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Evaluation of Least-Squares Collocation and the Reduced Point Mass method using the International Association of Geodesy, Joint Study Group 0.3 test data.

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Least-Squares collocation (LSC) and Reduced point masses (RPM)

- Both methods use radial base functions for constructing approximations to T=W-U:
- LSC: Reproducing Kernels (in all obs. Points)
- $K(P, Q_k) = \sum_{i=2}^{\infty} \sigma_i \left(\frac{{R_B}^2}{r_P r_{Q_k}} \right)^{i+1} P_i(cos\psi_{Pk})$
- RPM: Reduced point mass potentials in grid
- $\mathrm{M}(P, Q_k) = \sum_{i=N+1}^{\infty} \kappa \left(\frac{{R_M}^2}{r_P r_{Q_k}} \right)^{i+1} P_i(cos\psi_{Pk})$

Approximation of anomalous potential, $\tilde{T}$.

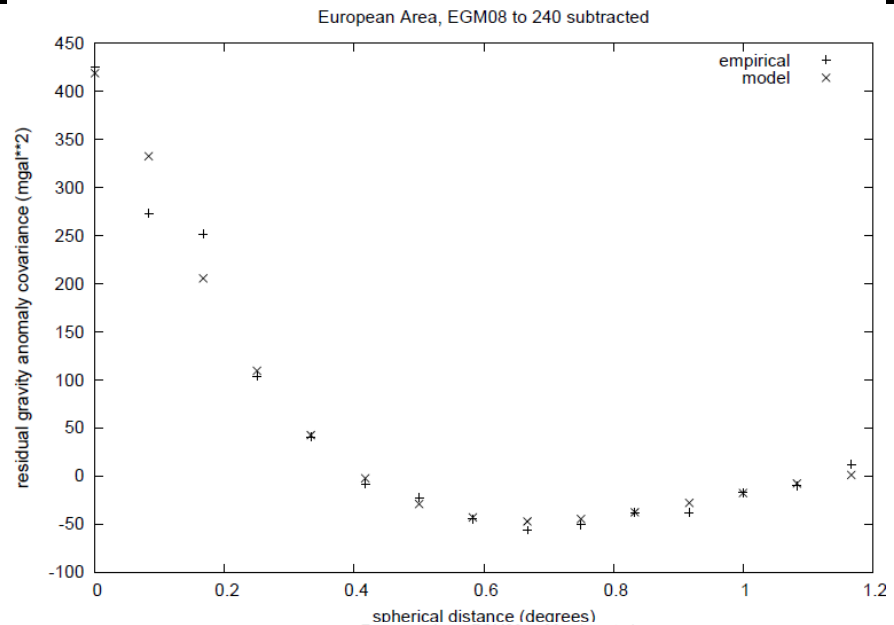
- $\tilde{T}$, harmonic function = linear combination of base-functions on which the observation functional has been applied wrt. Q.
- Requires global data-coverage, but JSG 0.3 data are regional, so egm2008 coefficients up to degree N used as observations.
- Equivalent to EGM is subtracted and later added. EGM96 error-degree variances used to represent the error (arbitrary choice).

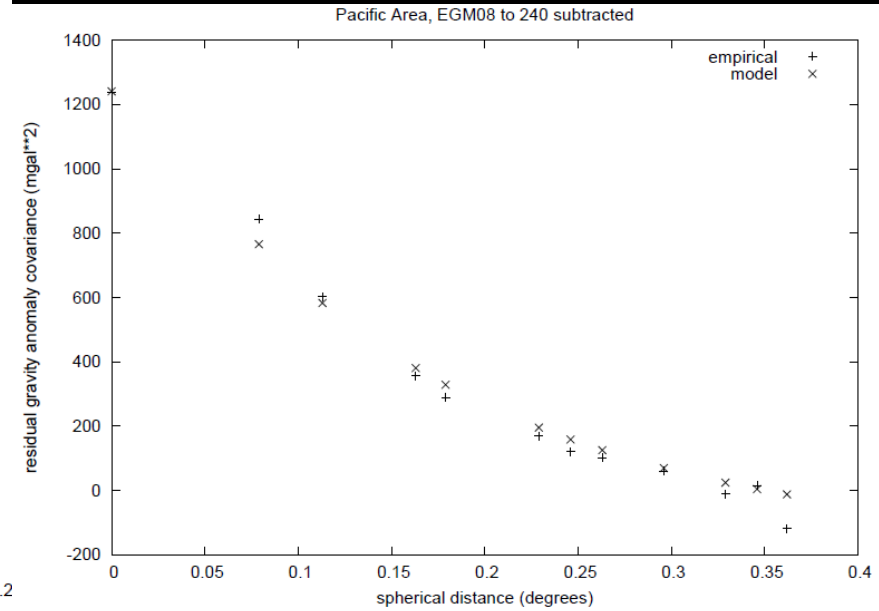
Reproducing kernel determination / Covariance fitting:

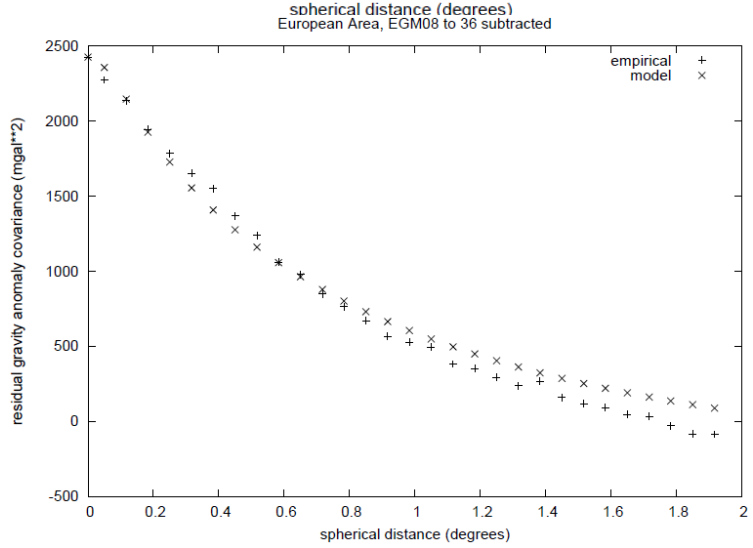
- Covariance functions with N=241 and N=37 estimated in the two test areas and used to determine analytic representation as a reproducing kernel.

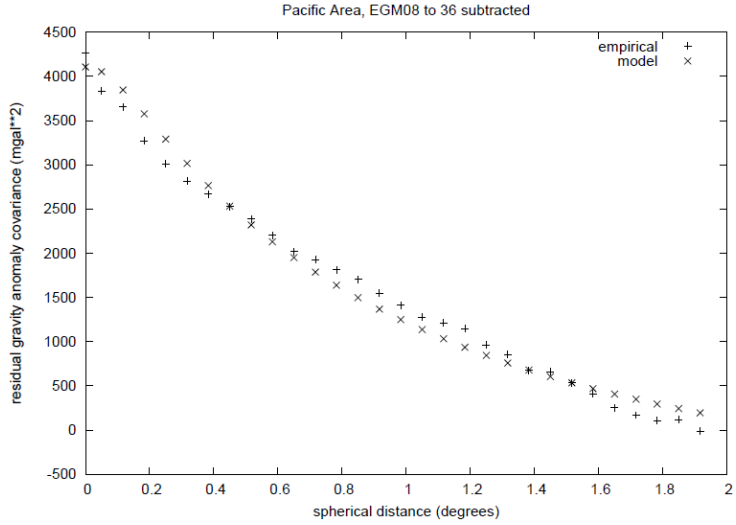
- Fitting difficult in Pacific area due to extreme values. (In practice, residual topographic effects would have been used to smooth values).

Empirical covariances and Reproducing kernel model:









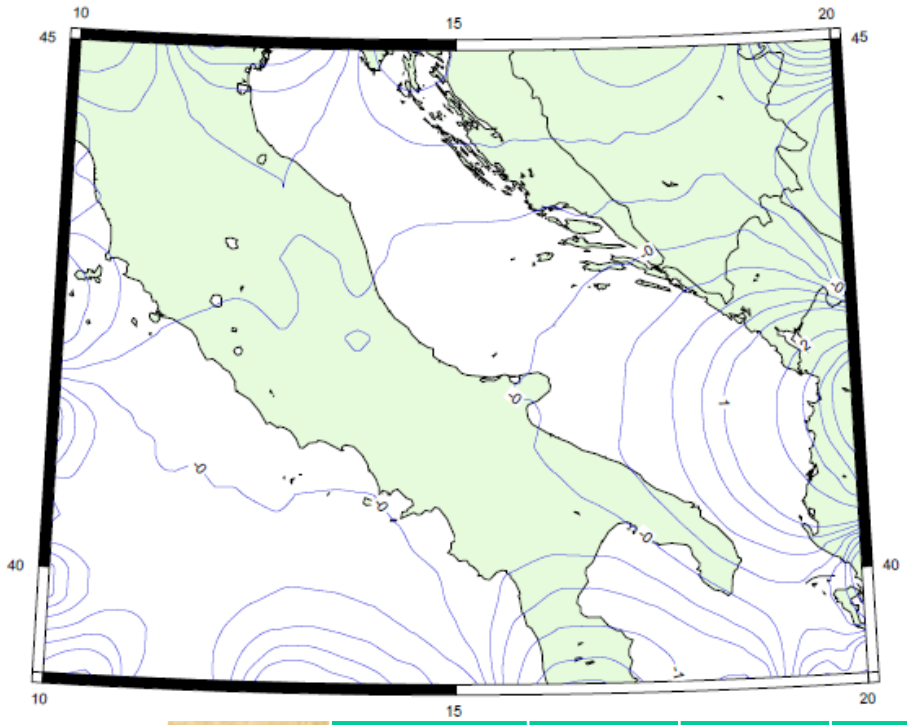
RPM grid and depth selection:

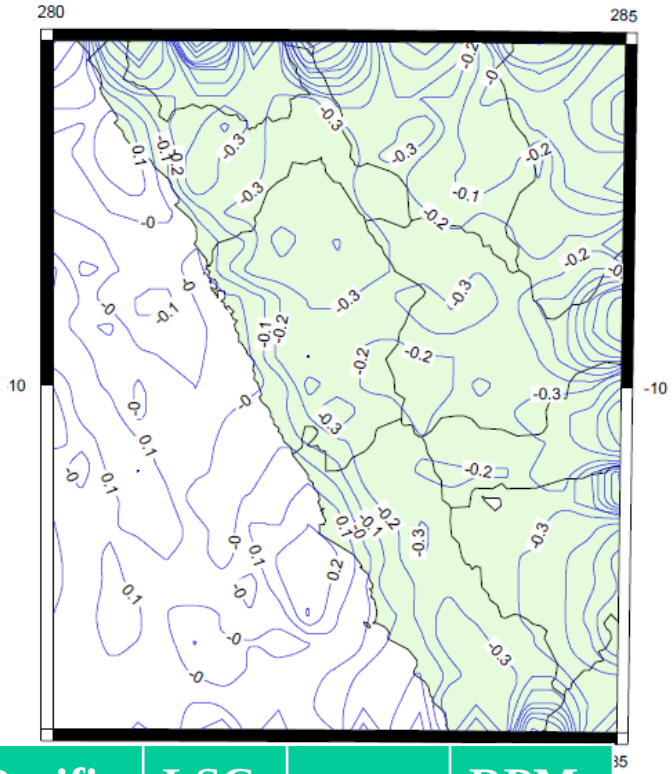
- For RPM we have to fix the position
- Grid spacing: 0.25° x 0.50°
- Depth of the sources (Bjerhammer sphere) is 20km

Calculations/estimations of $\tilde{T}$:

- Low and high resolution T_r ground data used (11335 values).
- Airborne data used at altitude (as only source)
- T_{rr} ”GOCE” data used at satellite positions
- ”GRACE” δT values used at satellite position
- Ground T_r computed from ”GRACE” and ”GOCE” data – results not shown.
- Detailled results available at http://cct.gfy.ku.dk/jsg03.htm

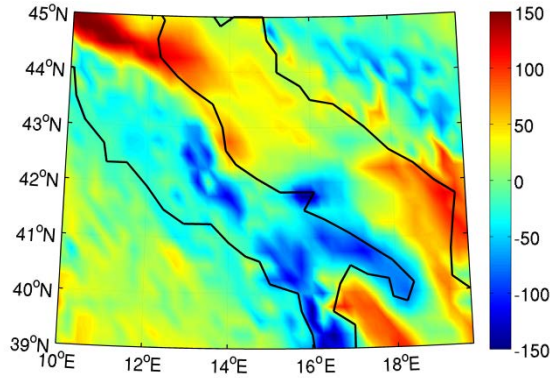
Results: Differences prediction from T_r of $\tilde{T} - T$ "observed", m^2/s^2



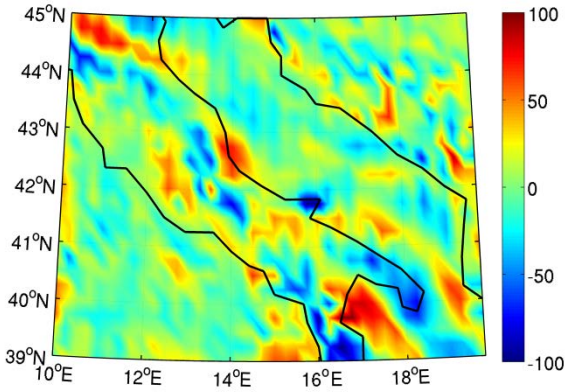


	Europe	LSC		RPM	Pacific	LSC		RPM
	"obs"	Diff	Error	Diff	"obs"	Diff	Error	Diff
Mean	-0.01	-0.03	0.80	0.07	-0.13	-0.06	0.79	-0.40
St.dev	4.08	0.82		1.44	4.82	0.24		2.94

Results: Differences prediction from T_r of $\tilde{T} - T$ "observed", m^2/s^2

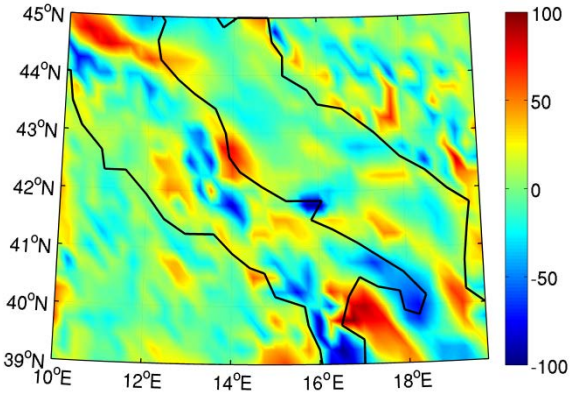


Observations (contribution up do d/o 240 is subtracted)



Difference

(Obs - RPM prediction)



Difference

(Obs - Col prediction)

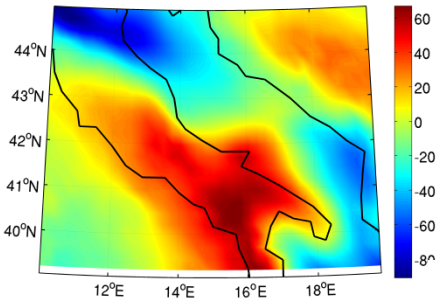
Differences prediction of $\tilde{T}$ (m^2/s^2):

From Airborne T_r, EGM2008 to 240 subtracted:

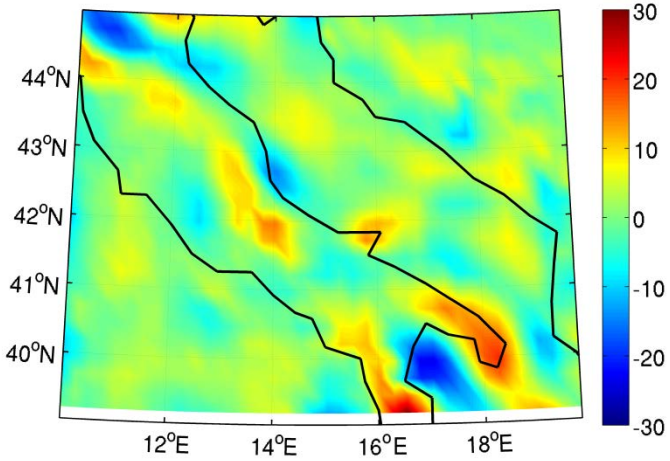
	Europe (LSC)			Pacific (LSC)		
	Obs	Dif	Err	Obs	Diff	Err
Mean	-001	-0.11	2.97	-0.13	-0.08	1.65
St.dev	4.08	2.73		4.82	1.92	

Differences prediction of $\tilde{T}$ (m^2/s^2):

From GOCE T_{rr}, and EGM08 to 36:

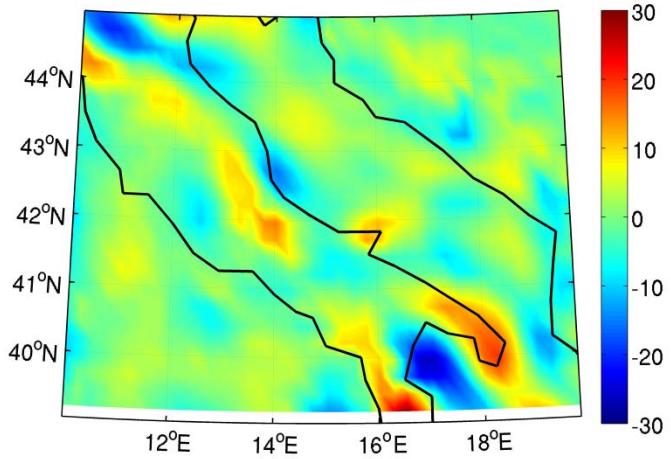


	Europe (LSC)			RPM	Pacific (LSC)			RPM
	Obs	Diff	Err	Diff	Obs	Diff	Err	Diff
Mean	-0.11	-0.26	6.80	0.06	4.62	-0.18	8.41	-0.74
St.dev	32.55	6.03		5.78	49.07	6.66		6.92



Difference

(Obs - RPM prediction)



Difference

(Obs - Col prediction)

Differences prediction of $\tilde{T}$ (m^2/s^2):

From GRACE potential differences, and EGM08 to 36:

	Europe (LSC)			Pacific (LSC)		
	Obs	Dif	Err	Obs	Diff	Err
Mean	-0.11	-1.76	28.75	4.62	0.19	24.64
St.dev	32.55	19.15		49.07	18.75	

Conclusion (1)

- Good agreement between differences and error-estimates for LSC. Errors large at borders to lower resolution data.
- Results unbiased considering error estimates.
- Good agreement for LSC and RPM
- Results in Europe of 8 cm, Pacific 2 cm excellent, but Pacific error-estimate larger.

Conclusion (2)

- Improvement in results if Topography or observations of EGM08 coefficients to higher degree was used (JSG decision)

- RPM must be further developed

1. in order to use potential differences (GRACE) defined by JSG 0.3.
2. to account for errors in EGM used.

- RPM experiments with grid point selection needed.