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Books

Advanced Physical Geodesy

H. Moritz, Herbert Wichmann, West Germany, xiii + 500 pp., 1980, $57.00.

Reviewed by C. C. Tscherning

Physical geodesy, the study of the gravitational field of the earth, has seen a tremendous progress in the last 20 years. Much of this progress is due to the author. He is professor at the Technical University at Graz, Austria, and adjoint professor at The Ohio State University. Presently, he is president of the International Association of Geodesy.

The progress has taken place both in the theoretical foundation and in the practical field; however, the book under review deals nearly exclusively with the advances in theory. The author has furthermore restricted himself to the treatment of time invariant phenomena.

The book is divided into four parts. Part A provides a brief general geodetic and mathematical background. It includes basic gravity field concepts, reference ellipsoid, spherical harmonics, and takes a first look at Hilbert spaces. These spaces are the natural generalizations of finite dimensional euclidian spaces to spaces of infinite dimension, and thereby gives the framework for the treatment of the earth's gravity potential *(V)*. The fact that this potential may only be expressed as the sum of an infinite series necessitates the use of such spaces. The author then has the tools to treat the problem of the convergence of the spherical harmonic expansion of *V*, i.e., the problem of whether this expansion is valid inside a sphere including all masses.

The absurdity of this question is pointed out, and the appropriate answer is given in the form of the so-called Runge's theorem: It is possible to find an arbitrary good approximation to *V* expressed as a linear combination of a finite number of solid spherical harmonics. That the coefficients of this finite series may not anymore be related to the true earth's density distribution is a drawback, which is not mentioned.

Part B gives an elementary introduction to the approximation method called least squares collocation. The author proceeds inductively by first explaining how this method generalizes least squares interpolation to a prediction or approximation method. It is also explained how the method is adapted to permit the use of noisy data and the determination of parameters. The use of the method is illustrated by examples from physical geodesy: geoid determination, prediction of gravity anomalies or deflections of the vertical, and determination of spherical harmonics.

One difficulty associated with the rigorous use of the method stems from the fact that a system of linear equations (with full coefficient matrix) must be solved which has as many unknowns as the number of observations used. The use of a stepwise solution method is advocated by the author. Such methods may give computational savings, but not the one described by the author. The use of the method requires that some a priori information about the magnitude of the coefficients of the spherical harmonic series is available in the form of a so-called empirical covariance function. The global and local structure of this function is discussed in the last sections of part B.

The least squares collocation method may very well be understood and applied based on a careful study of part B. However, the relation to other techniques, the understanding of the difficulties associated with the method and of a number of implicitly introduced assumptions require additional mathematical and statistical tools. They are introduced in part C, which also gives a rigorous treatment of least squares collocation based on the theory of Hilbert spaces with a reproducing kernel.

One of the theoretical difficulties associated with least squares collocation is caused by the fact that the (anomalous) gravity potential has an infinite norm in the Hilbert space, where the empirical covariance function is used as a reproducing kernel (p. 219). The author claims that this is merely a mathematical subtlety. He is hopefully right, because otherwise numerical problems may show up when large amounts of data are used.

The least squares collocation method very much resembles prediction methods used for ergodic stochastic processes on a sphere. The deterministic character of the gravity potential makes this type of interpretation questionable. But the author manages to give a very clarifying discussion of this difficult problem.

The theoretical treatment of the geodetic boundary value problem has been revolutionized in the last 20 years. The starting point was the first rigorous formulation of the problem by the Russian scientist Molodensky. Very briefly explained, the problem is to determine the unknown surface of the earth *(S)* given on *S* the gravity vector, *g*, and the gravitational potential, *W*, which is the sum of the gravity potential, *V*, and the centrifugal potential. An important aspect of the problem is that *W* is not a harmonic function (but *V* is) and that the boundary itself is an unknown.

Part D is nearly completely devoted to the treatment of this 'Molodensky's problem,' which may seem very far away from geode practice. But one must agree with the auth that theory should preferably be more advanced than practice. (Unfortunately, we a in many cases in the opposite situation.) V ous solution methods are discussed, some c which involve a series development. Its con vergence is not assured, but the author see convinced, that this will not cause any prob lems in practice.

The first proof of existence and uniquen of the solution to the boundary value prob lem was given by the Swedish mathematici Lars Hörmander, who used only a few restricting conditions on the smoothness of a idealized earth surface. The proof is very c ficult, but the author has managed to expl the result and outline the proof in a clear a enlightening manner.

A new approach, where the three compo nents of the gravity vector are used as coor nates is also described. The advantage is here, that the boundary S becomes known, but *W* will have to fulfill a very complicate partial differential equation. However, less advanced mathematical tools than were us by Hörmander may now be used involving fewer restrictions on *S* than were used before, and the new approach has thereby formed the basis for new research in theor cal physical geodesy.

Many of the important results presented the book have been obtained by using app priate but rather abstract mathematical co cepts. Some readers might be scared by th which also in some way justifies the term 'a vanced' occurring in the title of the book. However, the author has managed to mak even the most abstract mathematical conce understandable for readers with (only) an dergraduate course in linear algebra and analysis.

It is a pleasure to read the book. It is cl and concise, except maybe at a few points where the author in order to assist the rea in passing a difficult point uses a heuristic type of explanation. The book is directed graduate students and research workers i geodesy and gravity. It may be very usefu a textbook in a second course in physical g odesy. It will not in any way replace the b *Physical Geodesy*, which was published in 19 which the author wrote together with the W. A. Heiskanen, but it treats in a compre hensive way (in my opinion) all substantial theoretical advances that have occurred si that book.

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