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Correlation Between Time Dependent Variations of Doppler-Determined Height and Sunspot Numbers

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Ellipsoidal heights h' have been determined using Doppler-satellite data collected between 1971 and 1982 in nearly 800 points in North America where the height H above mean sea level is also known. The approximate geoid height N was computed for each point using Rapp's (1978) set of potential coefficients complete to degree and order 180. An estimate of the ellipsoidal height h was then obtained using $h = H + N$. The differences $h' - h$ (which numerically were less than 4.0 m) were used to form mean values of height differences for periods from 1 month to 1 year. Yearly mean values, for which the error contribution from the geoid is small owing to the geographical distribution of the points, were then studied with respect to their time dependence. A strong correlation was observed. Yearly mean $(h' - h) \simeq 0.95s - 86$ (cm), where s is the mean smoothed sunspot number. Using this relationship as an estimate of the error in h' for the individual points due to uncompensated ionospheric effects, the standard deviation (1 sigma) of the yearly means of the differences decreased from 55 to 20 cm. A statistical test of the corrected differences showed that they can be represented as white noise at the 95% significance level.

1. INTRODUCTION

Since 1971, Doppler satellite observations have been used for the determination of positions. The accuracy of the derived coordinate differences for sets of coordinates determined using observations made within the same time period ("campaign") is around 50 cm (see, e.g., *Hothem et al.* [1978]).

However, the analysis of time variations of positions have shown unreasonably large rate of station height change of from 10 to 30 cm/yr [*Anderle and Malyevac*, 1982]. Changes are also noticed in latitude but not of the same magnitude. The reason for these changes has been attributed mainly to ionospheric effects (see *Paquet and Dehant* [1982] or *Strange et al.* [1982]).

A detailed analysis of the ionospheric effects is given by *Clynch and Renfro* [1982] and is illustrated by *Anderle and Malyevac* [1983]. In addition to the dominant diurnal dependence, the effect also depends (when considering a single station) on the solar activity, the geomagnetic field, the location of the station, and the orbits of the satellites used. However, even when modeling all these effects for the base tracking network, a significant portion remains [*Anderle and Malyevac*, 1983]. This could be caused by a mismodeling, and this will be discussed in section 6.

A direct modeling of these effects on all other observations is difficult. It requires a redistribution of the recomputed precise ephemerides of the tracked satellites to the agencies which were responsible for the observations, and these must then be processed again. This will probably not be possible in practice, so it is desirable for other methods for correcting the results to be found.

The height coordinate is very important; it can be used for the determination of the geoid height N at locations with known orthometric height (height above mean sea level) H:

$$N = h - H \tag{1}$$

The discrepancies between results from various Doppler observation campaigns have been especially annoying. For example, the Doppler-derived geoid height difference between two stations in Denmark only 8 km apart observed in 1977 and 1979, respectively, was determined to be more than 1 m; the gravimetrically determined values differed by less than 10 cm.

Until now, the changes in height have been analyzed for individual stations, such as the stations which form a part of the tracking station networks. The changes show similar patterns but also considerable differences, as could be expected, since the geomagnetic field varies considerably with position. No definite conclusions have been made concerning the physical reasons causing the height changes. The reasons for this have probably been the rather limited number of stations available with longer time spans of observations within an area where the geomagnetic field has a reasonably small variation.

If the geoid height is known in addition to the orthometric height, the ellipsoidal height h may be computed for all points within an area, and the differences between these and the Doppler-determined heights (which we, in the following, will denote h') may be analyzed with respect to their time variations. In the following, we shall describe how this was done and give the result obtained using a large set of points in North America.

2. TIME VARIATION OF DOPPLER-DETERMINED HEIGHTS

A data set of Doppler determined coordinates and orthometric heights for North America has been collected at the National Geodetic Survey (see Figure 1). A subset of this set will be used when readjusting the North American Datum (NAD).

In some cases the coordinates of a station have been deter-

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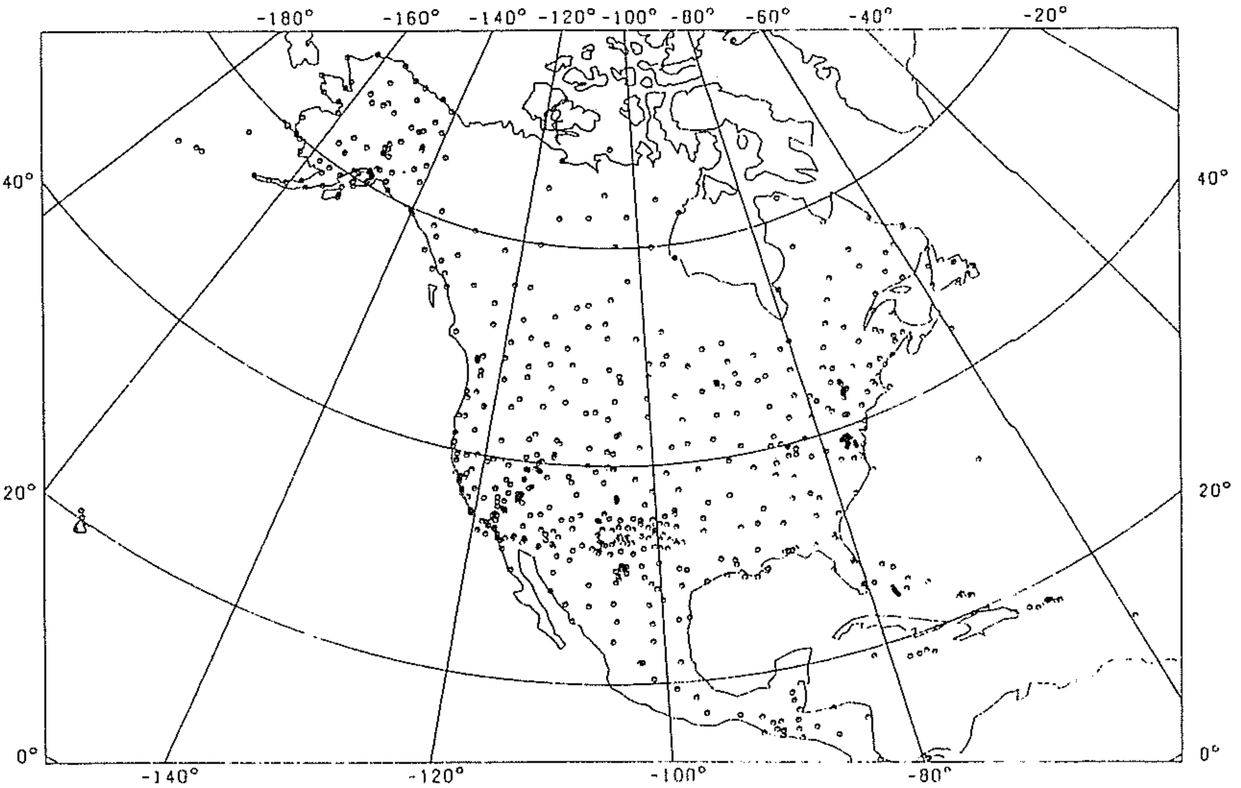

Fig. 1. Available Doppler-determined positions with orthmetric heights in North America.

mined several times, generally as a result of a reoccupation but in a few cases as the result of a recomputation or the simultaneous occupation of one site by two receivers (see Table 1).

For each record, the following values were available: station number, number of satellite passes, period of observation, latitude, longitude, and h', H, $h' - H$.

A scale change of -0.5 ppm, a shift of the Z coordinates of 4.0 m and a rotation around the Z axis of 0.5 arc sec were applied to the original Doppler-determined coordinates in order to bring the Doppler stations into a geocentric system with correct scale.

For all stations, an estimate of the geoid height N was computed using *Rapp*'s [1978] potential coefficient set complete to degree and order 180. An estimate of h was then obtained using $h = H + N$. The program published by *Tscherning et al.* [1983] was used for the computation of N.

The geoid height differences computed in this manner have an uncertainty of between 1.0 and 1.5 m in land areas. This uncertainty is largest in mountainous regions because the potential coefficients have been determined using mean free air gravity anomalies, which were not (and cannot be) reduced to the ellipsoid (R. H. Rapp, private communication, 1983). We use a value of 1.4 m for this uncertainty. Since the uncertainty in the orthometric height generally will be below 0.5 m, the uncertainty in the differences between computed ellipsoidal heights should then be approximately 1.5 m. (We state error estimates for differences, so the error in the reference system parameters such as the semimajor axis of the ellipsoid need not be considered.)

TABLE 1. Occupancy of Stations With Known Othometric Height

Number of Times Occupied	Number of Stations
1	762
2	45
3	6
4	5
>4	5

In order to exclude the effect of gross errors in the various quantities, we chose to reject all observations with difference $h' - h$ numerically greater than 4.0 m. Seventy-six such measurements were rejected. (The largest difference was -28 m.) The mean value of the remaining 792 observations was -0.29 m with a standard deviation of 1.59 m, which agrees well with the a priori estimate of 1.5 m. The small mean value indicates that the reference system parameters fit well with the datum shift given above. A rejection level of 5.0 m gave a mean value of 0.09 m and a standard deviation of 1.77 m.

A manual inspection of the rejected values revealed that most of the values were associated with points which were geographically close to each other (Hawaii, Caribbean Sea, southern coast of Alaska) and had very similar values or were individual points frequently located at high altitude, such as in the mountains of central Mexico or in the Rocky Mountains. The first type probably had an error in $h' - h$ originating from inaccuracy in the knowledge of the geoid, while the last type of error probably had its origin in the orthometric heights. This was confirmed in most cases, since neighboring stations had values which were well below the 4.0-m rejection level.

The distribution of the monthly means of the differences for months with more than one observation is shown in Figure 2. Totally, 120 months in the period January 1971 to May 1982 had at least one observation; approximately seven observations were available per month in the mean. Even though a large scatter is present, an increase of the differences from

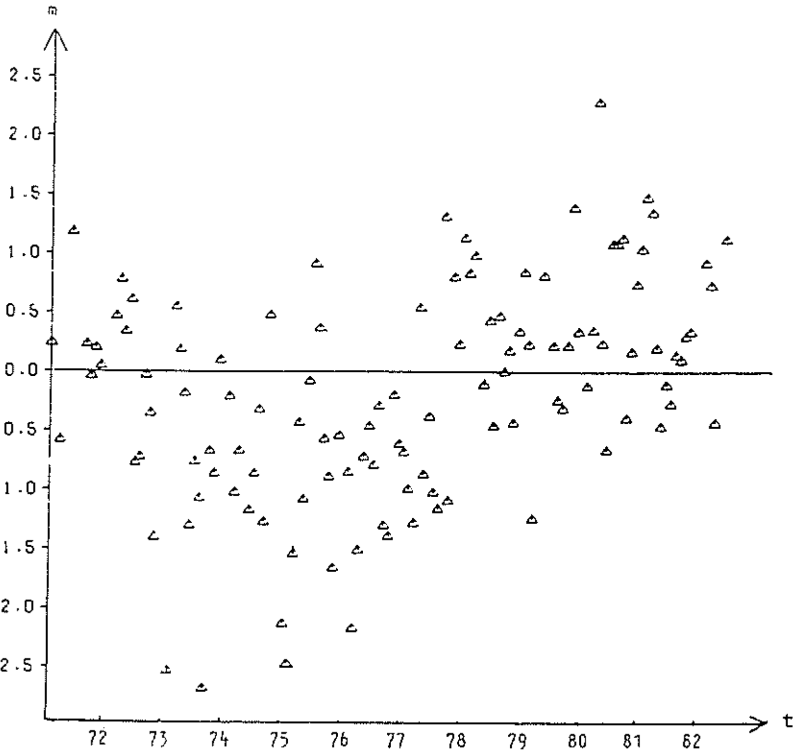

Fig. 2. Monthly means of differences between *Rapp* [1978] geoid heights and Doppler-derived values.

1976 to 1979 is clearly seen. This trend becomes clearer, and the splined curve connecting the values becomes smoother when means over longer time periods are formed (see Figure 3). This is because the points observed within a year are distributed over a very large area, and the mean value of the geoid heights in this area is very well determined from the low order potential coefficients. Also, observations obtained in the same 1 or 2 months frequently are observed at locations which are geographically close to each other. Table 2 shows the yearly mean values computed from monthly mean values.

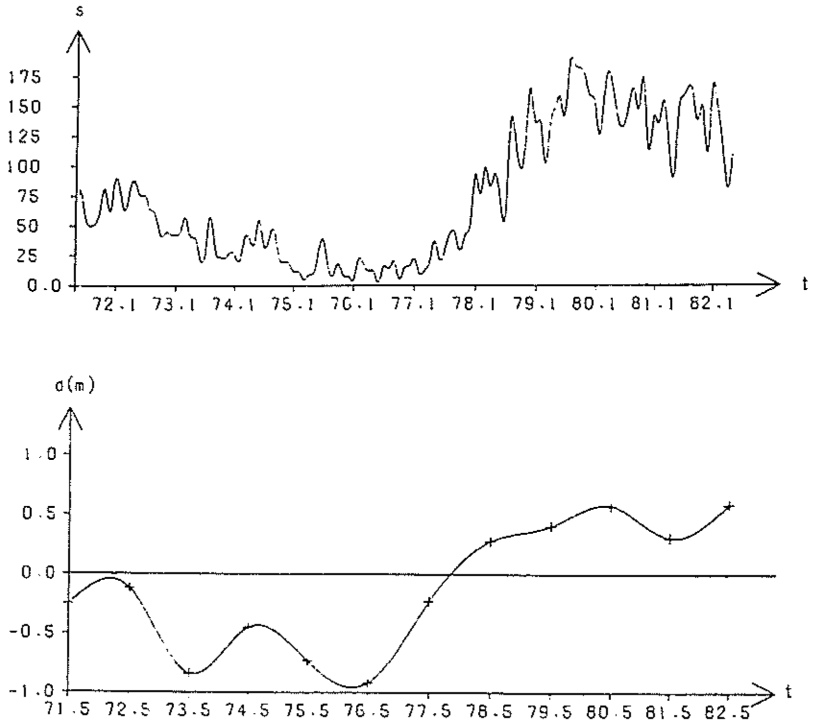

Fig. 3. Sunspot activity *s*, 1972–1981 and spline curve of yearly means of monthly means of differences between *Rapp* [1978] geoid heights and Doppler-derived values.

TABLE 2. Yearly Mean Values of $h' - h$ Based on Monthly Mean Values and Means of Smoothed Sunspot Numbers

Year	Mean, m	σ(Mean), m	S.D., m	Number of Months	Number of Observations	Mean Sunspot Numbers
1971	−0.25	0.47	1.32	8	24	69
1972	−0.12	0.25	0.74	9	53	68
1973	−0.84	0.31	1.04	11	60	40
1974	−0.45	0.41	1.30	10	57	33
1975	−0.73	0.29	0.96	11	148	18
1976	−0.92	0.17	0.57	12	127	13
1977	−0.23	0.28	0.97	12	58	32
1978	0.28	0.15	0.50	11	80	93
1979	0.41	0.33	1.05	10	62	148
1980	0.58	0.23	0.81	12	60	154
1981	0.31	0.20	0.64	10	54	142
1982	0.59	0.35	0.69	4	9	130

S.D., standard deviation.

3. ANALYSIS OF THE CORRELATION WITH SUNSPOT NUMBERS

Since the Doppler height error is related to the state of the ionosphere, which again is related to the number of sunspots, a direct comparison of the mean values of the height differences was made with the corresponding mean sunspot numbers given in Table 2 (Solar Geophysical Data, 1983). A clear correlation is seen in Figures 3 and 4.

Probably the electron density in the ionosphere is linearly related to the sunspot number, and the density is again related to the error in the distance from the satellite to the observing instrument [see *Clynch and Renfro*, 1982, equation (9)].

A regression analysis was then made using a model

$$h' - h = a\, s(t) + b + v \tag{2}$$

where $s(t)$ is the raw or smoothed sunspot number for the period when the observation was made (see Figure 5), a and b

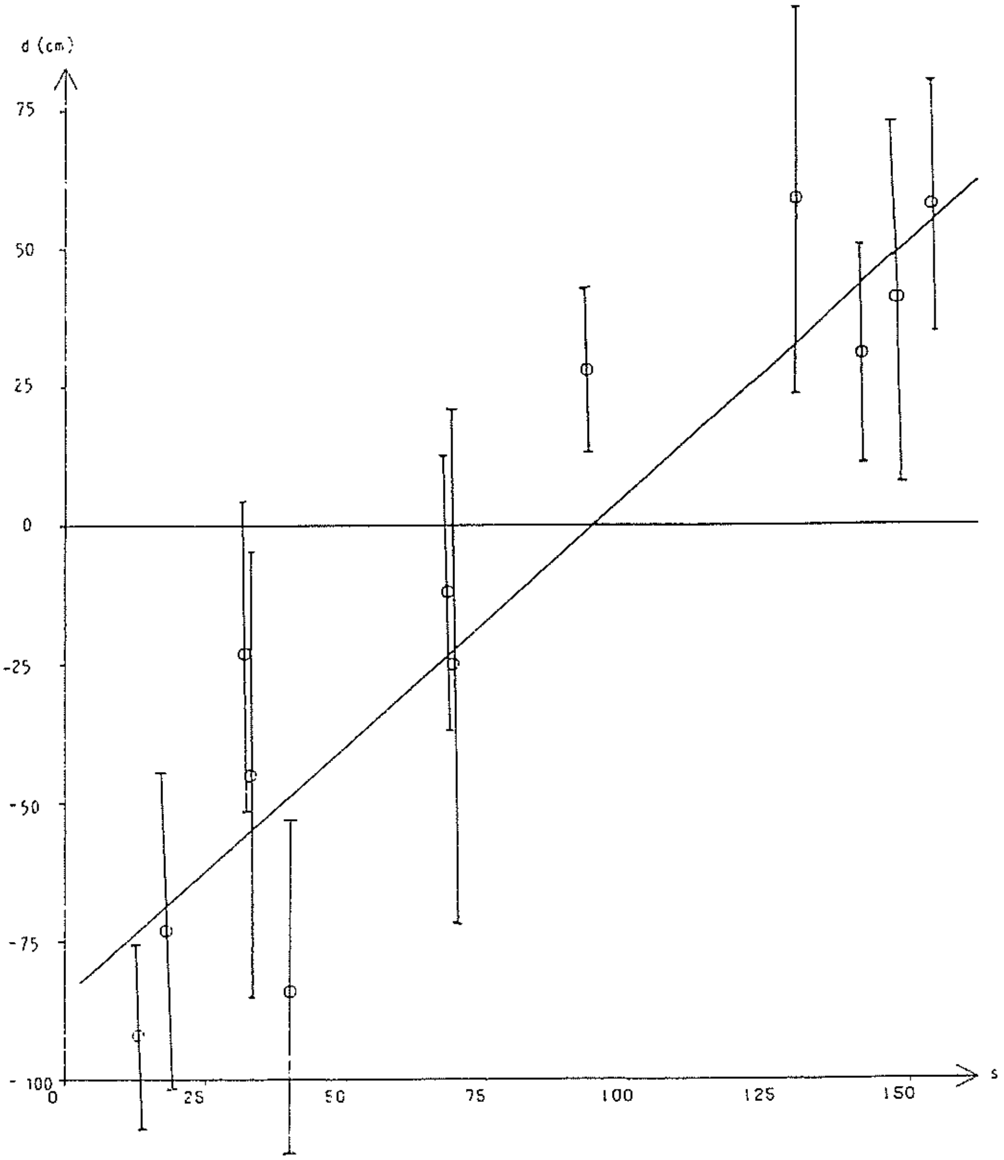

Fig. 4. Yearly means of monthly means of height $h' - h$ plotted with 1σ error bars as a function of yearly means of sunspot numbers, with the regression line, yearly mean $(h' - h) \simeq 0.95s - 86$ cm.

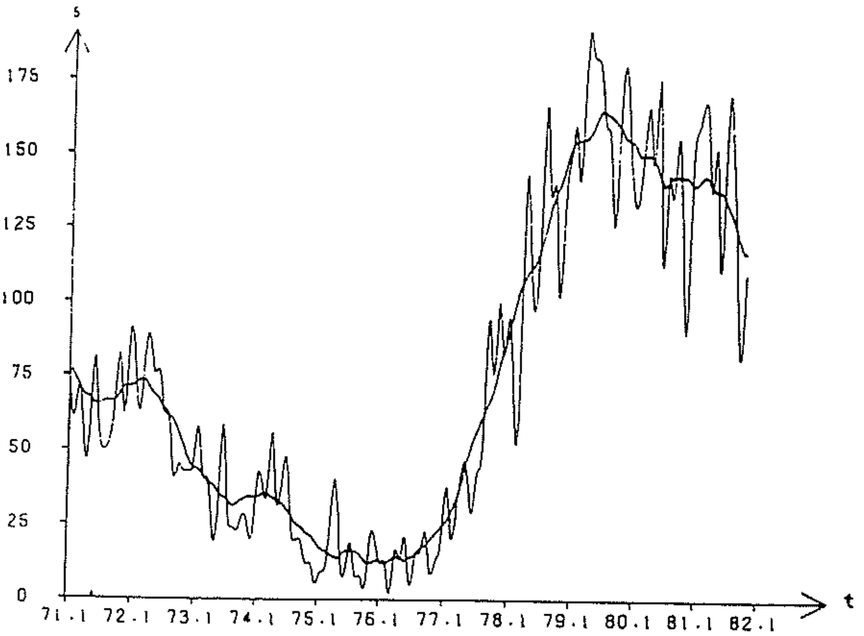

Fig. 5. Smooth and raw sunspot numbers, 1971–1982.

are constants, and v is the residual. A number of regression analyses were made using either individual observations, means of individual observations, or means of mean values (see Table 3).

The monthly and yearly means of the residual values are shown in Figure 6. The standard deviation of a is 0.12 cm and of b is 6 cm for the yearly means of the monthly means. Hence the parameter a is different from zero at the 99.95% significance level.

The decrease (given in the last row of Table 3) in the standard deviation of the yearly means of the monthly means from 55 to 20 cm is remarkable. This shows that we have extracted most of the information in the data. We will try to express this quantitatively in two ways: First, in a traditional statistical manner and, second, by analyzing the observations which have taken place at the same station.

4. STATISTICAL ANALYSIS OF THE RESIDUALS

The observations $h' - h$ and the residuals v can be regarded as a time series. To show that all information has been removed, it is necessary to prove (with a certain probability) that the residuals behave as a white noise stochastic process. In order to do this, we follow the procedure described by *Grenander and Rosenblatt* [1967, chap. 6]. This was inspired by a similar analysis of leveling height refraction errors presented by *Remmer* [1980].

First, one must show that v is a normal, stationary stochastic process. The monthly means of the residuals divided into yearly samples were used for this purpose. Fractile diagrams, means, and standard deviations were calculated and are given in Table 4.

A Student t test reveals that the null hypothesis that each of the mean values are equal to zero cannot be rejected at the 95% level. The test for the identity of the variances (see, e.g., *Hald* [1952, p. 291]) gave a less satisfactory result, namely, that the hypothesis cannot be rejected at 40% level. This is also what could be expected when regarding the standard deviations in Table 4. Gravitational models were changed for generating precise ephemerides in 1973 and 1977. Also, new station coordinates were introduced in 1977. Therefore we

TABLE 3. Results of Regression Analysis Using Smoothed or Raw Sunspot Numbers, Individual Observations, or Various Types of Mean Values

		Smoothed Sunspot Numbers			Raw Sunspot Number			
	S.D. of $h' - h$	S.D. of v	a	b	S.D. of v	a	b	Degrees of Freedom f
Individual observations	159	153	0.86	−85	153	0.77	−80	790
1-month means	102	90	0.90	−96	91	0.79	−78	118
2-month means of monthly means	85	69	0.94	−84	71	0.85	−78	63
3-month means of monthly means	74	57	0.90	−82	59	0.82	−77	43
6-month means of monthly means	59	32	0.83	−85	33	0.89	−82	21
12-month means of individual observations	56	26	0.94	−85	30	0.90	−81	10
12-month means of monthly means	55	20	0.95	−86	20	0.92	−84	10

All units are in centimeters. S.D., standard deviation.

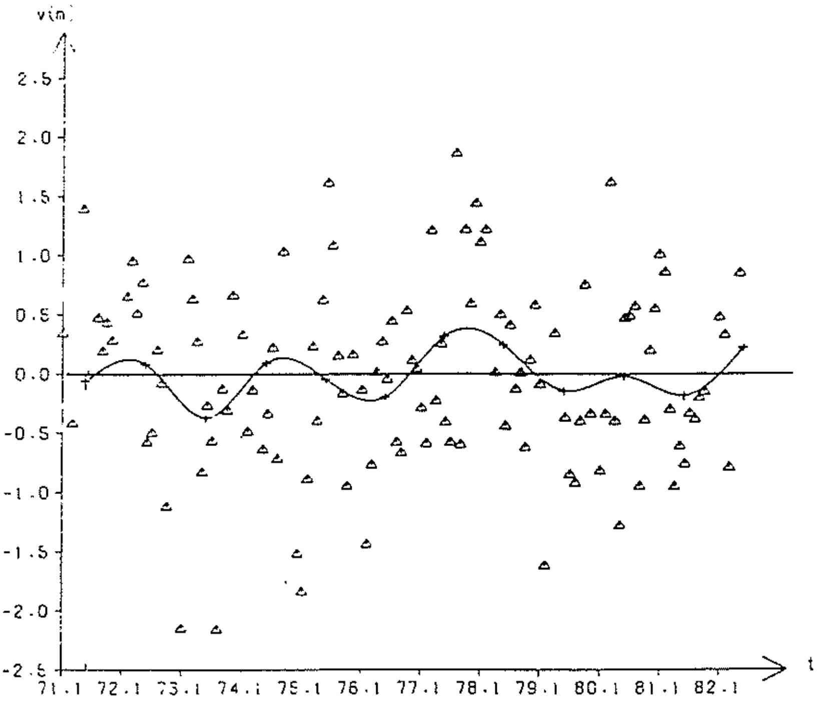

Fig. 6. Monthly and yearly means of reduced geoid height differences.

shall accept the hypothesis that we are dealing with a stationary stochastic process.

The next step is to check whether the spectral distribution function estimate, $F^*(\lambda)$ resembles a straight line (λ is equal to frequency of interest times the sampling interval; thus $\lambda = \pi$ represents the Nyquist frequency). The computation of $F^*(\lambda)$ is based on the covariance function of the process. This function was computed for both $h' - h$ and v. The covariance function of the height differences showed strong correlations with time differences between 0 and 0.5 years and for time differences between 10.6 and 10.8 years, which is the period of the sunspot activity also found by *Dehant and Paquet* [1983]. The v process showed much smaller correlations in general.

Estimates of the spectral distribution function were then computed as described by *Grenander and Rosenblatt* [1967] and the confidence interval for the hypothesis that we have a white noise process was calculated for the 95% level for both $h' - h$ and v. The hypothesis was rejected for $h' - h$ and accepted for v. The estimated spectral distribution functions are shown in Figures 7 and 8. We conclude that we have removed most of the signal by modeling the correlation with sunspot numbers.

TABLE 4. Means and Standard Deviations for Yearly Means of Monthly Means of Residuals

	Sunspot Numbers				
	Smooth		Raw		
Year	Mean	S.D.	Mean	S.D.	Months
1971	−5	133	−1	139	8
1972	9	71	3	71	9
1973	−36	105	−36	102	11
1974	10	130	7	124	10
1975	−4	98	−4	95	11
1976	−19	57	−19	58	12
1977	32	91	35	95	12
1978	25	58	25	61	11
1979	−14	103	−19	109	10
1980	−2	82	−1	93	12
1981	−18	64	−15	69	10
1982	22	71	25	96	4

All units are in centimeters. S.D., standard deviation.

5. ANALYSIS FOR THE STATIONS WITH MORE THAN ONE OBSERVATION

A consequence of our analysis in section 4 should be that height differences after a correction using equation (2) should be zero. This could be tested using 100 differences (cf. Table 1).

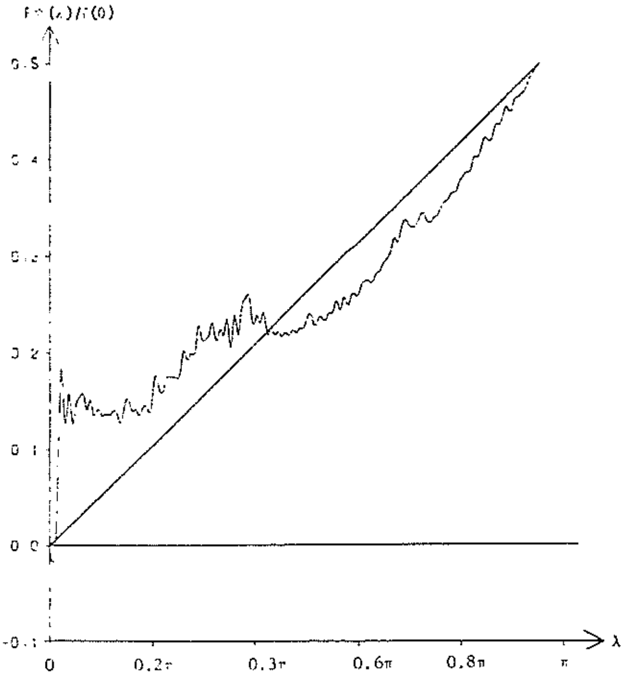

Fig. 7. Spectral distribution estimate $F^*(\lambda)$ divided by variance $C(0)$ for height differences, $h' - h$. (Note that $\lambda = \pi$ corresponds to the Nyquist frequency).

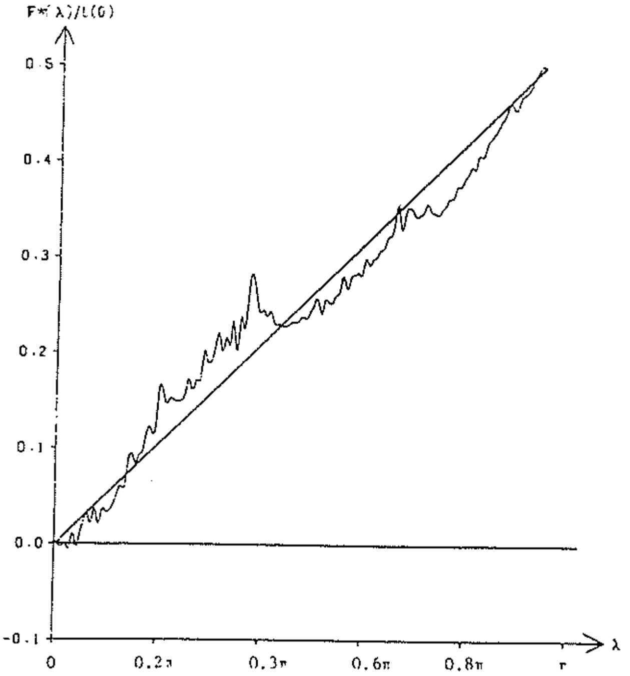

Fig. 8. $F^*(\lambda)/C(0)$ from reduced height differences v. (Note that $\lambda = \pi$ corresponds to the Nyquist frequency).

The differences $h'(t) - h'(t')$ were computed so that the time t always was at least 1 month larger than t'. (Thus the use of some duplicate measurements was avoided.) The result is given in Table 5. The mean difference of 14 cm is not significantly different from zero at the 95% significance level.

6. Investigation of Other Effects

The analysis of ionospheric effects by *Clynch and Renfro* [1982] found a dependence on geomagnetic latitude. The effect should be largest for points near the geomagnetic equator. This was investigated by dividing station heights in groups each covering 10° of geomagnetic latitude. Some small differences were observed, which may indicate a dependence on the geomagnetic latitude, but a more detailed analysis is required.

Thus the differences for these stations is probably caused by errors in the orbits. Note that this does not exclude that stations near the geomagnetic equator show height variations due to local ionospheric conditions, which are significantly different from these found here.

In the analysis made by *Anderle and Malyevac* [1983] the error was supposed to be correlated with the square of the value of the electron density in the ionosphere. A correlation analysis using quadratic sunspot numbers gave a decrease in the standard deviations of the yearly means of the monthly means to 25 cm, i.e., not as significant as when using the regular sunspot numbers. On the other hand, the difference is so small that no clear distinction between a linear and quadratic model is apparent.

A similar analysis was also made using the residuals v. Here no quadratic effect could be seen.

TABLE 5. Mean Values and Standard Deviations of 100 Doppler Height Differences Between Values in the Same Point and at Time Intervals Larger Than 1 Month

	Original Differences	Reduced Differences
Mean value	56	14
Standard deviation	102	93
Mean of absolute differences	91	71

All units are in centimeters. Raw sunspot numbers are used.

TABLE 6. Yearly Mean Height (Minus Adopted Height) in Uccle and the Height Difference Predicted from Equation (2) Using $a = 0.9$ and $b = -84$ cm

	h'	h'(Equation (2))	Difference
Year			
1972	−12	−15	3
1973	−22	−47	25
1974	−30	−52	22
1975	−25	−69	44
1976	−8	−72	64
1977	−5	−59	54
1978	88	86	86
1979	112	59	53
Mean	12		44
S.D.	55		26

All units are in centimeters.

7. Transfer of the Results to Other Geographical Areas

If the errors, as suggested in the previous section, are due mainly to orbit errors, then the result should be applicable for other stations having a reasonable high geomagnetic latitude, such as stations in northern Europe. In order to investigate this, we used the yearly height variation for the station Uccle in Belgium, given by *Dehant and Paquet* [1983] (see Table 5). The decrease in standard deviation corresponds very well to what was found for North America (cf. Table 3). Also, the discrepancy with gravimetrically determined geoid height differences between the two Danish stations mentioned in section 1 increased from 1.49 to 0.33 m.

8. Conclusion

A strong correlation between sunspot numbers and Doppler height errors has been found. The correlation may be used to correct older observations for the major part of the ionospheric effect. It is possible that the result may not only be valid for North America but also for other regions of high geomagnetic latitude.

The results presented here could possibly be improved using a better geoid computed using the dense gravity coverage of the United States and most of Canada. On the other hand, local geomagnetic variations and other effects may still contribute a high degree of random noise, so that the improvement may not be significant.

The errors in latitude and longitude depend strongly on the gradient in the ionosphere and may be more difficult to correct. However, methods should be developed to correct for these errors using all stations with more than one observation.

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References

Anderle, R. J., and C. A. Malyevac, Current plate motions based on Doppler satellite observations, *Tech. Rep. NSWC TR 82-369*, Nav. Surface Weapons Cent., Dahlgren, Va., 1982.

Anderle, R. J., and C. A. Malyevac, Plate motions from Doppler satellite observations, revised version of paper presented at XVIII

General Assembly, Int. Union of Geod. and Geophys., Hamburg, 1983.

Clynch, J. R., and B. A. Renfro, Evaluation of ionospheric residual range error model, in *Proceedings, 3rd International Geodetic Symposium on Satellite Doppler Positioning*, pp. 517–538, New Mexico State University, Las Cruces, 1982.

Dehant, V., and P. Paquet, Modeling of the apparent height variation of a TRANET station, *Bull. Geod.*, *57*, 354–364, 1983.

Grenander, U., and M. Rosenblatt, *Statistical Analyses of Stationary Time Series*, Almqvist and Wiksell, Stockholm, 1967.

Hald, A., *Statistical Theory With Engineering Applications*, John Wiley, New York, 1952.

Hothem, L. D., W. E. Strange, and M. White, Doppler satellite system, *J. Surv. Map. Div., Am. Soc. Civ. Eng.*, *104*(SU1), 79–91, 1978.

Paquet, P., and V. Dehant, The coordinate evolution of a TRANET station over 9 years, in *Proceedings, 3rd International Geodetic Symposium on Satellite Doppler Positioning*, pp. 539–556, New Mexico State University, Las Cruces, 1982.

Rapp, R. H., A global 1 deg × 1 deg. anomaly field combining satellite, GEOS-3 Altimeter and terrestrial data, *Rep. 278*, Dep. of Geod. Sci., Ohio State Univ., Columbus, 1978.

Remmer, O., Refraction in leveling, *Geod. Inst. Den. Medd.*, *54*, 48 pp., 1980.

Strange, W. E., L. D. Hothem, M. White, I. V. Stillfried, and S. Schulte, Time variability of Doppler results at Ukiah Latitude Observatory and other stations, in *Proceedings 3rd International Geodetic Symposium on Satellite Doppler Positioning*, New Mexico State University, Las Cruces, pp. 557–574.

Tscherning, C. C., R. H. Rapp, and C. C. Goad, A comparison of methods for computing gravimetric quantities from high degree spherical harmonic expansions, *Manusc. Geod.*, *8*, 249–272, 1983.

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