37)
   37 FORMAT(' NCOV TOO BIG, FIXED TO 181.')
  104 MT = MT+1
      IF (MV.NE.0.AND.MV.NE.3) GO TO 112
C INPUT OF INTEGERS KP AND KQ SIGNIFYING THE KIND OF QUANTITIES BETWEEN
C WHICH WE WANT TO COMPUTE THE COVARIANCES. THE VALUES OF KP,KQ MUST BE
C EQUAL TO THE EQUATION NUMBERS OF REF(A), WHICH DEFINES THE QUANTITIES
C (1) - (9), (12) AND (14). THE VALUES 10, 11, 13, 15 CORRESPOND TO
C THE QUANTITIES GIVEN IN REF (C), EQ. (3) - (6).
C ON THE SAME PUNCH CARD INPUT OF THE HEIGHTS OF P AND Q AS WELL,(IN
C METERS),AND OF A LOGICAL VARIABLE LAST3, TRUE WHEN THIS KIND OF COVA-
C RIANCES ARE THE LAST ONES TO BE COMPUTED WITH THE CHOOSED FORM OF THE
C TABLE. THREE SETS OF VALUES MAY BE PUNCHED ON ONE CARD, CF. FORMAT
C STATEMENT 15.
      READ(5,16)(KX(K),KY(K),IP(K),IQ(K),LA(K),K = 1, 3)
   16 FORMAT(3(2I3,2I8,L2))
      MV = 0
  112 MV = MV+1
      KP(MT) = KX(MV)
      KQ(MT) = KY(MV)
      HP(MT) = IP(MV)
      HQ(MT) = IQ(MV)
      LAST3 = LA(MV)
C
      KI(6) = KP(MT)
      KI(7) = KQ(MT)
      LNEW = KP(MT).NE.KPP.OR.KQ(MT).NE.KQQ
C COMPUTATION OF CONSTANTS NEEDED FOR THE COVARIANCE COMPUTATION, WHICH
C ARE INDEPENDENT OF T AND THE HEIGHTS BY THE CALL OF COVBX.
      IF (LNEW) CALL COVBX
      KPP = KP(MT)
      KQQ = KQ(MT)
```

```
C
      IF (LTEST.AND.LNEW)
     *WRITE(6,17)(CI(K),K=1,7),(KI(K),K=6,25),(SIGMA(K),K=1,N1)
   17 FORMAT('0CI:',7D11.4,/,' KI:',20I3,/,' SI:',5D11.4,/,59(4X,5D11.4/
     *))
C
      DO 120 M = 1, NCOV
      IF (MT.EQ.2) COV(M,1) = (M-1)*DT
      RV = (M-1)*RT
      T = DCOS(RV)
C RV IS EQUAL TO THE SPHERICAL DISTANCE BETWEEN P AND Q IN UNITS OF RAD-
C IANS.
      U = DSIN(RV)
      IF (LPOLE) GO TO 105
      SQ = U*CA
      CQ = DSQRT(D1-SQ *SQ)
      SD = U*SA/CQ
      CD = T/CQ
      GO TO 106
C
  105 SD = D0
      CD = D1
      IF (RV.GT.PI/D2) CD = -D1
      CQ = T
      I = 1
      IF (ALFA.LT.D0) I = -1
      SQ = U*I
C
C TRANSFER OF COORDINATE INFORMATION TO THE SUBROUTINE ACCORDING TO
C THE SPECIFICATIONS GIVEN IN THE SUBROUTINE.
  106 CR(1) = T
      CR(2) = HP(MT)
      CR(3) = HQ(MT)
      CR(4) = D0
      CR(5) = SQ
      CR(6) = D1
      CR(7) = CQ
      CR(8) = SD
      CR(9) = CD
      CR(10) = GM/(RE+HP(MT))**2
      CR(11) = GM/(RE+HQ(MT))**2
      IF (LTEST)WRITE(6,18)SQ,CQ,SD,CD
   18 FORMAT('0SQ,CQ,SD,CD=',4D12.5)
      CALL COVCX(COV(M,MT))
C
      IF (.NOT.LTEST) GO TO 120
      KK = KI(8)+1
      WRITE(6,19)((CR(I*8+K+3),K=1,8),I=1,KK)
   19 FORMAT(' CR:',8D11.4,/,4(4X,8D11.4,/))
  120 CONTINUE
C
      IF (.NOT.(LAST3.OR.MT.EQ.7)) GO TO 104
C
C OUTPUT OF A TABLE OF COVARIANCES.
      WRITE(7,30)
      WRITE(6,30)
   30 FORMAT(' ')
      WRITE(6,20)IDEG,MIN,SEC
      WRITE(7,20)IDEG,MIN,SEC
```

```
   20 FORMAT( ' TABLE OF COVARIANCES:',/,
     *' BETWEEN QUANTITIES OF KIND KP AND KQ, EVALUATED IN P,Q,',/,
     *' HAVING SPHERICAL DISTANCE PSI, HEIGHTS HP, HQ',/,
     *' AND AN AZIMUTH OF',I5,' D',I3,' M',F6.2,' SEC FROM P TO Q.')
      WRITE(6,30)
      WRITE(7,30)
      WRITE(6,21)(KP(I),I=2,MT)
      WRITE(7,21)(KP(I),I=2,MT)
   21 FORMAT('      KP=  ',6(I6,5X))
      WRITE(6,22)(KQ(I),I=2,MT)
      WRITE(7,22)(KQ(I),I=2,MT)
   22 FORMAT('      KQ=  ',6(I6,5X))
      WRITE(6,23)(HP(I),I=2,MT)
      WRITE(7,23)(HP(I),I=2,MT)
   23 FORMAT('      HP=  ',6(1X,F10.1))
      WRITE(6,24)(HQ(I),I=2,MT)
      WRITE(7,24)(HQ(I),I=2,MT)
   24 FORMAT('      HQ=  ',6(1X,F10.1))
      WRITE(6,26)
      WRITE(7,26)
   26 FORMAT(' PSI')
C
      DO 113 K = 1, NCOV
      RM = COV(K,1)
      IE = IDINT(RM/60.0D0)
      RM = RM-60.0D0*IE
      WRITE(7,25)IE,RM,(COV(K,I), I = 2, MT)
  113 WRITE(6,25)IE,RM,(COV(K,I), I = 2, MT)
   25 FORMAT(I4,F6.2,6F11.5)
C
      MT = 1
      IF (.NOT.LAST3) GO TO 104
C
      IF (.NOT.LAST2) GO TO 102
C
      IF (.NOT.LAST1) GO TO 100
      STOP
      END
      SUBROUTINE RAD(RA,IDEG,MIN,SEC,MODE)
      IMPLICIT REAL*8(A-H,O-Z)
      DATA RS,PI/206264.806D0,3.1415926535D0/
      IG = 1
      IF (IDEG.LE.0 .AND. MODE.EQ.1) IG = -1
      IDEG = IDEG*IG
      IF (MODE.NE.1 .OR. MIN .GE. 0) GO TO 20
      IG = -1
      MIN = MIN*IG
C
   20 GO TO (30,40,50,60),MODE
   30 RA =IDEG*3600.0D0+MIN*60.0D0+SEC
      GO TO 70
   40 RA =IDEG*3600.0D0+SEC*60.0D0
      GO TO 70
   50 RA = SEC*3600.0D0
      GO TO 70
   60 RA = SEC*3240.0D0
   70 RA = RA/RS
   80 IF (DABS(RA).LE.PI) GO TO 90
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```
      RA = RA-2.0D0*PI*DSIGN(RA,1.0D0)
      GO TO 80
   90 RETURN
      END
      SUBROUTINE COVAX(SM)
C THE SUBROUTINE COMPUTES THE COVARIANCE BETWEEN TWO QUANTITIES OF A
C KIND SPECIPIED THROUGH THE VALUE OF TWO INTEGER VARIABLES (STORED IN
C KI(6) AND KI(7), SEE BELOW). THE QUANTITIES ARE EVALUATED IN TWO
C POINTS, P AND Q, THE COORDINATES OF WHICH ARE GIVEN IMPLICITLY BY THE
C VALUES OF CR(1) - CR(9).
C
C THE COVARIANCE FUNCTION USED IS DEFINED ACCORDING TO A DEGREE-VARIANCE
C MODEL AND A SET OF EMPIRICAL (POTENTIAL) DEGREE-VARIANCES. THE DEGREE-
C VARIANCE MODEL IS SPECIFIED THROUGH THE VALUES OF KI(1)-KI(5),CI(8)-
C CI(10) AND THE PARAMETERS N1 AND LOCAL OCCURRING IN THE COMMON BLOCK
C /CMCOV/. EMPIRICAL ANOMALY DEGREE-VARIANCES WILL HAVE TO BE STORED IN
C SIGMA0 WHEN LOCAL IS FALSE, AND ARE USED FOR THE COMPUTATION OF RESI-
C DUAL POTENTIAL DEGREE-VARIANCES, (SEE REF(A), EQ.(16)).
C
C THE SUBROUTINE HAS THREE ENTRIES, COVAX, COVBX AND COVCX, WHICH HAVE
C TO BE CALLED IN THIS SEQUENCE.
C
C BY THE CALL OF COVAX, THE KIND OF COVARIANCE FUNCTION TO BE USED IS
C DETERMINED. THE VALUE OF KI(5) WILL DETERMINE THE DEGREE-VARI-
C ANCE MODEL (1,2 OR 3, CF.REF(A),EQ.(17)) THAT WILL BE USED. THE QUAN-
C TITIES K(2),K(3) MUST BE STORED IN KI(3),KI(4), AND BE EQUAL TO ZERO
C WHEN NOT USED (EG.,KI(3),KI(4) BOTH ZERO WHEN KI(5)=1). THE QUANTITY
C A(I) MUST BE STORED IN CI(8) IN UNITS OF (M/SEC)**4, AND THE SQUARE OF
C THE RATIO BETWEEN THE RADIUS OF THE BJERHAMMAR-SPHERE (RB) AND THE
C MEAN RADIUS OF THE EARTH (RE) MUST BE STORED IN CI(10).
C
C THERE ARE THEN THREE POSSIBILITIES:
C (1) ONE OF THE DEGREE-VARIANCE MODELS IS USED WITHOUT MODIFICATIONS.
C     THE SUMMATION LIMIT P OF REF.(A),EQ.(20) IS THEN FIXED TO 3.
C     BECAUSE THIS IS EQUIVALENT TO REQUIRING THE FIRST 3 DEGREE-VARIAN-
C     AREA /CMCOV/ MUST BE EQUAL TO 3 AND .TRUE., RESPECTIVELY.
C     CES TO BE ZERO, THE VARIABLES N1 AND LOCAL STORED IN THE COMMON
C (2) A NUMBER (N1) OF THE ANOMALY DEGREE-VARIANCES (DEGREE ZERO TO
C     N1-1) ARE PUT EQUAL TO EMPIRICAL DETERMINED QUANTITIES. THE ANO-
C     MALY DEGREE-VARIANCE OF DEGREE K WILL HAVE TO BE STORED IN
C     SIGMA0(K+1) IN UNITS OF MGAL**2 WHEN CALLING COVAX. LOCAL MUST BE
C     EQUAL TO .FALSE.. COVAX WILL CONVERT THE ANOMALY DEGRE5-VARIANCES
C     INTO POTENTIAL DEGREE-VARIANCES.
C (3) THE N1 FIRST DEGREE-VARIANCES (DEGREE 0 - N1-1) ARE EQUAL TO ZERO.
C     THIS MEANS, THAT THE VALUES OF A (N1-1)-ORDER LOCAL COVARIANCE
C     FUNCTION WILL BE COMPUTED. LOCAL MUST HAVE THE VALUE .TRUE..
C IN ALL CASES N1 MUST BE LESS THAN 300.
C
C THE COVARIANCES WILL GENERALLY BE COMPUTED BY CLOSED EXPRESSIONS, BUT
C THEY MAY IN CERTAIN CASES BE USELESS IN BIG ALTITUDES OF NUMERICAL
C REASONS, CF. REF(A), SECTION 4. IN THEESE CASES MUST THE LOGICAL VARI-
C ABLE LSUM BE TRUE AND THE VARIABLE HMAX MUST HAVE ASSIGNED A VALUE
C EQUAL TO THE CRITICAL ALTITUDE. WHEN LSUM IS TRUE AND THE HEIGHT OF
C P OR Q IS GREATHER THAN HMAX, WILL THE SERIES REF(A), EQ.(16), ABBRE-
C VIATED TO DEGREE N2-1 BE USED FOR THE COMPUTATION OF THE COVARIANCES.
C THE VALUES OF LSUM, N2 AND HMAX WILL (IN THE SAME WAY AS FOR THE PARA-
C METERS SPECIFYING THE DEGREE-VARIANCE MODEL) BE TRANSFERRED TO COVAX
C THROUGH THE COMMON AREA /CMCOV/, BUT AN ARRAY SM IS TRANSFERRED AS A
C PARAMETER IN THE CALL IN ORDER TO ENABLE VARIABLE DIMENSIONING (SPECI-
C FIED BY THE VARIABLE N2 IN /CMCOV/).
C
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C THE CALL OF COVAX WILL ALSO INITIALIZE CERTAIN VARIABLES USED IN
C SUBSEQUENT COMPUTATIONS.
C
C THE CALL OF COVBX WILL FIX CERTAIN CONSTANTS USED FOR THE COMPUTA-
C TIONS, WHICH ARE INDEPENDENT OF THE POINTS P AND Q. WHEN COVBX IS CAL-
C LED, THE KIND OF QUANTITIES BETWEEN WHICH THE COVARIANCE IS TO BE
C COMPUTED MUST BE SPECIFIED.  THIS IS DONE BY STORING IN KI(6) AND
C KI(7) INTEGERS EQUAL TO THE EQUATION NUMBERS OF REF.A, EQ.(1) - (9)
C (12) AND (14) , AND 10, 11, 13, 15 CORRESPONDING TO REF.(C), EQ.
C (3) - (6).
C
C THE CALL OF COVCX WILL RESULT IN THE COMPUTATION OF THE COVARIANCE ,
C WHICH IS TRANSFERRED TO THE CALLING PROGRAM THROUGH THE VARIABLE COV.
C THE RESULT WILL ALSO BE TRANSFERRED IN THE COMMON CW.
C INFORMATION RELATED TO THE COORDINATES OF P AND Q MUST BE STORED IN
C THE ARRAY CR WHEN COVCX IS CALLED, SEE BELOW.
C REFERENCES:
C (A) TSCHERNING,C.C.: COVARIANCE EXPRESSIONS FOR SECOND AND LOWER ORDER
C      DERIVATIVES OF THE ANOMALOUS POTENTIAL, REPORTS OF THE DEP. OF
C      GEODETIC SCIENCE NO. 225,1975.
C (B) TSCHERNING,C.C. AND R.H.RAPP: CLOSED COVARIANCE EXPRESSIONS
C      FOR GRAVITY ANOMALIES, GEOID UNDULATIONS, AND DEFLECTIONS OF
C      THE VERTICAL IMPLIED BY ANOMALY DEGREE-VARIANCE MODELS. DEP-
C      ARTMENT OF GEODETIC SCIENCE, THE OHIO STATE UNIVERSITY,
C      REPORT NO. 208, 1974.
C (C) KRARUP, T. AND C.C.TSCHERNING: EVALUATION OF ISOTROPIC COVARIANCE
C      FUNCTIONS OF TORSION BALANCE OBSERVATIONS. SUB. BULL. GEOD., 1983.
C
      IMPLICIT REAL *8(A-H,O-Z), LOGICAL (L)
C
      COMMON /CW/ CV(2,2) HD(36)
      COMMON /CMCOV/CI(12),CR(51),SIGMA0(300),SIGMA(300),HMAX,KI(25),N1,
     *N2,LOCAL,LSUM
C THE COMMON BLOCK CONTAINS THE VALUES OF PARAMETERS USED FOR THE COM-
C PUTATIONS AND RETURN VALUES OF FUNCTIONS AND CONSTANTS, WHICH HAVE
C BEEN USED IN THE COMPUTATIONS.
C PARAMETERS USED FOR THE COMPUTATIONS:
C   CI(8) = THE CONSTANT A(I) OF REF.(A), EQ.(17) IN UNITS OF (M/SEC)**4
C   CI(10) THE SQUARE OF THE RATIO BETWEEN THE BJERHAMMAR-SPHERE RADIUS
C   (RB) AND THE MEAN RADIUS OF THE EARTH (RE).
C   CR(1) COSINE OF THE SPHERICAL DISTANCE BETWEEN P AND Q,
C   CR(2),CR(3) THE HEIGHT OF P, Q, RESPECTIVELY, (UNITS METERS),
C   CR(4),CR(5) SINE OF THE LATITUDE OF P, Q, RESPECTIVELY,
C   CR(6),CR(7) COSINE OF THE LATITUDE OF P, Q, RESPECTIVELY,
C   CR(8),CR(9) SINE AND COSINE OF THE LONGITUDE DIFFERENCE,
C   CR(10),CR(11) THE REFERENCE GRAVITY IN P, Q, RESPECTIVELY (WHEN
C   USED, OTHERWISE STORE 1.0D0), (UNITS M/SEC**2).
C   SIGMA0(1)-SIGMA0(N1) MUST CONTAIN THE EMPIRICAL ANOMALY DEGREE-
C   VARIANCES IN UNITS OF MGAL**2.
C   KI(3) = K(2) OF DEG.VAR. MODEL 2 OR 3,
C   KI(4) = K(3) OF DEG.VAR. MODEL 3, CF. REF.(A), EQ.(17).
C   KI(5) = THE DEG.VAR. MODEL NUMBER, (EQUAL TO 1, 2 OR 3),
C   KI(6),KI(7) THE INTEGER SPECIFYING THE KIND OF QUANTITY WHICH IS
C   ASSOCIATED WITH P, Q, RESPECTIVELY,
C   N1 = THE NUMBER OF EMPIRICAL DEGREE-VARIANCES USED (LOCAL =.FALSE.)
C   OR (ORDER+1) OF THE LOCAL COVARIANCE FUNCTION USED (LOCAL=.TRUE.).
C   HMAX, N2, LSUM. HMAX IS THE HEIGHT ABOVE WHICH THE LEGENDRE SERIES
C   OF MAXIMAL DEGREE N2-1 WILL BE USED FOR THE COMPUTATION OF THE CO-
C   VARIANCES WHEN LSUM IS TRUE. N2 MUST BE GREATHER THAN 2 AS WELL AS
C   GREATHER THAN N1.
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C RETURN VALUES:
C   CI(1)-CI(7), THE QUANTITIES C(J,Q) OF REF.(A), EQ.(47), WITH
C   CI(1) - CI(KI(5)+1) = C(J,Q), CI(5) = C(KI(5)+2,Q),
C   CI(6) = C(KI(5)+3,Q), CI(7) = C(KI(5)+4,Q),
C   CI(9) = RB**2, CI(11),CI(12) QUANTITIES USED TO GIVE THE COMPUTED
C   COVARIANCES THE PROPER UNITS.
C   CR(ND*8+12), THE VALUES OF THE ND'TH DERIVATIVE OF THE SUM OF THE
C   FINITE LEGENDRE-SERIES, CF.REF.(A), EQ.(20),(48) AND (52).
C   CR(ND*8+13) - CR(ND*8+19), THE VALUES OF THE ND'TH DERIVATIVES OF
C   THE FUNCTIONS F(-2), F(-1), F(KI(3)), F(KI(4)), S0, S1, S2, CF. REF.
C   (A), EQ. (42), (41), (39), (39), (30), (34) AND (35).
C   SIGMA0(1) - SIGMA0(N1) THE POTENTIAL DEGREE-VARIANCE CORRECTIONS,
C   CF. REF.(A), EQ.(16), (AFTER THE CALL OF COVAX).
C   SIGMA(4) - SIGMA(N1), THE POTENTIAL DEGREE-VARIANCES MULTIPLIED BY
C   THE FACTORS GIVEN IN REF.(A), TABLE 1.
C   SIGMA(1) - SIGMA(3), THE DEGREE-VARIANCES OF DEGREE 0,1,2 MINUS
C   TERMS OF THE SAME DEGREES ACQUIRED FROM REF.(A), EQ.(34),(35),(41)
C   AND (42).
C   KI(8),KI(9) THE NUMBER OF DIFFERENTIATIONS IN RADIAL DIRECTION AND
C   WITH RESPECT TO T = COS(SPHERICAL DIST.) TO BE PERFORMED.
C   KI(10) - KI(15) THE CONSTANTS I,K,J,M,J1,M1 OF REF.(A), SECTION 2.
C   KI(16) - KI(19) THE QUANTITIES M(1) - M(4) OF REF.(A), EQ.(25)-(29).
C   KI(20),KI(21) THE EXPONENT OF THE REFERENCE GRAVITY,
C   KI(22),KI(23) THE EXPONENT OF THE RADIAL DISTANCE AND
C   KI(24),KI(25) SUBSCRIPTS OF THE RESULT STORED IN CV (COMMON CW).
C
      DIMENSION K7(15),K9(15),K11(15),K13(15),K15(15),K17(15),K19(15),
     *K21(15),K23(15),C11(15),K8(15),L(7),LN(7),CX(6,8),SM(N2)
     *,C(6),V(6),U(6),G(6),P(6),R(6),SS1(4),RM(6),Q(6)
     *,J2(2),I(3),I(4) I4(2), CZ(6),D(36)
C THE ARRAY SM IS USED TO STORE THE DEGREE-VARIANCES WHEN THE LOGICAL
C VARIABLE LSUM IS TRUE. IN CASE THE SUBSCRIPT LIMIT IS CHANGED IS IT
C NECESSARY TO CHANGE THE VALUE OF THE VARIABLE N2 ACCORDINGLY.
C
      EQUIVALENCE (CX(1,1),C(1)),(CX(1,2),V(1)),(CX(1,3),U(1)),
     *(CX(1,4),G(1)),(CX(1,5),P(1)),(CX(1,6),R(1)),(CX(1,7),SS1(1)),
     *(CX(2,8),SS2)
C
      DATA D0,D1,D2,D3,RE/0.0D0,1.0D0,2.0D0,3.0D0,6371.0D3/
     *,K7/5*0,6*1,4*2/,K9/5*1,2,3,2,3,2,3,2,2,3,4/,K11/11*0,2,3,3,6/,
     *K13/11*1,2,3,3,6/,K15/0,1,-1,-1,1,0,0,-1,-1,2,2,0,0,0,0/,
     *K17/3*0,2,2,10*0/,K19/1,4*0,1,1,8*0/,K21/0,2,1,2,2,1,1,8*2/,
     *K23/5*1,2,1,2,1,2,1,1,2,1,1/,K8/0,1,1,2,2,0,0,4*1,4*0/,
     *C11/1.0D0,1.0D9,1.0D5,2*1.0D9,2*-206264.806D0,
     *5*1.0D9,2.0D9,2*1.0D9/,D/36*0.0D0/,J2/3,2/,I3/6,3/,I4/4,2/
C THE ARRAYS K7 - K23 CONTAINS TABLES OF QUANTITIES RELATED TO THE KIND
C OF COVARIANCES (1 - 14) WHICH MAY BE COMPUTED. THE ELEMENT WITH SUBS-
C CRIPT 15 IS DUMMY (RESERVED FOR PROGRAM EXTENSIONS). THEIR ACTUAL VA-
C LUES WILL AFTER CALL OF COVBX BE STORED IN THE ELEMENTS OF THE ARRAY
C KI HAVING SUBSCRIPTS 8 - 25.
C K7 CONTAINS THE ORDER OF DIFFERENTIATION WITH RESPECT TO T,K8 THE
C ORDER OF DIFFERENTIATION WITH RESPECT TO THE RADIUS, CF.REF(A),TABLE
C 1. K9,K11,K13 THE KIND OF DIFFERENTIATIONS TO BE COMPUTED WITH RESPECT
C TO THE LATITUDE (2) AND THE LONGITUDE (3), CF.REF(A),SECTION 3. K15
C AND K17 CONTAINS AN INTEGER, WHICH WILL BE ADDED TO THE DEGREE. THE
C SUM WILL THEN BE MULTIPLIED WITH THE DEGREE-VARIANCE OF THE CORRESPON-
C DING DEGREE WHEN A FIRST AND/OR SECOND DIFFERENTIATION WITH RESPECT
C TO THE RADIAL DISTANCE HAS TAKEN PLACE.
C C11 CONTAIN QUANTITIES USED TO GIVE THE COVARIANCES THE PROPER UNITS.
C
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      KT = KI(5)
      KT1 = KT+1
      IF (KT.LT.3) GO TO 15
      DO 16 K = KT, 2
   16 KI(K+2) = D0
   15 KI(1) = -2
      KI(2) = -1
C
      IF ((KT.LT.3).OR.(KT.EQ.3.AND.KI(4).GT.KI(3))) GO TO 17
C ASSURING, THAT KI(4).GT.KI(3), BECAUSE THIS FACT IS USED IN SUB-
C SEQUENT COMPUTATIONS.
      K = KI(3)
      KI(3) = KI(4)
      KI(4) = K
   17 II = KI(3)
      JJ = KI(4)
      SM(1) = D0
      SM(2) = D0
      N3 = N1
      A = CI(8)
      S = CI(10)
      RB2 = S*(RE**2)
      CI(9) = RB2
      RB2 = RB2*1.OD-10
      T = D0
      Q(1) = D0
      RM(1) = D0
C
      SIGMA0(1) = D0
      SIGMA0(2) = D0
      IF (LOCAL) SIGMA0(3) = D0
      IF (.NOT.LOCAL) SIGMA0(3) = SIGMA0(3)*RB2/S**4
      IF (N1.LT.4) GO TO 14
      DO 13 K = 4, N1
      IF (.NOT.LOCAL) T = SIGMA0(K)*S**(-K-1)*RB2
      GO TO (10,11,12),KT
   10 KK = 1
      GO TO 13
   11 KK = K+II-1
      GO TO 13
   12 KK = (K+II-1)*(K+JJ-1)
   13 SIGMA0(K) = (T-A*(K-2)/((K-3)*KK))/(K-2)**2
   14 RETURN
C
C
      ENTRY COVBX
C BY THE CALL OF COVBX ALL QUANTITIES NECESSARY FOR THE COMPUTATION OF
C THE COVARIANCE, BUT INDEPENDENT OF THE POSITION OF THE POINTS P AND Q,
C ARE COMPUTED.
      RB2 = CI(9)
      CI(11) = D1
      IF (KI(6).EQ.16.OR.KI(7).EQ.16) GO TO 19
C
      DO 20 M = 1, 2
      K = KI(M+5)
C FOR M = 1, K IS EQUAL TO THE KIND EVALUATED IN P AND FOR M = 2 EQUAL
C TO THE KIND EVALUATED IN Q.
      KI(M+9) = K9(K)
      KI(M+11) = K11(K)
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      KI(M+13) = K13(K)
      KI(M+15) = K15(K)
      KI(M+17) = K17(K)
      KI(M+19) = K19(K)
      KI(M+21) = K21(K)
      KI(M+23) = K23(K)
C
   20 CI(11) = CI(11)*C11(K)
C
      KQ = K
      KP = KI(6)
      KI(8) = K7(KP)+K7(KQ)
      KI(9) = K8(KP)+K8(KQ)
   19 ND = KI(8)
      NR = KI(9)
C ND AND NR ARE THE NUMBER OF DIFFERENTIATIONS WITH RESPECT TO T AND THE
C RADIAL DISTANCES, RESPECTIVELY.
C
C UPDATING THE DEGREE-VARIANCES, CF. REF(A), TABLE 1.
      SIGMA(1) = D0
      SIGMA(2) = D0
      IF (LSUM) N1 = N2
      DO 21 M = 3, N1
      B = D1
      DO 22 I = 1, 4
   22 IF (KI(I+15).NE.0) B = B*(M+KI(I+15)-1)
      IF (M.LE.N3) SIGMA(M) = SIGMA0(M)*B
      IF (.NOT.LSUM.OR.M.EQ.3) GO TO 21
      DO 48 K = 1, KT1
   48 B = B/(M+KI(K)-1)
C STORING THE MODIFIED DEGREE-VARIANCES OF DEGREE M-1 IN SM(M) AND AD-
C DING THE DEGREE-VARIANCE CORRECTIONS FOR M .LE. N3.
      SM(M) = B*A
      IF (M.LE.N3)  SM(M) = SM(M)+SIGMA(M)
   21 CONTINUE
      IF (N1.GT.2) SM(3) = SIGMA(3)
      IF (LSUM) N1 = N3
C
C EVALUATION OF THE QUANTITIES C(J,NR), CF.REF(A), TABLE 2.
      DO 23 K = 1, 7
   23 CI(K) = D0
C
      DO 25 K = 1, KT1
      CI(K) = D1
      DO 25 KQ = 1, KT1
   25 IF (K.NE.KQ) CI(K) = CI(K)/(KI(KQ)-KI(K))
C CF.REF(A),EQ.(19). WE WILL THEN COMPUTE THE QUANTITIES GIVEN IN REF(A)
C TABLE 2.
      IF (NR.LT.2) GO TO 29
      KP = KI(16)+KI(17)+KI(18)+KI(19)
      IF (NR.EQ.4) M = KI(16)*(KI(17)+KI(18)+KI(19))+KI(17)*(KI(18)+
     *KI(19))+KI(18)*KI(19)
C
      GO TO (26,27,28),KT
   26 CI(NR+3) = D1
      IF (NR.GT.2) CI(NR+2) = KP+3
      IF (NR.EQ.4) CI(NR+1) = M+3*KP+7
      GO TO 29
   27 IF (NR.GT.2) CI(NR+2) = D1
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```
      IF (NR.EQ.4) CI(NR+1) =-KI(3)+3+KP
      GO TO 29
   28 IF (NR.EQ.4) CI(NR+1) = D1
   29 IF (NR.EQ.0) GO TO 31
C
      DO 30 KP = 1, 4
      DO 30 K  = 1, KT1
   30 IF (KI(KP+15).NE.0) CI(K) = CI(K)*(KI(KP+15)-KI(K))
C
C THE LOGICAL ARRAYS L AND LN REGISTER WHICH TERMS THAT WILL HAVE TO
C BE EVALUATED , RESPECTIVELY NOT EVALUATED IN REF.(A), EQ. (47).
   31 DO 38 K = 1, 7
      L(K) = DABS(CI(K)).GT.1.0D-15
   38 LN(K) = .NOT.(L(K))
C
      DO 32  K = 3, 7
      DO 32  M = 1, 3
      IF (M.EQ.1.AND.K.GT.5.OR.(M+KI(K)-1).EQ.0.AND.K.LT.5.OR.LN(K))
     *GO TO 32
      GO TO (34,34,35,35,34,36,37),K
   34 B = D1
      GO TO 33
   35 B = D1/(M+KI(K) -1)
      GO TO 33
   36 B = (M-1)
      GO TO 33
   37 B = (M-1)*(M-1)
   33 SIGMA(M) = SIGMA(M)-A*CI(K)*B
   32 CONTINUE
      SIGMA(3) = SIGMA(3)-A*CI(2)
C
      ND1 = ND+1
      ND2 = ND+2
      RETURN
C
      ENTRY COVCX(COV)
C COMPUTATION OF THE COVARIANCE IN A SPECIFIC PAIR OF POINTS. THE VALUE
C IS RETURNED THROUGH THE PARAMETER COV.
C THE COVARIANCES COMPUTED WILL BE IN UNITS CORRESPONDING TO THE KIND OF
C QUANTITIES, I.E. FOR KIND (1) METERS, (2) EOTVOS (E), (3) MGAL,
C (4),(5) E, (6),(7) ARCSECONDS, (8) - (14) E.
C THE FOLLOWING QUANTITIES MUST BE STORED IN THE ELEMENTS OF THE ARRAY
C CR WHEN COVCX IS CALLED: (1) COSINE TO THE SPHERICAL DISTANCE BETWEEN
C P AND Q, (2),(3) THE HEIGHT OF P, Q RESPECTIVELY, (4),(5) SINE OF THE
C THE LATITUDE OF P, Q, RESPECTIVELY, (6),(7) COSINE OF THE LATITUDE OF
C P, Q, RESPECTIVELY, (8),(9) SINE AND COSINE OF THE LONGITUDE DIFFER-
C ENCE. THE REFERENCE GRAVITY WILL HAVE TO BE STORED IN CR(10),CR(11)
C FOR P, Q RESPECTIVELY (WHEN USED, OTHERWISE STORE 1.0).
      T = CR(1)
      HP = CR(2)
      HQ = CR(3)
      SP = CR(4)
      SQ = CR(5)
      CP = CR(6)
      CQ = CR(7)
      SD = CR(8)
      CD = CR(9)
      RP = RE+HP
      RQ = RE+HQ
C IN HEIGH ALTITUDES AND WHEN LSUM IS TRUE WILL THE COVARIANCE BE COM-
C PUTED BY A SUMMATION OF THE LEGENDRE-SERIES ABBREVIATED TO DEGREE
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C N2-1.
      LSUMC = LSUM .AND. (HP.GT.HMAX .OR. HQ.GT.HMAX)
C COMPUTATION OF THE CONSTANT USED TO CONVERT THE COVARIANCE INTO
C PROPER UNITS.
      CI(12) = CI(11)/(RP**KI(22)*RQ**KI(23)
     **CR(11)**KI(21)*CR(10)**KI(20))
C
      S = RB2/(RP*RQ)
      S2 = S*S
      ST = S*T
      T2 = T*T
      P2 = (D3*T2-D1)/D2
      P3 = (D3*ST+D1)/D2
C
C INITIALIZING ARRAY ELEMENTS. NOTE THE USE OF THE EQUIVALENCING.
      DO 50  K = 1, 8
      DO 50 M = 1, ND2
   50 CX(M,K) = DO
      DO 51 K = 1, ND2
      C(K) = DO
   51 D(K) = DO
      DO 52 K = 1, 40
   52 CR(K+11) = DO
C
C SUMMATION AND DIFFERENTIATION OF THE LEGENDRE SERIES, CF.REF(A),EQ.
C (49) AND (51).
      IF (LSUMC) N1 = N2
      K1 = N1
      K2 = N1+1
      K = N1-1
      DO 54 M = 1, N1
      GI = (D2*K+D1)*S/K1
      GJ = -K1*S2/K2
      K2 = K1
      K1 = K
      K = K-1
      IF (.NOT.LSUMC) SI = SIGMA(K2)
      IF (LSUMC) SI = SM(K2)
      I2 = 0
      I1 = 1
      DO 53 I = 2, ND2
      B = D(I)
      D(I) = C(I)
      C(I) = GI*(D(I)*T+I2*D(I1))+GJ*B+SI
      SI = DO
      I2 = I1
   53 I1 = I
   54 CONTINUE
      LOLDP = (KI(6).EQ.12) .OR. (KI(6).EQ.14)
      LOLDQ = (KI(7).EQ.12) .OR. (KI(7).EQ.14)
      IF (LSUMC) N1 = N3
C
C COMPUTATION OF THE QUANTITIES D(1)-D(36),CF.REF(A),SECTION 3.
C (MODIFIED ACCORDING TO REF.(C)).
      IF (ND.EQ.0) GO TO 55
C
```

```
      D(1) = D1
      CS = CP*SQ
      SC = SP*CQ
      SCC = SC*CD
      CC = CP*CQ
      CCS = CC*SD
      CSC = CS*CD
      D(2) = CS-SCC
      D(7) = SC-CSC
      CPSD = CP*SD
      CPCD = CP*CD
      CQSD = CQ*SD
      CQCD = CQ*CD
      D(3) = CQSD
      D(13)=-CPSD
C
      IF (ND.EQ.1) GO TO 55
      SS = SP*SQ
      D(8) = CC+SS*CD
      D(9) = -SQ*SD
      D(14)= SP*SD
      D(15)= CD
      IF (LOLDP) GO TO 91
      D(4) = D(2)+D(3)
      D(6) = D(3)-D(2)
      GO TO 92
   91 D(4) = -T
      D(6) = -CQCD/CP
   92 IF (LOLDQ) GO TO 93
      D(19)= D(13)+D(7)
      D(31)= D(13)-D(7)
      GO TO 94
   93 D(19)= -T
      D(31)= -CPCD/CQ
C
   94 IF (ND.EQ.2) GO TO 55
      IF (LOLDP) GO TO 95
      D(10) = D(9)+D(8)
      D(12) = D(9)-D(8)
      D(16) = D(15)+D(14)
      D(18) = D(15)-D(14)
      GO TO 96
   95 D(10) = -D(7)
      D(12) = SQ*CD/CP
      D(16) = CPSD
      D(18) = SD/CP
   96 IF (LOLDQ) GO TO 97
      D(20) = D(14)+D(8)
      D(32) = D(14)-D(8)
      D(21) = D(15)+D(9)
      D(33) = D(15)-D(9)
      GO TO 98
   97 D(20) = -D(2)
      D(21) = -CQSD
      D(32) = SP*CD/CQ
      D(33) = -SD/CQ
```

```
   98 IF (ND.EQ.3) GO TO 55
      IF (.NOT.(LOLDP.AND.LOLDQ)) GO TO 99
      D(22) = T
      D(24) = CQCD/CP
      D(34) = CPCD/CQ
      D(36) = CD/CC
      GO TO 55
   99 IF (.NOT.LOLDQ) GO TO 100
      D(22) = D(21)+D(20)
      D(24) = D(21)-D(20)
      D(34) = D(33)+D(32)
      D(36) = D(33)-D(32)
      GO TO 55
  100 D(22) = D(16)+D(10)
      D(34) = D(16)-D(10)
      D(24) = D(18)+D(12)
      D(36) = D(18)-D(12)
   55 IF (LSUMC) GO TO 75
C
C COMPUTATION OF THE FUNCTIONS L=R(1), N=1/RN, M=RM(2), FO=P(2), CF.
C REF.(A), EQ. (31)-(33),(40) AND (77A).
      RL2 = D1-D2*ST+S2
      RL = DSQRT(RL2)
      R(1) = RL
      RL1 = D1/RL
      RN = D1/(D1+RL-ST)
      RL2 = D1/RL2
      RNL = RN*RL1
      RM(2) = D1-RL-ST
      P(2) = S*DLOG(D2*RN)
      RL3 = RL2*RL1
      RL5 = RL3*RL2
      S3 = S2*S
      R(2) = -S*RL1
      IF (ND.EQ.0) GO TO 56
C
C COMPUTATION OF THE DERIVATIVES WITH RESPECT TO T.
C CF. REF.(A), EQ. (77B),(69A),(57).
      R(3) = -S2*RL3
      RM(3) = -R(2)-S
      P(3) = S2*(RNL+RN)
      IF (ND.EQ.1) GO TO 56
C
C CF. REF.(A), EQ. (77C),(69B),(58).
      R(4) = -D3*S3*RL5
      RM(4) = -R(3)
      P(4) = S3*(RL3+(D1+(D2+RL1)*RL1)*RN)*RN
      IF (ND.EQ.2) GO TO 56
C
C CF. REF.(A), EQ. (77D),(69C),(59).
      RL4 = RL2*RL2
      RL7 = RL5*RL2
      S4 = S2*S2
      R(5) = -15.0D0*S4*RL7
      RM(5) = -R(4)
      P(5) = S4*(D3*RL5+((D3+D3*RL1)*RL3+D2*(D1+(D3+(D3+RL1)*RL1)*RL1)
     **RN)*RN)*RN
      IF (ND.EQ.3) GO TO 56
```

```
C
C CF. REF.(A), EQ. (69D),(60).
      S5 = S4*S
      RL6 = RL4*RL2
      RM(6) = -R(5)
      P(6) = D3*S5*((5.0D0*RL7+((4.0D0+5.0D0*RL1)*RL5+((4.0D0+(8.0D0
     *+4.0D0*RL1)*RL1)*RL3+(2.0D0+(8.0D0+(12.0D0+(8.0D0+D2*RL1)*RL1)
     **RL1)*RL1)*RN)*RN)*RN)*RN)
C
   56 IF (LN(2)) GO TO 58
C COMPUTATION OF THE FUNCTION F-1 AND ITS DERIVATIVES, CF. REF.(A),
C EQ. (41) AND (61) - (65).
      U(2) = S*(RM(2)+T*P(2))
      IF (ND2.LT.3) GO TO 58
      DO 57 K = 3, ND2
   57 U(K) = S*(RM(K)+T*P(K)+(K-2)*P(K-1))
C
   58 IF (LN(1)) GO TO 60
C COMPUTATION OF THE FUNCTION F-2 AND ITS DERIVATIVES, CF. REF.(A) EQ.
C (42), AND (65)- (68).
      DO 59 K = 2, ND2
      GO TO (61,61,62,63,64,65),K
   61 CY = S*(D1-T2)/4.0D0
      GO TO 59
   62 CY = -ST/D2
      GO TO 59
   63 CY = D3*P(2)-S/D2
      GO TO 59
   64 CY = 9.0D0*P(3)
      GO TO 59
   65 CY = 18.0D0*P(4)
   59 V(K) = S*(RM(K)*P3+S*((K-2)*D3*RM(K-1)/D2+P2*P(K)+D3*T*P(K-1)*
     *(K-2)+CY))
C
   60 IF (LN(3)) GO TO 73
C COMPUTATION OF THE FUNCTION F1 AND ITS DERIVATIVES, CF. REF.(A) EQ.
C (36), REF.(B), EQ.(101) AND REF.(A), EQ.(70),(71).
      Q(2) = DLOG(D1+D2*S/(D1-S+RL))
      IF (ND.EQ.0) GO TO 66
      Q(3) = S2*RNL
      IF (ND.EQ.1) GO TO 66
      Q(4) = S3*((RL1+D1)*RN+RL2)*RNL
      IF (ND.EQ.2) GO TO 66
      Q(5) = S4*(D3*RL4+((D2+D3*RL1)*RL2+(D2 +(4.0D0+D2*RL1)*RL1)*RN)
     **RN)*RNL
      IF (ND.EQ.3) GO TO 66
      Q(6) = D3*S5*(5.0D0*RL6+((D3+5.0D0*RL1)*RL4+((D2+(6.0D0+4.0D0*RL1)
     **RL1)*RL2+(2.0D0+(6.0D0+(6.0D0+D2*RL1)*RL1)*RL1)*RN)*RN)*RN)*RNL
C
C COMPUTATION OF THE FUNCTION F2 AND ITS DERIVATIVES, CF. REF.(A), EQ.
C (3),(72)-(75).
   66 P(2) = (RL-D1+T*Q(2))/S
      IF (ND.EQ.0) GO TO 68
      DO 67  K = 3, ND2
   67 P(K) = (R(K-1)+T*Q(K)+(K-2)*Q(K-1))/S
   68 I1 = II-1
      K1 = 1
```

```
      J1 = I1
      IF (I1.GE.2) GO TO 49
      DO 49 M = 2, ND2
      IF (I1.EQ.0) G(M) = Q(M)
      IF (I1.EQ.1) G(M) = P(M)
   49 CONTINUE
      IF (L(4)) J1 = JJ-1
      IF (J1.LE.1) GO TO 71
C
C CF. REF.(A), EQ. (38),(76).
      DO 71  K = 2, J1
      DO 69  M = 2, ND2
      B = Q(M)
      Q(M) = P(M)
   69 P(M) = (R(M-1)+(2*K-1)*((M-2)*Q(M-1)+T*Q(M))-K1/S*B)/(K*S)
      IF (K.NE.I1) GO TO 71
      DO 70 M = 2, ND2
   70 G(M) = P(M)
   71 K1 = K
C
   73 IF (LN(6)) GO TO 72
C CF. REF.(A), EQ. (34),(55).
      SS1(2) = S2*(T-S)*RL3
      IF (ND.GT.0) SS1(3) = S2*(RL3+D3*(T-S)*S*RL5)
C
C CF. REF.(A), EQ. (35).
   72 IF (L(7)) SS2= S2*((T+S)*RL3+D3*S*(T2-D1)*RL5)
C
C ADDING THE DIFFERENT TERMS, CF. REF.(A), EQ. (22),(47).
C TIPLIED BY RB**2 IN UNITS OF MGAL**2, THE INTEGERS K(2),K(3) OF EQ.
   75 DO 78 M = 2, ND2
C CF. REF.(A), EQ. (50),(52).
      C(M) = S*C(M)
      CR(M*8 -4) = C(M)
      DO 78 K = 1, 7
      IF (LN(K)) GO TO 78
C STORING THE TERMS FOR TRANSFER TO THE CALLING PROGRAM USING THE COMMON
C AREA /CMCOV/.
      CR(M*8+K -4) = A*CX(M,K+1)*CI(K)
      IF (K.EQ.5) CR(M*8+K-4) = -CR(M*8+K-4)
      C(M) = C(M)+CR(M*8+K -4)
   78 CONTINUE
C
C INTEGERS SPECIFYING THE KINDS OF DIFFERENTIATION WITH RESPECT TO THE
C LATITUDES AND/OR THE LONGITUDES, CF. REF.(A), SECTION 3.
      I = KI(10)
      J = KI(12)
      K = KI(11)
      M = KI(13)
      J1 = KI(14)
      M1 = KI(15)
      IF (.NOT.(LOLDP.OR.LOLDQ)) GO TO 110
C
      IJ = I+J
      IF (I.GT.3) IJ = 5
      KM = K+M
      IF (K.GT.3) KM = 5
C
C COMPUTATION OF THE DERIVATIVES OF ORDER ND WITH RESPECT TO THE LATI-
C TUDES AND THE LONGITUDES, CF. REF.(A), EQ. (43) - (46).
```

```
      GO TO (80,81,82,83,84),ND1
   80 COV = C(2)
      GO TO 85
   81 COV =  C(3)*D(I+6*(K-1))
      GO TO 85
   82 COV = D(I)*D(J1)*D(6*(K-1)+1)*D(6*(M1-1)+1)*C(4)+D(IJ+6*(KM-1))
     **C(3)
      GO TO 85
   83 COV =  D(IJ+6*(KM-1))*C(3)+(D(IJ)*D(6*(KM-1)+1)+D(I+6*(K-1))
     **D(J1+6*(M1-1))+D(I+6*(M1-1))*D(J1+6*(K-1)))*C(4)
     *+D(I)*D(J1)*D(6*(K-1)+1)*D(6*(M1-1)+1)*C(5)
      GO TO 85
   84 COV  = D(IJ+6*(KM-1))*C(3)+(D(IJ+6*(K-1))*D(6*(M-1)+1)
     *+D(I+6*(KM-1))*D(J)+D(J+6*(KM-1))*D(I)+D(IJ+6*(M-1))
     **D((K-1)*6+1)+D(IJ)*D(6*(KM-1)+1)+D(I+6*(K-1))*D(J+6*(M-1))
     *+D(I+6*(M-1))*D(J+6*(K-1)))*C(4)+(D(IJ)*D(6*(K-1)+1)*D(6*(M-1)+1)
     *+D(I+6*(K-1))*D(J)*D(6*(M-1)+1)+D(I+6*(M-1))*D(J)*D(6*(K-1)+1)
     *+D(J+6*(K-1))*D(I)*D(6*(M-1)+1)+D(J+6*(M-1))*D(I)*D(6*(K-1)+1)
     *+D(6*(KM-1)+1)*D(I)*D(J))*C(5)+D(I)*D(J)*D(6*(K-1)+1)*D(6*(M-1)
     *+1)*C(6)
C
C GIVING THE COVARIANCE THE PROPER UNITS.
   85 COV = COV*CI(12)
C
      GO TO 199
  110 CF=CI(12)
      IF (KI(6).EQ.13) CF=CF/D2
      IF (KI(7).EQ.13) CF=CF/D2
      DO 111 IX = 2, ND2
  111 CZ(IX-1) = C(IX)*CF
      CV(1,2) = D0
      CV(2,1) = D0
      CV(2,2) = D0
      GO TO (112, 113, 114, 115, 115), ND1
  112 CV(1,1) = CZ(1)
      GO TO 198
  113 IF (I.EQ.1) GO TO 116
      CV(1,1) = CZ(2)*D(3)
      CV(2,1) = CZ(2)*D(2)
      GO TO 198
  116 CV(1,1) = CZ(2)*D(13)
      CV(1,2) = CZ(2)*D(7)
      GO TO 198
  114 IF (I.GT.1) GO TO 117
      CV(1,1) = CZ(3)*D(19)*D(31)      + CZ(2)*
      CV(1,2) = CZ(3)*D(7)*D(13)*D2
      GO TO 198
  117 IF (K.GT.1) GO TO 118
      CV(1,1) = CZ(3)*D(4)*D(6)        + CZ(2)*
      CV(2,1) = CZ(3)*D(2)*D(3)*D2
      GO TO 198
  118 CV(1,1) = CZ(2)*D(15)+CZ(3)*D(13)*D(3)
      CV(2,2) = CZ(2)*D(8) +CZ(3)*D(2)*D(7)
      CV(1,2) = CZ(2)*D(9) +CZ(3)*D(3)*D(7)
      CV(2,1) = CZ(2)*D(14)+CZ(3)*D(13)*D(2)
C FIRST ORDER HORIZONTAL DERIVATIVES IN BOTH P AND Q.
      GO TO 198
  115 CONTINUE
```

```
      DO 119 IX = 1, 2
      DO 120 JX = 1, 2
      IF (ND.EQ.4) GO TO 121
C SECOND ORDER HORIZONTAL DERIVATIVE IN P OR Q.
      IF (KI(6) .GE. 12) GO TO 122
      CF = JX
      I = J2(IX)
      J1 = 1
      K = I4(JX)
      M1 = I3(JX)
      GO TO 123
  122 CF = IX
      I = I4(IX)
      J1 = I3(IX)
      K = J2(JX)
      M1 = 1
  123 K6 = 6*(K-1)
      M6 = 6*(M1-1)
      CV(IX,JX) = (CZ(3)*(D(I+K6)*D(J1+M6)+D(J1+K6)*D(I+M6))
     * +CZ(4)*D(I)*D(J1)*D(K6+1)*D(M6+1))*CF
      GO TO 120
  121 I = I4(IX)
      J = I3(IX)
      K = I4(JX)
      M = I3(JX)
      K6 = 6*(K-1)
      M6 = 6*(M-1)
      CV(IX,JX) = (CZ(3)*(D(I+K6)*D(J+M6)+D(I+M6)*D(J+K6))
     *  +CZ(4)*(D(J)*(D(I+K6)*D(M6+1)+D(I+M6)*D(K6+1))
     *          +D(I)*(D(J+K6)*D(M6+1)+D(J+M6)*D(K6+1)))
     *  +CZ(5)*D(I)*D(J)*D(K6+1)*D(M6+1))*IX*JX
  120 CONTINUE
  119 CONTINUE
  198 COV = CV(KI(24),KI(25))
  199 RETURN
      END
/*
 F
 0.9996170E+00 4.2528000E+02   24     0 F     2     2 F     0 0.0000000E+00 T
  0.00  0.00  7.50
    4  30.00      0 00 00.00 F
  1  1        0         0 F  2  1       0         0 F  5  1       0         0 F
 12  1        0         0 F 14  1       0         0 T
    4  30.00      0 00 00.00 F
  3  3     1000      1000 F  2  3    1000      1000 F  5  3    1000      1000 F
 12  3     1000      1000 F 14  3    1000      1000 T
    4  30.00      0 00 00.00 T
 13 13        0         0 F 13 15       0         0 F 15 13       0         0 F
  1 15        0         0 F  3 15       0         0 F  6 15       0         0 F
  7 15        0         0 F 10 15       0         0 F 11 15       0         0 F
 10 10        0         0 F 10 11       0         0 F 11 11       0         0 F
 10 13        0         0 F 11 13       0         0 F  3 10       0         0 T
/*
```

Appendix 2. Output from program.

TEST OF COVARIANCE FUNCTION SUBROUTINES, VERS. SEP 83.

COVARIANCES BETWEEN QUANTITIES OF KIND KP,KQ ARE COMPUTED. THE KINDS AND CORRESPONDING UNITS ARE AS FOLLOWS: (E=EOTVOS): (1) THE HEIGHT ANOMALY (METERS), (2) THE NEGATIVE RADIAL DERIVATIVE DIVIDED BY THE RADIAL DISTANCE (E), (3) THE GRAVITY ANOMALY (MGAL), (4) THE RADIAL DERIVATIVE OF (3) (E), (5) THE SECOND ORDER RADIAL DERIVATIVE (E), (6),(7) THE LATITUDE AND THE LONGITUDE COMPONENTS OF THE DEFLECTIONS OF THE VERTICAL (ARCSECONDS), (8),(9) THE DERIVATIVES OF (3) IN NORTHERN AND EASTERN DIRECTION, RESPECTIVELY (E), (10),(11) THE DERIVATIVE OG (2) IN THE SAME DIRECTIONS (E), (12) - (15) THE SECOND ORDER DERIVATIVES WITH RESPECT TO LATITUDE, IN NORTHERN AND EASTERN DIRECTION * 2, WITH RESPECT TO LONGITUDE, AND IN EASTERN MINUS NORTHERN DIRECTIONS, RESPECTIVELY (E).

PARAMETERS SPECIFYING THE MODEL DEGREE-VARIANCES:
S,A = 0.9996170D+00 0.4252800D+03
K0-K3,N,KT= -2 -1 24 0 2 2

EMPIRICAL ANOMALY DEGREE-VARIANCES IN UNITS OF MGAL**2:
0.00 0.00 7.50

MODEL DEGREE-VARIANCE CORRECTIONS:
0.0000D+00 0.0000D+00 0.3048D+05

TABLE OF COVARIANCES:
BETWEEN QUANTITIES OF KIND KP AND KQ, EVALUATED IN P,Q,
HAVING SPHERICAL DISTANCE PSI, HEIGHTS HP, HQ
AND AN AZIMUTH OF 0 D 0 M 0.00 SEC FROM P TO Q.

KP=	1	2	5	12	14
KQ=	1	1	1	1	1
HP=	0.0	0.0	0.0	0.0	0.0
HQ=	0.0	0.0	0.0	0.0	0.0
PSI					
0 0.00	926.59371	1.15780	23.19983	-10.44212	-10.44212
0 30.00	925.47496	1.12971	12.93102	-4.43459	-6.23701
1 0.00	922.88515	1.10161	10.45730	-3.25429	-4.99980
1 30.00	919.25573	1.07576	9.10668	-2.64768	-4.30748

TABLE OF COVARIANCES:
BETWEEN QUANTITIES OF KIND KP AND KQ, EVALUATED IN P,Q,
HAVING SPHERICAL DISTANCE PSI, HEIGHTS HP, HQ
AND AN AZIMUTH OF 0 D 0 M 0.00 SEC FROM P TO Q.

KP=	3	2	5	12	14
KQ=	3	3	3	3	3
HP=	1000.0	1000.0	1000.0	1000.0	1000.0
HQ=	1000.0	1000.0	1000.0	1000.0	1000.0
PSI					
0 0.00	1551.47275	2.65154	911.69904	-453.19798	-453.19798
0 30.00	791.63914	1.45146	53.79638	4.37320	-55.26665
1 0.00	568.76144	1.09365	22.32984	4.95074	-25.09328
1 30.00	450.43329	0.90066	13.03381	4.00347	-15.23596

TABLE OF COVARIANCES:
BETWEEN QUANTITIES OF KIND KP AND KQ, EVALUATED IN P,Q,
HAVING SPHERICAL DISTANCE PSI, HEIGHTS HP, HQ
AND AN AZIMUTH OF 0 D 0 M 0.00 SEC FROM P TO Q.

KP=	13	13	15	1	3	6
KQ=	13	15	13	15	15	15
HP=	0.0	0.0	0.0	0.0	0.0	0.0
HQ=	0.0	0.0	0.0	0.0	0.0	0.0
PSI						
0 0.00	3542.3031	0.00000	0.00000	0.00000	0.00000	0.00000
0 30.00	-22.76298	0.00000	0.00000	-1.80242	-64.26477	-0.03132
1 0.00	-6.06705	0.00000	0.00000	-1.74551	-31.17693	0.31006
1 30.00	-2.69672	0.00000	0.00000	-1.65981	-19.71971	0.31302

TABLE OF COVARIANCES:
BETWEEN QUANTITIES OF KIND KP AND KQ, EVALUATED IN P,Q,
HAVING SPHERICAL DISTANCE PSI, HEIGHTS HP, HQ
AND AN AZIMUTH OF 0 D 0 M 0.00 SEC FROM P TO Q.

KP=	7	10	11	10	10	11
KQ=	15	15	15	10	11	11
HP=	0.0	0.0	0.0	0.0	0.0	0.0
HQ=	0.0	0.0	0.0	0.0	0.0	0.0
PSI						
0 0.00	0.00000	0.00000	0.00000	3546.2886	0.00000	3546.2886
0 30.00	0.00000	-11.43705	0.00000	-12.31156	0.00000	11.74628
1 0.00	0.00000	-3.08325	0.00000	-3.10227	0.00000	2.76736
1 30.00	0.00000	-1.38205	0.00000	-1.34157	0.00000	1.16285

TABLE OF COVARIANCES:
BETWEEN QUANTITIES OF KIND KP AND KQ, EVALUATED IN P,Q,
HAVING SPHERICAL DISTANCE PSI, HEIGHTS HP, HQ
AND AN AZIMUTH OF 0 D 0 M 0.00 SEC FROM P TO Q.

KP=	10	11	3
KQ=	13	13	10
HP=	0.0	0.0	0.0
HQ=	0.0	0.0	0.0
PSI			
0 0.00	0.00000	0.00000	0.00000
0 30.00	0.00000	-23.41755	-63.72408
1 0.00	0.00000	-5.75287	-29.30766
1 30.00	0.00000	-2.45676	-18.02297

\- - -

NOTE: THE PROGRAM HAS BEEN RUN ON A RC8000-COMPUTER, USING 10.5 SIGNIFICANT DIGITS AND NO DOUBLE PRECISION TRIGONOMETRIC FUNCTIONS. DIFFERENCES BETWEEN THE NUMBERS HERE, AND THE NUMBERS OBTAINED ON OTHER COMPUTERS CAN BE EXPECTED, ESPECIALLY FOR PSI = 0.