

3.2 Estimation of and modelling of the covariance function.

Underlying the definition of the empirical covariance function $C(P,Q)$, cf. PG (Chapter 7), is the hypothesis, that we work in spherical approximation. We will do this here. Also we will only consider covariance functions, which are equal to the mean value of the product of point values $T(P)$ and $T(Q)$, where the mean is taken over all pairs of points which have the same spherical distance, and contingently also the same azimuth, α , from P to Q . If the mean is computed for a local area $\varphi_0 < \varphi < \varphi_1$, $\lambda_0 < \lambda < \lambda_1$ then we have a local covariance function. Otherwise we have a global covariance function. In both cases the points are supposed to be in fixed heights r and r' ,

$$C(P,Q) = \frac{1}{2\pi A} \int_0^{2\pi} \int_{\varphi_0}^{\varphi_1} \int_{\lambda_0}^{\lambda_1} T(\varphi, \lambda, r) T(\varphi', \lambda', r') \cos \varphi \, d\lambda \, d\varphi \, d\alpha,$$

where A is the area of the block bounded by φ_0, φ_1 , and λ_0, λ_1 , the spherical distance between P and Q is φ and $\varphi_0 < \varphi' < \varphi_1$, $\lambda_0 < \lambda' < \lambda_1$.

The evaluation is done by replacing the integrals by sums,

$$\hat{C}(\varphi_q, \alpha_k) = \frac{1}{p} \sum_{i=1}^n \sum_{j=1}^m T(\varphi_i, \lambda_j, R) T(\varphi'_i, \lambda'_j, R), \quad (3.9)$$

where $T(\varphi', \lambda', R)$ is put equal to zero if the point is outside the area, p is the number of products with Q in the area, $r = r' = R$,

$$\varphi_i = (\varphi_1 - \varphi_0)(i - \frac{1}{2})/n + \varphi_0, \quad \lambda_j = (\lambda_1 - \lambda_0)(j - \frac{1}{2})/m + \lambda_0,$$

$\alpha_k = k \, 2\pi/v$, $\varphi_q = q \, \varphi_{\max}/w$, $0 \leq k < v$, $0 \leq q < w$, integers, and φ_{\max} the maximal spherical distance between the points in the block. Then

$$\hat{C}(\varphi_q) = \frac{1}{v} \sum_{k=1}^v \hat{C}(\varphi_q, \alpha_k). \quad (3.10)$$

In practice not values of T , but of linear functionals L_p of T will be available. Hence $C(L_p, L_q)$ is estimated, and we will in the most important cases be able to find $C(P,Q)$ from this function.

