Mathematical and statistical methods in Physical geodesy,

Exercises prepared by C.C.Tscherning, Geophysical Department, University of Copenhagen, Spring semester 1991. Revised July, 1993 and Nov. 1997.

Introduction.

These exercises are associated with my lectures on "Mathematical and statistical methods in Physical Geodesy" prepared in the spring of 1991. They illustrate the use of the GRAVSOFT package of computer programs and related data.

The compiled programs and the data are expected to be found in the same directory. However, explicit path-names are found in the text, which are used on the SGI workstations of the Department of Geophysics. Please disregard these names if you are not working on this computer.

Important data to be used in the exercises are:
Potential coefficients, here OSU91A or the older GPM2,
height data from New Mexico , nmdtm, nmdtm5,
gravity data -   -   -   , nmfa   and
deflections for the vertical, nmdfv, and
altimeter data from the New England sea Mount Area (altimetry).
sea gravity from this area (WATTSG).

A user working on the computers of the Department of Geophysics will have access to several other data sources, including the Nordic gravity data base, deflections of the vertical and altimeter data from the North Atlantic.

After having went through the exercises, a student should be able to compute geoid heights or deflections of the vertical from gravity data, with and without topographic reduction and to compute gravity anomalies from adjusted or unadjusted altimeter data. The student will also be able to use both the collocation method and the FFT method.

Results using the New Mexico and the New England Sea Mount data have been published in:

Forsberg, R. and C.C.Tscherning: The use of Height Data in Gravity Field Approximation by Collocation. J.Geophys.Res., Vol. 86, No. B9, pp. 7843-7854, 1981.

Kearsley, A.H.W., M.G.Sideris, J.Krynski, R.Forsberg and K.P.Schwarz: White Sands Revisited - A Comparison of Techniques to Predict Deflections of the Vertical. Report 30007, Division of Surveying Engineering, University of Calgary, 1985.

Tscherning, C.C. and P.Knudsen: Determination of bias parameters for satellite altimetry by least-squares collocation. Proceedings 1. Hotine - Marussi Symposium, Rome, June 3-6, 1985, pp. 833-852. Politecnico di Milano, 1986.

Detailled information of the GRAVSOFT package with many more references of technical character are found in:

Tscherning, C.C.: Geoid determination by least-squares collocation using GRAVSOFT. Lecture Notes "Int. School of the Determination and Use of the Geoid", Milano, Oct., 1994, pp. 135 - 164, published by International Geoid Service, 1994.

Tscherning, C.C., P.Knudsen and R.Forsberg: Description of the

GRAVSOFT package. Geophysical Institute, University of Copenhagen, Technical Report, 1991, 2. Ed. 1992, 3. Ed. 1993, 4. ed, 1994.

Tscherning, C.C., R.Forsberg and P.Knudsen: The GRAVSOFT package for geoid determination. Proc. 1. Continental Workshop on the Geoid in Europe, Prague, May 1992, pp. 327-334, Prague, 1992.

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Exercise 1. (Vers. June 1997).

Use the FORTRAN program GEOCOL for the computation of values of the geoid height, gravity anomaly, second order vertical gravity gradient (Tzz) and the meridian component of the deflection of the vertical in two grids with spacing 1 deg. and 0.1 deg. respectively, with 20 values in eastern and 20 values in southern direction.

Use one area with a smooth gravity field and one with a strongly varying field. The result should be computed in GRS80.

Use the spherical harmonic expansion OSU91A with coefficients up to degree 180 and up to 360. A file with the coefficients are stored on character form in the file:
/disk1/cct/cctf/osu91a1f

You may run geocol under UNIX using the following command:
geocol11 < input.ex1 > output.ex1 &
where the input file input.ex1 looks as follows (for geoid heights):

```
f                                           ; batch run mode
f t f f f t f                               ; logical variables
5                                           ; selects GRS80
OSU91A to degree 180                        ; name of set
3.986005D14  6378136.2 -484.18              ; GM, a, dummy C20.
180 f f t f                                 ; max deg., logical var.
(2i3,2d19.12)                               ; data format of set
/gfy/cct/cctf/osu91a1f                      ; name of file with coeff.
t f f                                       ; select grid output
40.0 59.0 8.0 27.0 1.0 2.0 11 -1 0.0 F T F  ; defines grid
grid11.ex1                                  ; name of file
T                                           ; stop message.
```

The grid values will be stored in a file with the name grid11.ex1 in your current directory. It may then be used as input to a plotting program.

The input line which defines the grid contains the following values:

```
40.0 59.0 8.0 27.0   -  minimum and maximum latitude and longitude,
 1.0 2.0   -  spacing in degrees in South and East direction,
      11   -  code for geoid height (13 gravity, 15 Tzz, 16 meridian
              component of deflection of the vertical).
      -1   -  indicates use of basic reference system
     0.0   -  altitude of grid points
F T F      -  logical variable set true of output to file.
```

So, the excercise may be carried out, changing these input values.

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Exercise 1., Appendix, valid for systems with AVS/UNIRAS.

```
                                             (Vers. JUNE 1997).

        In order to plot the grids produced in exercise 1, the
        program called geoplot21 may be used. It may be called as
        ../cct/dgeoplot/geoplot21 < ex1plot.inp >ex1plot.out &

        It may be run either batch or interactivly. If run batch,
        the following may be used:

        GEOID HEIGHTS FROM OSU91A, C.I. 1.0 m.  ; Figure text
        4                                       ; Projection
        26.0d0                                  ; Central meridian
        50.0d0                                  ; Base Parallel
        40.0d0  60.0d0                          ; 1. & 2. parall.
        39.0 60.0 7.0 47.0.                     ; Area boundaries
        6.0 4.0 10                              ; Meridians
        40.0 2.5 8                              ; Parallels
        1                                       ; Grid digits
        ciakyst                                 ; Coastline file
        t                                       ; Contouring - yes
        grid11.inp                              ; Grid file name
        3 2                                     ; 3 data elements, 2. used.
        T                                       ; Data gridded - yes.
        20 20                                   ; Grid size.
        1.0                                     ; Contour interval.
        f                                       ; no annotated contours
        f                                       ; No region used.
        2                                       ; 2D plot.
        f                                       ; no UNIPICT output
        t                                       ; Colour contouring.
        6                                       ; Color scale.
        0                                       ; Number of decimals for ci.
        f                                       ; No point file data.
        select MPOST; EXIT                    ; Select plotter unit.



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        Exercise 2.                                  (Vers. June 1997).


        Use the FORTRAN program SELECT to extract gravity and altimeter
        data.
        Nordic gravity data: /disk1/cct/gravity (80 character) or
        New Mexicio gravity data: ../cct/cctf/nmfa
        North Atlantic altimeter data: /disk1/cct/altsca or
        New England Sea Mount altimeter data: ../cct/cctf/altimetry

        (1) Select a 2 deg. * 2 deg. area in DK, N, S, SF or New Mexico
            and plot the positions using geoplot21.

        (2) Select a 2 deg. * 2 deg. area in New Mexico, The Nordic Area,
            or New England as close as possible to a grid with node
            distance 0.1. (In the Nordic Area use a 2 deg. * 4 deg. grid
            and spacing 0.1 deg, 0.2 deg., respectively). Use geoplot21
            to plot and contour the data.


        call of select:
        ../cct/dgravsoft/select

        Example of input to select in order to get gravity data
        in area bounded by 56.5, 58.5 in latitude and 17.0, 21.0
        deg. in longitude with spacing 0.2, 0.4 deg, and output
        to a file with the name gr5817z :

        /disk1/cct/gravity                    ; input file name
        gr5817z                               ; output file name
```



```
        1 6 2                                 ; input mode, type and NDATA.
        56.5 58.5 17.0 21.0 0.2 0.4           ; area boundaries and spacing.

        Input to select New England Altimetry:
        /disk1/cct/altimetry                  ; input file name
        alt3865                               ; output file name
        1 9 2                                 ; input mode, data type & NDATA
        38.0 38.2 -64.9 -64.5 0.0 0.0         ; area boundary and no spacing.



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        Exercise 3.                                        (Vers. June 1997).

        This exercise has the purpose of demonstrating the use of
        the programs for evaluation of terrain effects, TC and TCFOUR.
        These programs accepts as input files with grids of point or
        mean values of terrain heights, and files with coordinates
        identifying the point where the terrain effect is computed.

        For the New Mexico, White Sands area we have available terrain
        files, gravity and deflection files.
        nmdtm            grid file containing 0.5 minute values
        nmdtm5           grid file    ---     5.0 min mean values
        nmdtm30          grid file    ---    30.0 min mean values
        nmfa             point file (no., lat., lon., height, free-air gr)
        nmdfv            point file ( -    -     -      -   , ksi, eta)
                                  deflections of the vertical.

        For the program TC we must specify the terrain files used, the
        kind of terrain effect to be computed (rtm, isostatic etc)
        and limits for where certain computational simplifications
        are to be used (cylinders instead of boxes).

        Call of TC:  ../cct/ dgravsoft/tc

        Input:
        sfile                        ; name of file with station coordinates
        nmdtm                        ; name of point height file
        nmdtm5                       ; name of mean height file
        nmdtm30                      ; name of height reference file
        ofile                        ; output file name
        6                            ; computation of dg,ksi,eta,zeta
        4                            ; computation of rtm effects
        0                            ; computations at terrain level
        1                            ; terrain files standard.
        31.52 34.98 -107.98 -105.02  ; gives area limits
        18.0 999.0 2.67              ; distances where outer grids are
                                       to be used and density.
        -1                           ; indicates that all points are used.

        Repeat the calculation, now computing the values in a grid. This
        requires that the last -1 is changed to 0, and then followed by
        grid specification:
        Min, max latitude (deg), Min, max longitude (deg), spacing in
        latitude (min), longitude (min) and height (m). Example:

        32.5 34.5 -107.5 -105.5 6.0 6.0 1000.0

        The program TCFOUR may also be used for the calculation of
        terrain effects, however with values given in a grid.

        Call of TCFOUR: ../cct/dgravsoft/tcfour

        Input:
        nmdtm                         ; grid file name
        nmdtm30                       ; reference grid name
        nmtcgrid                      ; output file name
```

```
4 t                           ; gravity terrain eff., ref. grid
120.0 0.0                     ; distance
32.8 -106.8 25 25             ; sub-grid sw-corner and points.
```

Compare the result of the two computations.

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Exercise 4. (Vers. June 1997).

This exercise has the purpose of demonstrating the use of the program altcory, written by P.Knudsen. The program will cross-over adjust altimeter data, and produce files of cross-over values and corrected altimeter data.

Call of the program: ../../cct/cctf/altcory.o

Input example:

```
/disk1/cct/altimetry      : input file name
30.0 60.0 200.0 350.0     : areal boundaries (min/max lat,lon)
TEST                      : text to be reproduced
1  -1.0  0.0              : input mode, data spacing, distance.
```

The program produces 3 files
ALTX1, holding cross-over differences,
ALTX2 holding estimated biases and
ALTX3 holding the corrected observations.

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Exercise 5. (Vers. June 1997).

Use the FORTRAN program SELECT to extract gravity or altimeter data from a gravity data file or a file with altimeter data.
Nordic gravity data: /disk1/cct/gravity (80-character standard)
New Mexicio free-air gravity data, (no., lat., long., height, free-air gravity): ../cct/cctf/nmfa
North Atlantic altimeter data: /disk1/cct/altsca or
New England Sea Mount altimeter data: ../cct/altimetry
Note, that SELECT has predefined input format for the North Atlantic data (8) and for the New England data (9).

(1) Select a 2 deg. * 2 deg. area in the Nordic Area, New Mexicio or New England as close as possible to a grid with node distance 0.1 and 0.1 deg. respectively. Denote the files gr.ex5 or alt.ex5, respectively. (Use contingently the files obtained in exercise 2).

Call of select, see exercise 2.

(2) Compute using GEOCOL the values in a similar grid, but with one extra point in all directions using OSU91A, with N=360. See exercise 1 for the input to GEOCOL. However, the logical variables on the line specifying the grid should be T T F. In this case the data will be output on grid form.
Call geocol with: time ../cct/dgravsoft/geocol11 ,to see time used for execution.

(3) Use GEOCOL to compute the values from OSU91A in the points obtained in (1), and subtract the values from the observed values. The results must be output to files with the names grdif or altdif, respectively.

Input to GEOCOL:

```
f                                                ; batch run mode
f t f f f t f                                    ; logical variables
5                                                ; selects GRS80
OSU91A to degree 360                             ; name of set
3.986005D14  6378136.2 -484.1803 360 f f t f; GM, a of set
(2i3,2d19.12)                                    ; data format of set
/disk1/cct/cctf/osu91a1f                        ; name of file
f f t                                            ; select diff. as output
1 2 3 3 4 5 0 13 -1 0.0 t f f f f f t f f t ; codes describing input
gr.ex5                                           ; input file name (grav.)
 24                                              ; fortran unit number
gr.dif                                           ; output file name
5.0                                              ; bin sixe for histogram
T                                                ; stop message.
```

The names of the input and output files should be changed to for example alt.ex5 and alt.dif, if altimetry is used. If the input file contain error estimates of the individual observations, then they may be repeated in the output by using
f t t in the line defining the output. This is needed, if the difference values are to be used subsequently as input when colocation is used (see exercise 10).

The input line which defines the input contains the following values:
1 2 3 - Value 1 is station number, latitude is second and longitude third
3 - angles in decimal degrees
4 5 0 - height as 4.th element, data as 5.th and no second data element.
13 -1 gravity data and in same reference system as 5. If the file contains altimeter data, then : 11 -1
0.0 dummy height (used when records contain no height).
t f etc. logical cariables, see GEOCOL.

(4) Use the grid computed in (2) and the program GEOIP to carry out the same operation as in (3).
Call of GEOIP: ../cct/ dgravsoft/geoip and it will prompt you for the needed information. Use contingently mode=13 in order to have an error estimate associated with the original data element repeated in the output.

Compare the speed of computation. Compute the difference file using the program fc (../cct/cctf/fc). Note, the change in mean value and standard deviation.

Save the resulting files, because they will be used in a subsequent exercise.

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Exercise 6. (Vers. JUNE 1997.

This exercise has the purpose of demonstrating the use of EMPCOV for the computation of an empirical covariance function. In order to use this program, on must first select the spacing and lenght of the table representing the covariance function. If data spaced 6' apart is to be used for the estimation, this is also the natural spacing to be used, and the number of points could then be the size of the data area divided by the spacing. In this manner a reasonabe number of products is secured.

The program also produces a primitive histogram with 21 bins, so the binsize will clearly be e.g. standard deviation of the data divided by 3 or 4. Optionally the mean value may be subtracted from the data. This should only be done, if the data is well (regularily) distributed. Since we want to use the empirically estimated values in exercise 8, they must be stored on a file. This is secured by putting a logical variable true, and giving the name of the file holding the table.

Call of EMPCOV: ../cct/dgravsoft/empcov

Input: (EMPCOV will prompt you for the various quantities)

```
Free-air and residual gravity data   ; test describing data
6.0 20 3 f t t                       ; sampling interval etc.
msm.ex4.cov                          ; file holding the covfct.
420 9 t 3 13 0 5.0 f                 ; input file definition
2 2 0                                ; input record definition
msm.gr.dif                           ; input file name
t                                    ; end input indication
```

Note the correlation length of the resulting covariance functions and the distance to the first zero points.

Compute the covariance function also using reduced data obtained in exercise 3.

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Exercise 7. (Vers. Feb. 1991).

This exercise has the purpose of demonstrating the use of GEOGRID for the gridding of randomly distributed data. Use the difference file of gravity and altimeter data produced in exercise 5 and the correlation length found in exercise 6. Use nodes as close as possible to the nodes used with SELECT. Save the result to be used in exercise 9. Make a contour plot of the data using GEOPLOT21.

Call of GEOGRID: ../cct/dgravsoft/geogrid

Input to geogrid:

```
gr.dif                        ; input file name
gr.grid                       ; output file name
gr.err                        ; name of file with errors
3 2                           ; no. of data, used element
3 1                           ; no. points used for pred., method
20.0 1.0                      ; correl. length (km), noise
1 20.0                        ; mode (grid), interpol. dist.
58.0 60.0 0.1 8.0 12.0 0.2    ; grid specification.
```

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Exercise 8. (Vers. June 1997).

This exercise has the purpose of demonstrating the use of COVFIT for the evaluation of analytic covariance expressions and for the determination of parameters describing the analytic covariance function.

COVFIT operates in 4 modes:
1 − evaluation of an analytic expression
2 − establishing tabulated values, which may be used for fast calculation of analytic expressions
3 − comparison of values interpolated from the tabulated values with the analytically evaluated values

4 − determination of parameters describing the analytic expression, a = scale factor for error degree−variances, RB = radius of the Bjerhammar Sphere and A = scale factor on the model degree variances.

The first part of the exercise demonstrates the influence of the basic parameters on the shape of the covariance function. Here an important role is played by the maximal degree−and order of the reference field used, nmax.

Call of COVFIT: ../cct/dgravsoft/covfit11
and you will then be prompted for the various input items. Use the program for the calculation of 3 different covariance functions at zero and 2000 m altitude. The covariance functions are (geoid,geoid), (geoid,gravity anomaly), (gravity a., do.). This is indicated by using certain codes (1: geoid, 3: gravity anomaly). The covariance functions should be computed using the error degree variances of OSU91A in file ../../cctf/edgv.91, to degree 60, 120, 180 and 360. The scale factor (a) should in each case be 1.0. The scale factor A is determined implicitly by fixing the gravity anomaly variance. It should be fixed to 400.0 mgal*2. The Value of RB is given as the difference in KM between this quantity and the mean earth radius, i.e. a negative number. Use here −1.00, −3.00 and −7.00, in order to see the difference. The covariance finction should be tabulated in steps of 5.0 arc.min., and out to a spherical distance of 5.0 deg., i.e. totally 60 values.

Typical input will look like:

```
f                              ; batch run
1 F F T F                      ; mode (tabulation)
2                              ; degree variance model.
4                              ; integer in denominator.
-1.00 400.00 360 F T F         ; Bjh. rad (km), Variance, deg.
0  1  1.0  F                   ; model, start, scale and log.
../cct/cctf/edgv.osu91         ; file holding error deg. var.
60 5.0   0 0 0.0 t             ; number, spacing, azimuth, last
1 1 0 0 f 1 3 0 0 f 3 3 0 0 t  ; spec. of type and height.
```

In exercise 6 we calculated an empirical covariance function. In the second part of the exercise, we will now use these values as input to COVFIT. But first some starting values must be selected, such as the depth to the Bjerhammar sphere. Since the spherical harmonis coefficient set OSU91A was used, then the error degree variances from this model must be used again.
The program works iteratively, and we must also specify how many times we want to iterate (typically 10).

Typical input will look like:

```
f                              ; batch run
4 F F F F                      ; mode (fitting)
2                              ; degree variance model.
4                              ; integer in denominator.
-0.301000 375.00 360 F T F     ; Bjh. rad (km), Variance, deg.
0  1 0.2  F                    ; model, start, scale and log.
../cct/cctf/edgv.osu91      ; file holding error deg. var.
7  1.00  1.00  1.00            ; iterations and weights
1                              ; number of cov. tables to be input
15  3  3   0.000   0.000       ; steps, type, height
1   1.000 t                    ; error mode, weight and file?
375.0  375.0                   ; variances
61.75     64.75    0.05        ; min, max lat.  and step.
7.0      13.00    0.10         ; min, max long. and step.
ex6.cov                        ; name of file holding table
```

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Exercise 9. (Vers. June 1997).

This exercise has the purpose of demonstrating the use of the fast fourier transform for geoid or gravity compuation. The dataset(s) created in exercise 7 should be used, i.e. regular grids of gravity values or geoid heights, from which the contribution from a spherical harmonic expansion has been subtracted.

The program GEOFOUR must be used. Call of geofour:
../cct/dgravsoft/geofour

Input to geofour:

```
gr.grid       ; input file name
dd            ; height file (here dummy, i.e. not used)
geoid.grid    ; output file name
1             ; mode (1: gravity to geoid, 2: geoid to gravity)
0.0           ; Attenuation (damping) − here zero.
0 0 21 21     ; grid start point and size
2             ; size of cosine−tapered window in grid units.
```

Change the size of the window, in order to see border effects.
Plot the result (gravity or geoid) using GEOPLOT15.

The program GEOIP may subsequently be used to add back the contribution from the OSU91A field, if a grid of geoid heights is available in an area larger than the grid area used here.

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Exercise 10. (Vers. Apr. 1997).

This exercise has the purpose of demonstrating the use of of GEOCOL for geoid or gravity computation from gravity, respectively satellite altimetry. In order to speed up the calculations, all data are supposed to be at zero altitude.

We will use the datasets created in exercise 7 and the covariance function parameters extimated in exercise 8 using COVFIT. Here the major parameters are the scale factor used for the error degree variances (AA), the gravity variance, the depth to the Bjerhammar sphere, and the degree up to which the error degree−variances are used. In fact, the input used to specify the analytic expression for the covariance function are given in nearly the same manner as in COVFIT.

Call of GEOCOL:
../dgravsoft/geocol11

Input:

```
f                          ; batch run mode
F f T F f F t              ; logical variables
t f f f f  f               ;
msmpot                     ; name of restart file
msmneq                     ; name of normal equation file
F T f                      ; logical variables
5                          ; Coordinate system selection
2                          ; degree−variance model no.
4                          ; deg.var. numerator.
−13.618 77.38 360 F f t T  ; covariance spec. param.
0 5 0.0629                 ;       same
../cctf/edgv.osu91      ; name of file with error deg. var.
```



```
1 200    60.0              ; tabulation param.
0.0  2                     ;
1  3                       ; geoid and gravity cov. will be tabul.
−1  2  3  3  0  6  0 −13 −1; input data description. First 0 indi−
                             cates no height.
0.00 F F F F F f T f F T   ; selection of options.
gr.dif                     ; input data file.
4                          ; input unit
5.0                        ; histogram bin size.
T F                        ; logical var. indicating no more data.
F F F                      ; collocation start.
T T F                      ; task definition for prediction step.
39.0 58.0 8.0 27.0 41 41 11; grid N−W corner and spacing definition.
−1 0.0 F T F               ; coord.sys., height, output to file true.
ex10.geoid                 ; output file name.
t                          ; end indication.
```

Compare these values with the values computed in exercise 9.
Plot the geoid heights and the error estimate.

Repeat the computations using the gravity data at their proper heights. Note, that the covariance tabulation specification must be changed. (See GEOCOL).

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Exercise 11. (Vers. June 1997).

This exercise has the purpose of demonstrating the use of the restart file created when running GEOCOL in exercise 10. A file named nsmpot was created, which contain all necessary data, except the instructions for computation of predicted values.

This is done by appending to the end of the file the necessary instructions:

```
t f f                                          ; grid computation
32.5 34.5 −107.5 −105.5 0.2. 0.2 −1 0.0        ; grid specif.
f f f                                          ;
T                                              ; stop
```

which will compute deflections of the vertical in a grid with North−West corner 34.5,−107.5 and spacing 0.2 deg.

```
f f t                                          ; comparison
1 2 3 1 4 5 6 25 4 0.0 f f f f f t f f t       ; record descr.
nmdfv                                          ; input file
25                                             ; unit
0.5                                            ; histogram spacing
t                                              ; stop
```

where we will compare observed and predicted deflections of the vertical in NAD27 datum. This naturally requires, that msmpot holds a solution for the New Mexico dataset.