

Use of GOCE TRF vertical gravity gradients for spherical harmonic coefficient estimation.

Note by C.C.Tscherning, NBI, UCPH, Aug. 31, corrected Sep. 1, 2011..

Abstract: GOCE TRF vertical gradients have been used for gravity model coefficient estimation using data as close as possible to the points of equal area grids of 1° and 40' spacing, supplemented with gravity anomalies at the poles. The method of Least-Squares Collocation (LSC) was used without using spherical approximation.

The gravity anomalies were ground values “lifted” by prediction to the altitude of 15 km in order to avoid that they were located inside the used Bjerhammar-sphere having a radius of 6371.037 km.

A comparison with the coefficients of EGM96 showed that coefficients to degree 180 and 140 could be computed with a noise lower than the standard deviation of the coefficients for the 1° and 40' grids, respectively. The coefficients up to degree 36 were in both cases supposed to be known and their contribution was removed from the data using EGM96.

1. Introduction.

One of the GOCE products are gravity gradients in a terrestrial North-West-Up (NWU) frame, see HPF (2010). From these data the gradients of the corresponding anomalous potential T can be computed.

The second order radial and North components, T_{rr} and T_{xx} have successfully been used to compute ground gravity anomalies, see Tscherning and Arabelos (2011). The method of Least-Squares Collocation (Moritz, 1980) was used considering the coefficients of EGM96 (Lemoine et al., 1998) up to degree 36 as error-free observations (i.e. the error estimates were not accounted for).

Since the launch of GOCE a very large data-set has been collected and the collection is still ongoing. The dataset has a very good global distribution except at the poles. A screened data-set available from the ESA GOCE Virtual Data Archive up to May 2011 has been used. The screening removed all data not flagged as nominal and values which deviated more than 3 Eötvös from the used reference EGM96 to degree 36. This resulted in 18145494 values in total.

LSC may also be used to estimate spherical harmonic coefficients as well as error-estimates and error-correlations of such values, see (Tscherning, 2001) and Arabelos and Tscherning (2008). Meanwhile the program GEOCOL has been upgraded to take advantage of multiprocessing (Tscherning and Veichert, 2008), so that it became feasible to use large sets of data for coefficient estimation.

Below are described two experiments where T_{rr} and ground gravity at the poles were used.

2. Coefficient estimation experiments.

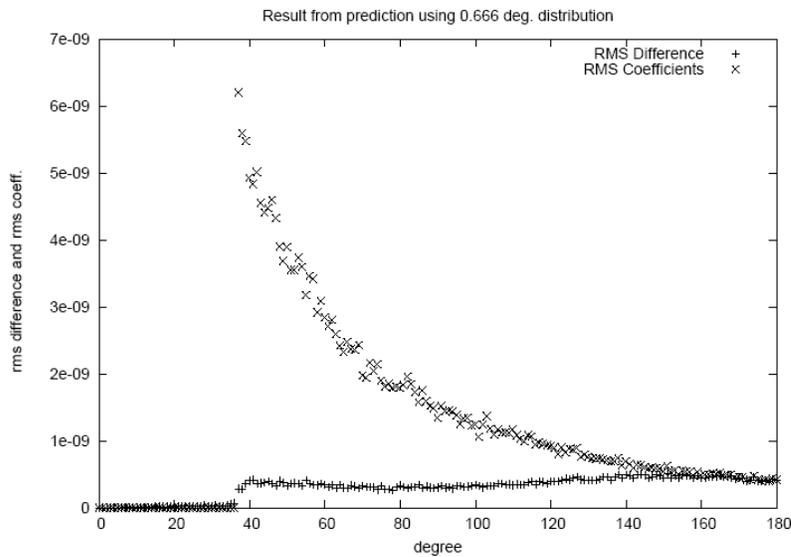
As shown in Arabelos and Tscherning (2008) it is of advantage to use data in a equal-area grid and not in a equal angular grid in order to obtain uniform error-estimates. Two equal area grids with (approximate) spacing of 1° and 40' was selected as a foundation for computational experiments. Programs written by D.Arabelos (see <http://cct.gfy.ku.dk/egg-c.htm>) were used in order to extract the data as close as possible to the grid points and so they still were ordered time-wise (so that bias parameter estimation could be handled by the software). Radial gradient data were extracted from the above mentioned screened data

set, resulting in 40946 and 91498 gradient observations for the 1° and $40'$ grids, respectively. Gravity anomaly data were extracted from the polar datasets used in (Tscherning and Arabelos, 2011).

A covariance function was estimated and an analytic representation by a reproducing kernel was determined using the method described in Knudsen (1987). This resulted in an estimate of the radius of the Bjerhammar-sphere of 6371.037 km, i.e. data at the poles would be inside the sphere. The selected values were therefore “lifted” to the altitude of 15 km. This resulted in values with larger error-estimates (3 mgal) than the 1 mgal originally associated with the ground data.

The two combined data-sets were then used to estimate spherical harmonic coefficients and their associated error-estimates. The processing of the $40'$ grid and the estimate of coefficients and errors up to and inclusive degree 100 took 154 hours on our standard server using 2 processors.

The estimated coefficients were compared with the EGM96 coefficients and with the computed error-estimates. The standard deviations of the differences and of the coefficients are shown in Fig. 1 and 2. They show that the two grids enable the estimation of coefficients up to degree 140 and 180, respectively, i.e. at the degree the signal and noise standard deviations becomes equal.



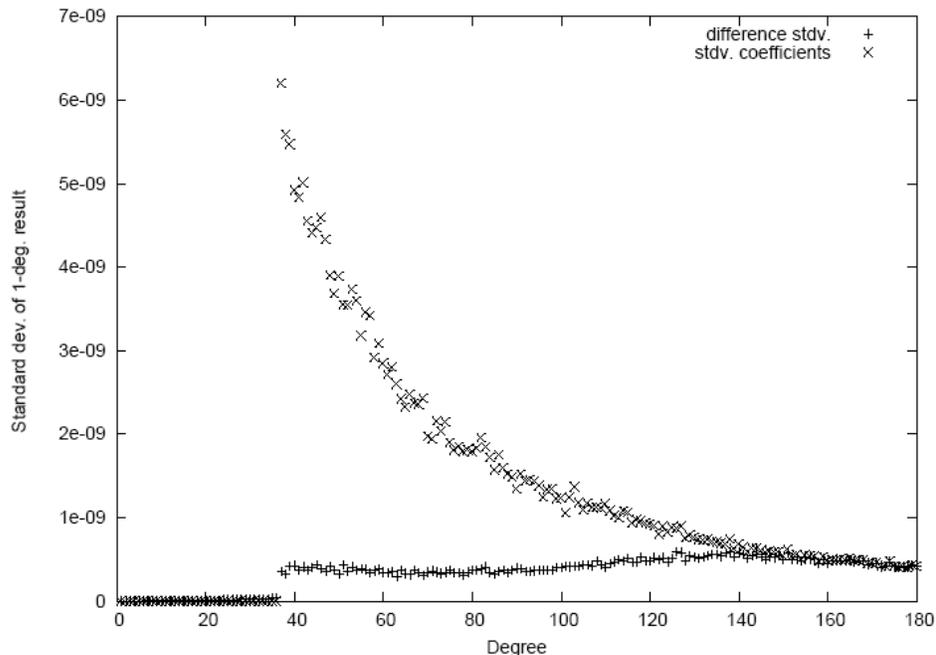


Fig. 1 and 2: Results from 40' and 1° grids.

The estimated errors were two or three times smaller than the standard deviations of the differences.

3. Future plans.

An (approximate) 30' grid of T_{zz} values has been prepared with 163963 values (plus gravity anomalies). The resulting normal equations exceeded the available disk space, but a larger disk is being installed. This will also enable the use of other gradients in the same point. Experiments which include new available data are also planned. They will give a more homogeneous global data coverage.

The computation of error-estimates seems to depend on the error associated with the observations and the selected covariance function representation. In the computations described the reference field errors were put to zero, and in new calculations we will study the effect of using the actual error-degree variances.

The GOCE HPF has already produced several excellent sets of spherical harmonic coefficients. This study has been carried out with the aim of showing the high quality of the HPF TRF gradient product. It is possible (using the software) to use the "original" L2 data and associated quarternions. In this manner the unavoidable error due to the rotation to the NWU frame can be avoided. Also the determination of bias parameters will be included in the computations. This corresponds to the cross-over analysis proposed by IFE (Mueller et al., 2004).

Acknowledgement: This is a contribution to the ESA sponsored GOCE HPF. The assistance of D.Arabelos, University of Thessaloniki is appreciated.

References:

- Arabelos, D. and C.C.Tscherning: Error-covariances of the estimates of spherical harmonic coefficients computed by LSC, using second-order radial derivative functionals associated with realistic GOCE orbits. *J.Geodesy*, DOI 10.1007/s00190-008-0250-9, 2008.
- HPF: GOCE Level 2 Product Data Handbook, GO-MA-HPF-GS-0110, Issue 4.3, 09.12.2010.
- Moritz, H.: *Advanced Physical Geodesy*. H.Wichmann Verlag, Karlsruhe, 1980.
- Mueller, J., H.Denker, F. Jarecki and I. Wolf: Computation of Calibration Gradients and Methods for In-orbit Validation of Gradiometric GOCE data. 2nd Int. GOCE User Workshop, Frascati 8-10 March 2004.
- Knudsen, P.: Estimation and Modelling of the Local Empirical Covariance Function using gravity and satellite altimeter data. *Bulletin Geodesique*, Vol. 61, pp. 145-160, 1987.
- Lemoine, F.G., S.C. Kenyon, J.K. Factor, R.G. Trimmer, N.K. Pavlis, D.S. Chinn, C.M. Cox, S.M. Klosko, S.B. Luthcke, M.H. Torrence, Y.M. Wang, R.G. Williamson, E.C. Pavlis, R.H. Rapp, and T.R. Olson, The Development of the Joint NASA GSFC and the National Imagery and Mapping Agency (NIMA) Geopotential Model EGM96, NASA/TP-1998-206861, Goddard Space Flight Center, Greenbelt, MD, July, 1998.
- Tscherning, C.C.: Computation of spherical harmonic coefficients and their error estimates using Least Squares Collocation. *J. of Geodesy*, Vol. 75, pp. 14-18, 2001.
- Tscherning, C.C. and D.Arabelos: Gravity anomaly and gradient recovery from GOCE gradient data using LSC and comparisons with known ground data. Proc. 4th International GOCE user workshop, 31 March - April 1, 2011. ESA SP-696, 2011.
- Tscherning, C.C. and M.Veicherts: Accelerating generalized Cholesky decomposition using multiple processors. Power point presentation, 2008. Available as http://cct.gfy.ku.dk/publ_cct/cct1914.pdf .