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Prediction Test using Least-Squares Collocation and Residual Terrain Reduction

by

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1. INTRODUCTION

In the following we describe the results of the prediction of gravity anomalies and deflections of the vertical in the New Mexico test area using least-squares collocation and digital terrain models. The computations have been carried out as described in Tscherning and Forsberg (1978), Forsberg and Tscherning (1981, 1981a).

2. THE ORGANIZATION OF THE COMPUTATIONS

As the data, which should be predicted was contained in the area bounded by $-107^\circ 45' < \lambda < -105^\circ 15'$, $31^\circ 45' < \varphi < 34^\circ 45'$ we only worked with the subset of gravity anomalies contained in this area. This was totally 3466 points. From these anomalies and from the deflections of the vertical we subtracted the contribution from the Rapp (1979) set of 180 x 180 spherical harmonic coefficients and we transformed the deflections to an approximate geocentric reference system. In this way the long wavelength contributions were removed from the data. We simultaneously computed an empirical gravity anomaly covariance function, to be used as guideline for the choice of covariance function in the approximation procedure.

The covariance function was then modelled by

$$K(P, Q) = \sum_{n=200}^{\infty} \frac{A \; R_E^2}{(n-1)(n-2)(n+24)} \left(\frac{R_B^2}{r_P \, r_Q}\right)^{n+1} P_n(\cos\psi_{PQ}),$$

where ψ_{PQ} is the spherical distance between P and Q, r_P, r_Q the radial distances of P, respectively Q from the origin, R_B the radius of the Bjerhammar sphere, R_E the mean radius of the Earth, P_n the Legendre-polynomical of order n. We used $A = 622\ \text{mgal}^2$, $R_E = 6371000\ \text{m}$, $R_E - R_B = 1250.0\ \text{m}$. This covariance function was used throughout the test. The covariance function was tabulized using the method of Sünkel (1979).

We also computed the effects of the topography, see the following section.

Because one of the time-consuming steps in collocation is the establishment and solution of the normal-equations, we devided the area in $1^\circ \text{x}\, 1^\circ$

blocks in a manner as we would have done in a routine computation. However, if a block only contained 1 deflection to be predicted, we extended one of the neighbouring blocks so that it included this point. We then used the data in each block located within a distance of up to $0\overset{\circ}{.}2$ from the deflections of the vertical. In order to get rid of points which are located very close to each other, and which will not contribute to the solution, we selected for each solution the point most near to each 3'x 3' cell. The final number of points used are shown in Table 2.

The normal equations were then established and the Cholesky-reduced matrices stored on disc. This had the effect, that we only had to establish and reduce each set of equations one time. They could then be used both for terrain-reduced and unreduced data, since we used the same covariance function in both cases.

3. UTILIZATION OF TOPOGRAPHIC DATA

30" x 30" point heights covering the area was utilized in a residual terrain model (RTM) reduction, where the topographic irregularities with respect to a smooth mean height surface are removed computationally using a rectangular prism integration procedure, described in Forsberg and Tscherning (1981). The density of the topography was set at 2.67 g/cm³.

In the test area the 30" x 30" point heights was averaged into a series of mean height grids with increasing element size (1' x 1', 2' x 2', 5'x 5', 15' x 15' and 30' x 30'), the coarser grids being used for the more remote topography in order to speed up the computations. The 30' x 30' mean height grid is used for defining the mean height surface (through a parabolic hyperboloid interpolation), and the gravitational effects of the residual topography for the fixed area bounded between latitudes 30.5 to 35 N and longitudes 105 to 108 W are then the "terrain effect", which is subtracted from the individual "observation data" (the gravity anomalies) before using collocation, and then added to the "predicted" data (the deflections) to get the final predictions.

In the vicinity of the individual stations (gravity, deflections etc.) special actions are taken due to the limited resolution of the terrain model:

the topographic information is densified using a bicubical spline interpolation procedure, and the unavoidable discrepancy between the terrain model station height and the actual station height is removed by either forcing the terrain model to give the correct value in the station (for gravity anomalies) or the station height from the terrain model is temporarily used for the terrain effect computation (deflections of the vertical). In the used type of residual reduction stations situated below the mean height surface are reduced to values inside the smoothed topography, and to get reduced values corresponding to the "outer", harmonic potential approximative "harmonic corrections" are used as described in Forsberg and Tscherning (1981).

With the system at the Geodetic Institute, the terrain effect computation time per point was c. 4 sec. The smoothing effect of the terrain reduction can be seen from Fig. 1 and Fig. 2, showing the observed* and terrain-reduced deflections of the vertical, respectively.

TABLE 1. STATISTICS OF THE OBSERVED* AND TERRAIN REDUCED DATA

	no of points	observed		terrain-reduced	
		mean	std. dev.	mean	std.dev.
gravity anomalies (mgal)	3466	-0.14	25.28	2.59	15.68
N-S deflection (arcsec)	441	-0.40	2.72	-0.27	1.56
E-W deflection (arcsec)	441	2.18	6.63	1.66	4.25

One deflection station turned out in the collocation approximation to be an outlier, so for this station, located on top of Socorro Peak (2100 m) near the NW-corner of the area, a more detailed terrain effect computation was done using additional heights read from a 1:250000 map. The computed terrain effect changed by 1.06" and 1.36" respectively for this extreme station, partly explaining the outlier. This example shows that for extreme stations even more detailed height information than the available 30" x 30" might be necessary to obtain the best results. (The outlier is not removed in the results presented in Table 3).

*) Contribution from potential coefficients subtracted.

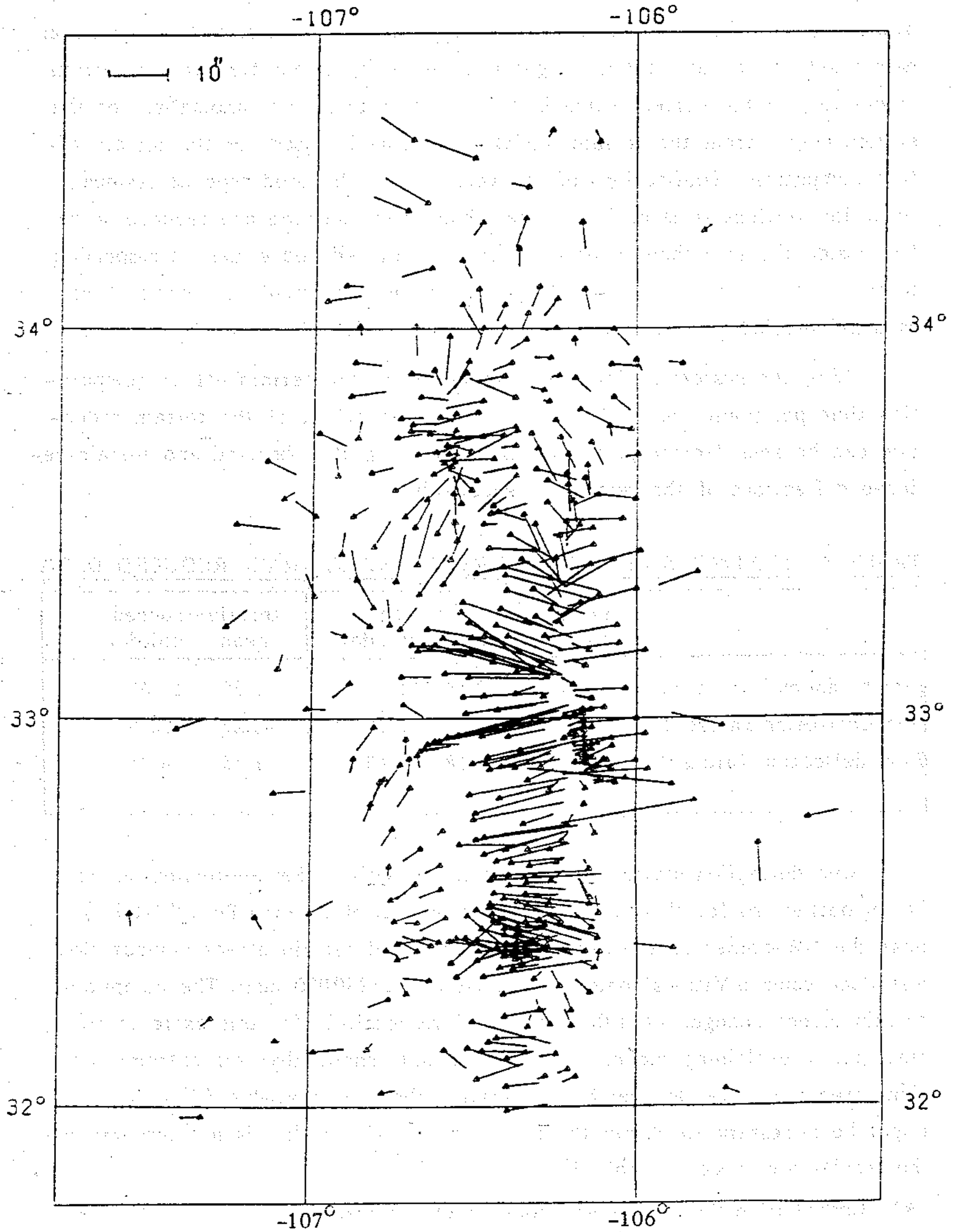

FIG. 1. OBSERVED DEFLECTIONS OF THE VERTICAL - MINUS CONTRIBUTION FROM POTENTIAL COEFFICIENTS

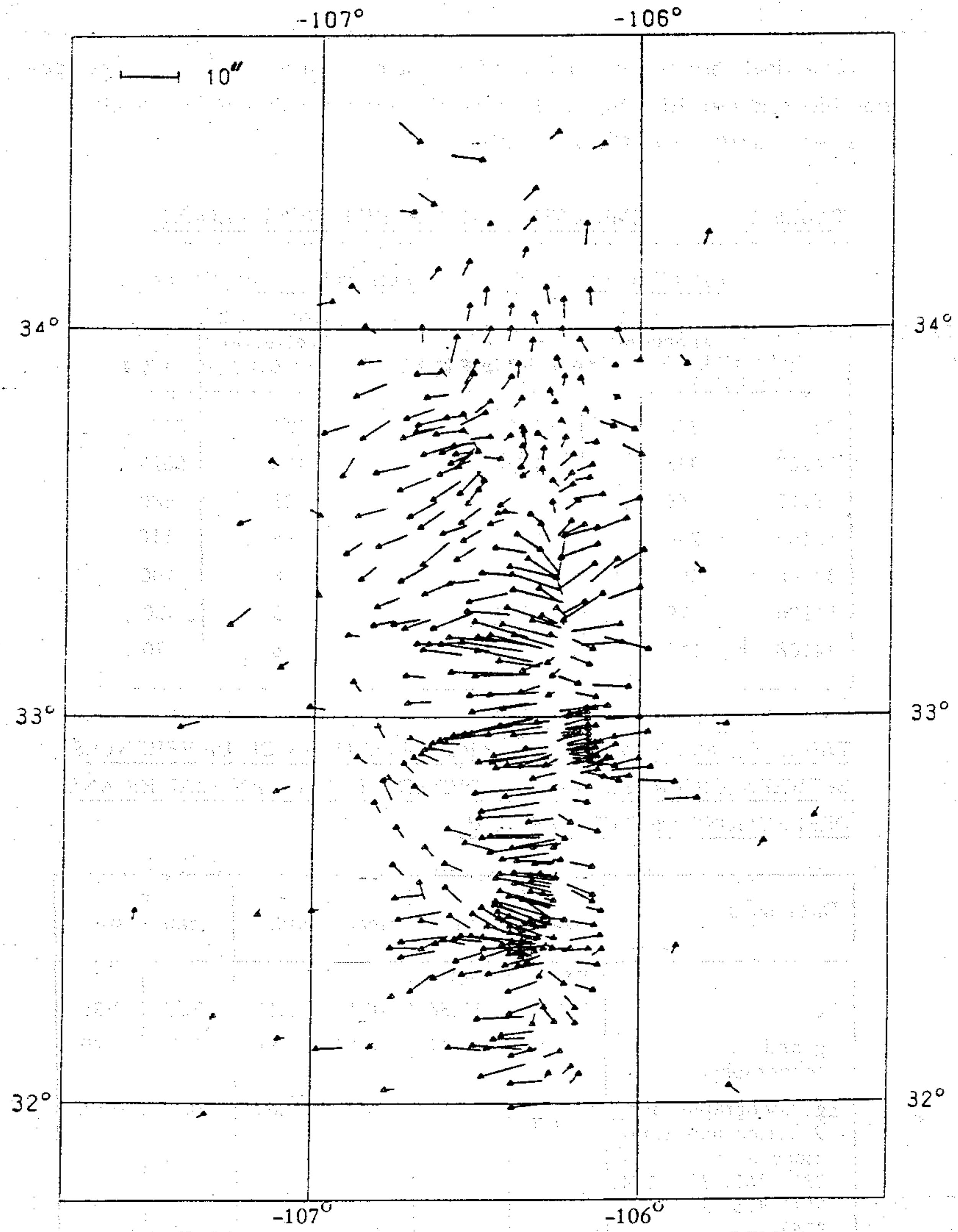

FIG. 2. TERRAIN REDUCED DEFLECTIONS OF THE VERTICAL

4. RESULTS

The final results are summarized in Table 2 and 3. Each $1^{\circ} \times 1^{\circ}$ computation block is identified by the latitude and longitude of the North-West corner, e.g. 34107 : $\varphi = 34^{\circ}$, $\lambda = -107^{\circ}$.

TABLE 2. TIME USED FOR THE DIFFERENT BLOCKS

Block	Establishment of Normal Equations		Prediction	
	Number of anomalies	time (s)	Number of deflections (ζ,η)	time
33107	548	3438	209	8395
34107	348	1760	176	5025
35107	269	855	28	480
33106	286	(c)1000	9	110
33108	294	1030	8	100
34106	79	100	5	20
34108	115	125	6	70

TABLE 3. MEAN AND STANDARD DEVIATIONS OF DIFFERENCES BETWEEN OBSERVED AND PREDICTED GRAVITY ANOMALIES AND DEFLECTIONS OF THE VERTICAL.

Data used	Δg		ξ		η	
	mean	std.	mean	std.	mean	std.
	mgal	mgal	"	"	"	"
Δg	2.01	17.86	-0.11	2.11	0.22	2.38
Δg and topography	-0.08	6.47	-0.18	0.83	0.61	0.99
Δg, topography and 9 deflections (stations no.: 238, 261, 284, 294, 304, 326, 333, 373 378)	-	-	-0.11	0.84	-0.12	0.98

These results are compatible with earlier results obtained in the central $1^{\circ} \times 2^{\circ}$ block of the same area (Forsberg and Tscherning, 1981, 1981a), even in spite of these previous tests used less of the available gravity data.

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COMPARISON OF GRAVITY FIELD APPROXIMATION TECHNIQUES

Method Used / Characteristic Property	Least squares Collocation	
Theory		
Spherical approximation not required.	no	
Parameter estimation possible.	yes	
Reduced data not needed.	yes	
Flexibility and Reliability		
All functionals can be evaluated.	yes	
All data types can be used.	yes	
Data quality (σ) taken into account.	yes	
Error estimates computable.	yes	
Global data coverage not required.	yes (no)	
Stable in clusters.	yes (no)	
New data can be added sequentially.	yes	
Behaviour in data holes known.	yes	
Computation and Accuracy		
Method is fully automized.	no	
Base functions are automatically selected.	~~yes~~ (no)	
No solution of linear equations necessary.	no	
Gridded data not needed.	yes	
Installation on small computers possible.	yes	
Program available to other users.	yes	
Time estimates:		
Data preparation	see paper	
100 values estimated		
I/O time		
Accuracy (1σ)		
Computer Used.	RC 8000	