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GEOID MODELLING USING COLLOCATION

by

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Abstract:

Collocation has been used for the construction of approximations to the anomalous gravitational potential in Scandinavia and Greenland combining geopotential coefficients to degree and order 180, gravity anomalies, deflections of the vertical, Doppler satellite and satellite altimeter derived geoid heights. The method was also simultaneously used to determine a longitude bias of the deflections corresponding to a rotation around the Earth's Z-axis of -0.50 +/-0.26 arcsec.

Approximations were also determined for testpurposes without using the (approximate) geoid heights determined by satellite altimetry. Using a semi-major axis of 6378136 m, the r.m.s. difference between observed and computed values was +/-0.4 m for the sea areas bordering Scandinavia up to a distance of 150 km from the coast, with an even better agreement in areas with available sea gravity data.

1. Introduction.

The gravitational potential (W) is equal to the sum of the gravity potential (V) and the centrifugal potential (Φ), $W = V + \Phi$.

W is in princple varying with time, but we will here regard W at a specific epoc. The geoid is then an equipotential surface of W, which agrees most closely with mean sea level at the same epoc. This surface will outside the oceans generally pass through the solid earth surface, a fact which makes it necessary to introduce the concept of the quasi-geoid.

Let U be an approximation to W, the normal potential, for which the reference ellipsoid is an equipotential surface with $U = W_0$, where W_0 is the value of W at the geoid. It is futhermore constructed so that the difference $T = W - U$, the anomalous potential, is a harmonic function, regular at infinity.

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To a point in space, P, corresponds a point Q located on the same normal to the ellipsoid as P, for which U(Q) = W(P), see Figure 1. The ellipsoidal height distance between P and Q is called the height anomaly $\zeta(P)$. This quantity can be used to construct another point P', as the point on the ellipsoidal normal having the height $\zeta(P)$. The surface formed by taking all points P' corresponding to P at the Earths surface is called the quasi-geoid. (That this construction is not always unique causes a small problem in physical geodesy).

Therefore, if we are able to determine T, we can determine the quasi-geoid. Obviously, the geoid and the quasi-geoid are nearly identical at the oceans.

The advantage of dealing with T and the corresponding quasi-geoid stems from the fact that T is a harmonic function. It may be determined by solving a boundary value problem. Or, approximations T' may be determined by numerical techniques for solving boundary value problems.

The purpose of this paper is then to describe some recent results obtained using collocation for the determination of an approximation T' and thereby of the quasigeoid. I also want to point out some of the problems which arize when using this method. In section 2 a very short review of collocation is given and the main problems arizing when using the method are described. The solutions to these problems and the results obtained in two areas, Greenland and Scandinavia are described in section 3. The results include comparisons with SEASAT-A sea surface heights.

2. The method of collocation and the problems arizing using it.

We suppose that we have observed values (x(i)) related to the gravity field, which may be expressed as the sum of a linear functional L_i applied on T, the scalar product of a vector A_i and a parameter vector X and the noise n(i),

$$x(i) = L_i(T) + A_i^T * X + n(i). \qquad (1)$$

The parameters may represent the effect of a datum-shift correction or some other systematic effect.

Collocation is then a technique, which determines a (harmonic) function T' and estimates of parameters X' so that we for n(i) = 0 has an exact agreement,

$$x(i) = L_i(T') + A_i^T * X' .$$

A unique solution is found (for example) by requiring T' to be an element of a reproducing kernel Hilbert space (consisting of harmonic functions, with norm $\|\cdot\|_H$) and X' to be an element of a linear vector space with inner product (and norm $\|\cdot\|$) and so that

$$\|T'\|_H^2 + \|X'\|^2 + \lambda(x^T * R^{-1} * x)$$

is minimalized. Here R = R(i,j) is the variance-covariance matrix of the noice vector n and λ is a positive constant.

If the so-called empirical covariance function is used as the reproducing kernel, then T' will be optimal in a least-squares sense, (see Moritz(1980)). We call this variant of collocation for least-squares collocation. (Also $\lambda = 1$ here). We will only discuss this form for collocation in the following.

The covariance function of two values $L_P(T) = T(P)$ and $L_Q(T) = T(Q)$ is here equal to

$$\mathrm{cov}(L_P, L_Q) = K(P,Q) = \sum_{n=2}^{\infty} \sigma_n \left(\frac{R_E^2}{r_P r_Q}\right)^{n+1} P_n(\cos\psi_{PQ}), \tag{2}$$

where r_P and r_Q are the radial distance of P, respectively Q, from the origin, ψ_{PQ} the spherical distance between P and Q, σ_n the so-called degree-variances and R_E is the mean radius of the Earth. σ_n is equal to the square sum of the n'th degree fully normalized spherical harmonic coefficients $\bar{C}_{nm}$, $\bar{S}_{nm}$ occurring in the spherical harmonic expansion of T.

We will denote one or two linear functionals applied on K by

$$L_i(K(P,Q)) = K(L_i, Q) = K_i(Q),$$

$$L_i L_j(K(P,Q)) = K(L_i, L_j) = K_{ij}.$$

Let us now regard a set of observations x(i), i = 1, ..., N, and put

$$\bar{C} = \{K_{ij}\} + \{R_{ij}\}, \quad A = \{A_{ij}\}, \quad x = \{x(i)\}.$$

Then

$$T'(P) = \sum_{i=1}^{N} b_i * K_i(P), \tag{3}$$

$$X' = (A^T * \bar{C}^{-1} * A)^{-1} * A^T * \bar{C}^{-1} * x, \tag{4}$$

$$\{b_i\} = \bar{C}^{-1}(x - A^T * X'). \tag{5}$$

Error estimates may also be computed for predictions, L(T'),

$$\sigma^2(L(T')) = (K(L,L) - \{K(L,L_i)\}^T * \bar{C}^{-1} * \{K(L_j,L)\}) , \tag{6}$$

in the simplest case with no parameters. Error expressions for the parameters can be found in Moritz(1980).

(Note, that we as a condition for obtaining a solution must require a least $\bar{C}$ to be positive definite).

The most severe problem should in principle be the determination of the empirical covariance function (2), as it involves the knowledge of the coefficients of the spherical harmonic expansion of T, the function we are trying to approximate. However, for the use of collocation as a kind of interpolation method (e.g. predicting gravity anomaly values from other gravity values) the results have not been very sensitive to the kind of model used to describe the variation of the degree-variances. But problems have been found, for example when computing geoid heights from gravity anomalies (see Arabelos (1980)).

Another problem arizes because of the global character of (2). The solution only becomes optimal in the mean for all data point configurations, which by a rotation of the coordinatesystem will coinside. The use of empirical covariance functions, which in a better way describes the local behavior of the gravity field may help in solving this problem (see Lachapelle & Schwarz (1980), Goad et al. (1983)). Futhermore, the main causes of the an-isotropies of the gravity field, namely the attraction of the topographic masses, should be removed (and later on restored) as discussed in Forsberg & Tscherning (1981, 1981a).

A severe problem in practice occurs due to the need for solving the system of linear equations eq.(5), which in principle has as many equations as the number of observations, N. However, using eq.(6) with varying data configurations, shows that the addition of new data not always will give any essential improvement of the needed result, such as the geoid in a specific local area.

The result will to a greath extend depend on the local data distributi Hence it is reasonable to try to construct local, but overlapping solutions as proposed in Tscherning(1975). Alternatively one may use a covariance function, which is put equal to zero for $\psi > \psi_0$, and then solve the equations using sparce-matrix techniques. (This alternative is presently being investigated).

But what should be the size of the overlap area ? And how should the the solutions be "glued" together?

Finally, the real practical problems are connected to the data:

(1) Lack of sufficient data,
(2) Data is disturbed of systematic effects or by random effects with (sometimes unknown) statistical characteristics,
(3) Data are disturbed by gross-errors,
(4) Data not properly related to a well-defined datum.

However, using collocation, we have the possibility to combine various data types and to model systematic and random effects in a consistent manner.

3. Recent (quasi)-geoid determinations in Greenland and Scandinavia.

The last years have given a very important improvement in our knowledge of the global gravity field variations, primarily due to the collection of sea surface heights by the satellites GEOS-3 and SEASAT, see e.g. Rapp (1981, 1982). Especially the 180 * 180 degree and order potential coefficient sets (Lerch et al. (1981), Rapp (1979, 1981)) have been valuable for the construction of local approximations to the gravity field. By subtracting out the effect represented by one of these 180 * 180 sets, the covariance function of the residuals will get a correlation distance much smaller than before (see Tscherning & Rapp (1974) and Tscherning (1982, Fig. 1)). In the results discussed below we have throughout used the Rapp (1979) solution, which seems to be the best one in the investigated areas (see also Tscherning & Forsberg (1982)).

3.1. Greenland.

In Greenland geoid undulations are determined at the coast as the difference between ellipsoidal heights (determined by Satellite Doppler technique) and heights above mean sea level determined by traditional geodetic techniques. The problem is to "extrapolate" these values to points far away from the coast, see Fig. 2, so that the ellipsoidal heights can be "converted" into heights above mean sea level, see Forsberg & Madsen (1981).

Besides the potential coefficients and the height anomalies at the coast we have point gravity values (20 -30 km spacing) and preliminary 10 km * 10 km mean topographic heights. Recently also SEASAT sea surface heights have become available, but we have not yet been able to take advantage of this information.

Now, the point is, that it without any special problems was possible to combine various data types in a rigorous way. The resulting geoid is shown in Fig. 2. The geoid is estimated to have a standard deviation of 1.1 m. The magnitude of the value is caused by the quality of the Doppler-derived geoid heights and the (probably) rather large error in the contribution from the potential coefficients in North Greenland, see Table 1.

Table 1. Survey of comparison between Doppler derived geoid heights and values computed from various data-combinations, 58 stations used.

Data used	Difference observed-computed (m). Mean	Standard deviation
None	28.85	9.31
Rapp-79 coeff.	-0.78	4.23
Rapp-79 & gravity	-0.80	3.01
Rapp-79, gravity, topography.	-0.42	2.43

The main advantage of using collocation here has been, that it was possible to combine all datatypes in a rigorous way.

3.2 Scandinavia.

The present author is responsible for the computation of a standard geoid for the area of the Nordic countries. The object is to compute (quasi) geoidal heights, for which the standard deviation of the differences are below 0.5 m. The project is described in detail in Tscherning (1982, 1982a).

Very much data is available, but it is rather irregularily distributed. In most of the Baltic Sea, no gravity values are available due to the political situation in this area. However, deflections of the vertical makes it possible to "extrapolate" the geoid from the coast to the sea. And fortunatly sea-surface heights determined from satellite radar altimetry have now also become available. They have, however, not been used to there full extend, because we first must investigate the magnitude of possible oceanographic effects. This may be done by computing geoid heights from independent data, such as topography, gravity, deflections of the vertical and doppler derived geoid undulations.

First approximations T' valid in blocks of approximatly 220 km * 220 km extent (or larger) were computed using gravity data and gravity data combined with deflections of the vertical. The deflections of the vertical were transformed to an approximate geocentric reference system using parameters given in Ordnance Survey (1981) and by T.Vincenty (1980, private communication).

A comparison with a set of Doppler derived geoid heights and a set of SEASAT sea surface heights spaced 1 deg. apart (Rapp, 1981, private communication) showed that the best fitting ellipsoid had to have a semi-major axis a = 6378136 m and that the Doppler coordinate system needed a shift of the equatorial plane of 3.5 m. Using these parameters the difference between 31 doppler derived geoid heights and geoid heights computed from Rapp's 1979 solution had a mean difference of 0.49 m and a standard deviation of 0.78 m.

The approximations were used to compute the geoid height in the points where SEASAT heights were available within the area of validity of the solutions, see Tscherning(1982a, Table 5). In the southernmost block, which had an East-West extend of 600 km, it was recognized that the computed geoid had a large tilt. This tilt was not present in the solution where only gravity data had been used. The tilt is illustrated in Fig. 3.

The reason for this tilt is, that the zero-meridian plane for the NWL9D-system is not well defined, see e.g. Hothem et al.(1982). Collocation was then used to determine a correction to the meridian plane, described as an unknown rotation ω_z around the Z-axis. First the value was determined using only gravity and deflection data, and a value of $\omega_z = -0.3''$ was found. However, this did only remove a part of the tilt. Secondly, SEASAT heights in the area bounded by $56 < \varphi < 58$, $7.8 < \lambda < 12$ was used and treated like geoid heights. (A comparison of these heights with geoid heights determined using only gravity data showed, that the oceanographic effects in this area probably were small, while large effects (up to 30 cm) are known to exist in the Danish Straits (Borre, 1970)).

Using the updated parameters, new comparisons with the SEASAT heights was made. (Note, that it was not necessary to compute the whole set of solutions to the equations (2) once more, as the reduced normal equations had been saved on magnetic tape). The comparison has not yet been finished, but the results obtained by January 10, 1983, are given in Table 2.

The computed geoid is shown in Fig. 4, and the estimated standard deviations in Fig. 5. Local empirical covariance functions were determined for each block, in order to reflect the local gravity field behavior in the best possible way.

Table 2. Results of comparisons between computed geoid heights and SEASAT sea surface heights.

Area				SEASAT heights		Differences		
Latitude		Longitude		mean	standard deviation	mean	standard deviation	number of
min	max	min	max	m	m	m	m	values.
61°.5	63°.5	14°	19°	20.81	1.59	-0.12	0.39	277
59.5	61.5	4	10	43.93	0.53	-1.34	0.42	65
59.5	61.5	14	19	22.31	1.13	-0.03	0.33	104
57.5	59.5	5	10	39.36	1.50	0.01	0.22	436
57.5	59.5	10	14	37.28	0.55	-0.11	0.19	57
57.5	59.5	14	19	24.31	2.06	-0.49	0.37	184
54.5	57.5	7	17	37.20	2.29	0.12	0.27	426

The results are very satisfactory in all areas, where sea-gravity is available, see Tscherning (1982a, Fig. 1). The large error in one of the blocks is primarily due to the fact that no gravity values east of longitude 5 deg. was used, and because the local very strong gravity variation. Fortunatly this is very well reflected in Fig. 5, which shows the estimated prediction error.

4. Conclusion.

The method of collocation has been used successfully for the determination of the geoid in areas of large extend. The problems arizing when using the method have not yet been completely solved. Whether the method will give better (or worse) results than other techniques will still have to be investigated.

An agreement with SEASAT sea surface heights is found at the 0.4 m level, with an even better agreement in areas with available sea gravity. An even better agreement can be expected if more data are used and the topographic variations are taken into account as we were forced to do in Greenland.

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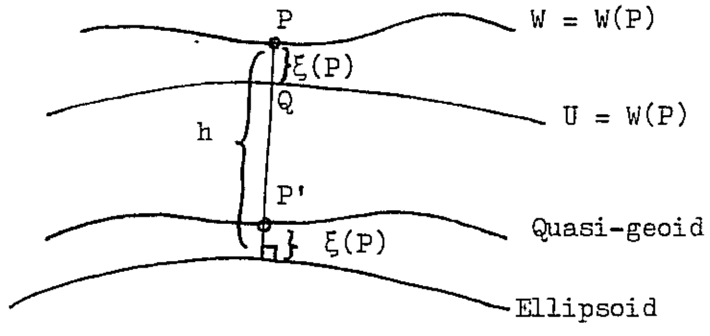

Fig. 1. The relationship between the quasi-geoid and the height-anomaly ξ(P).

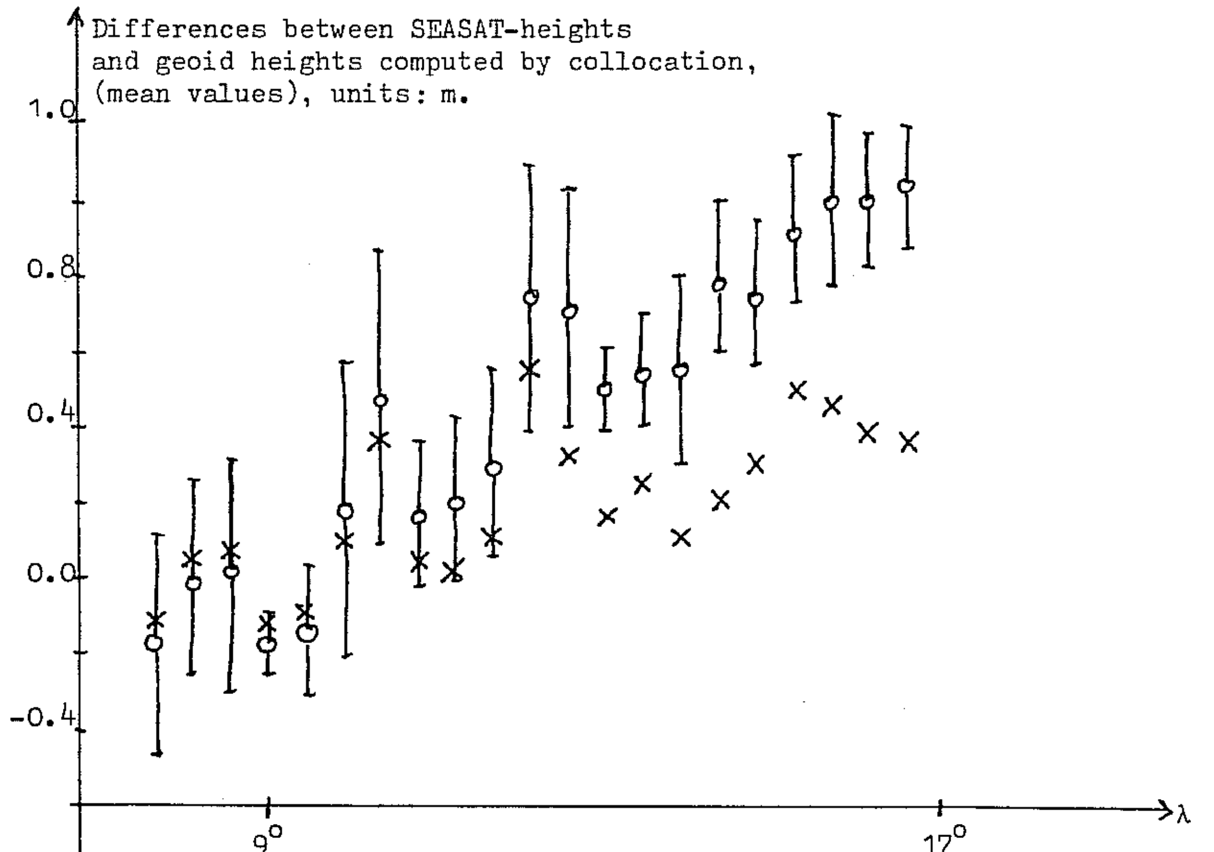

Fig. 3. Mean values of differences between SEASAT sea surface heights and geoid heights determined using (o) gravity anomalies and deflections of the vertical and (x) only gravity anomalies. The differences have been sampled in equidistant belts having a longitude extend of $0.^\circ25$ (at $\varphi = 56^\circ$) bounded by $54.^\circ5 \leq \varphi \leq 57.^\circ5$. The standard deviations for the differences in each class is shown as a bar for the first kind of differences (o). The $0.''5$ tilt of the computed astro-gravimetric geoid is seen clearly.

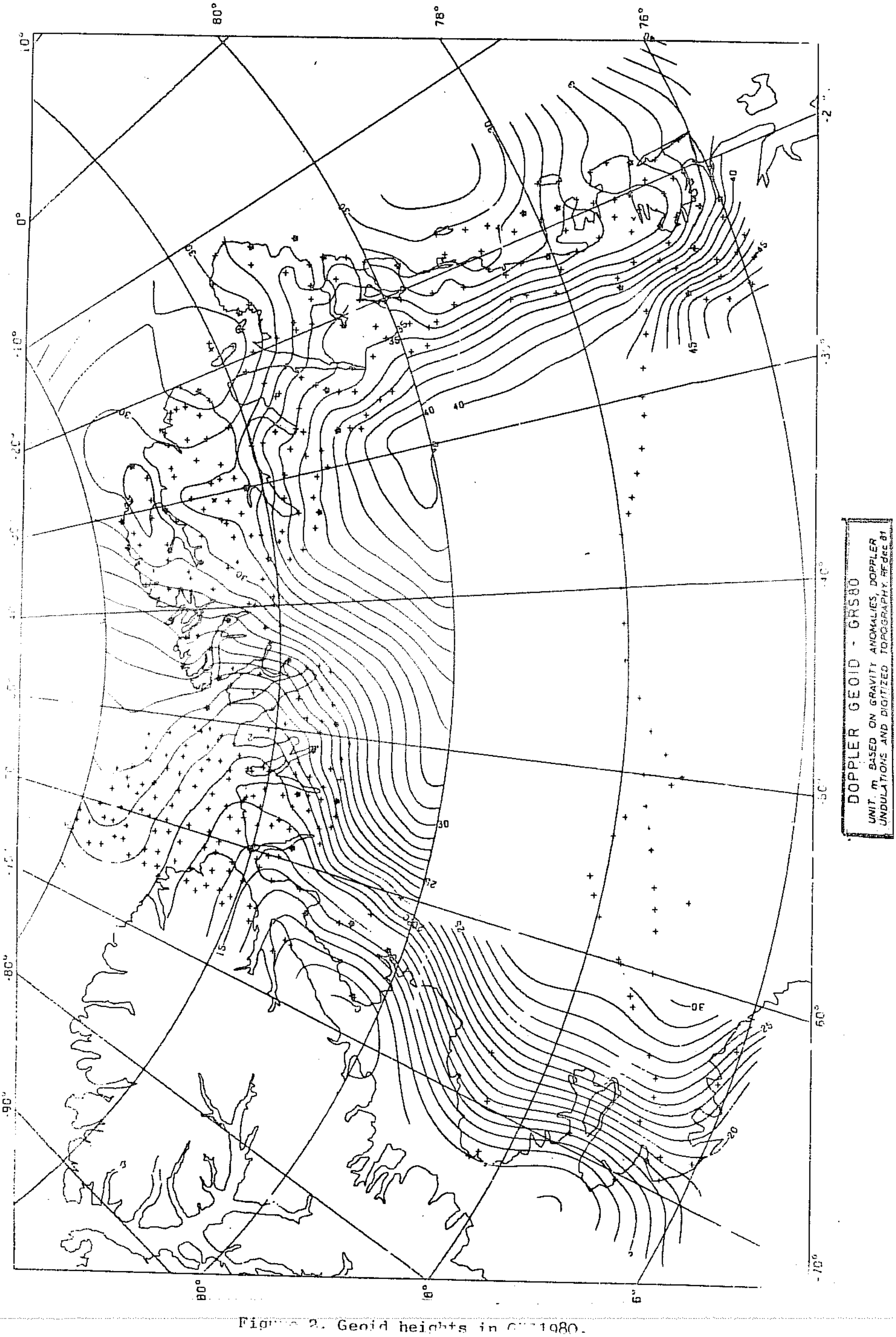

Figure 2. Geoid heights in GRS1980.

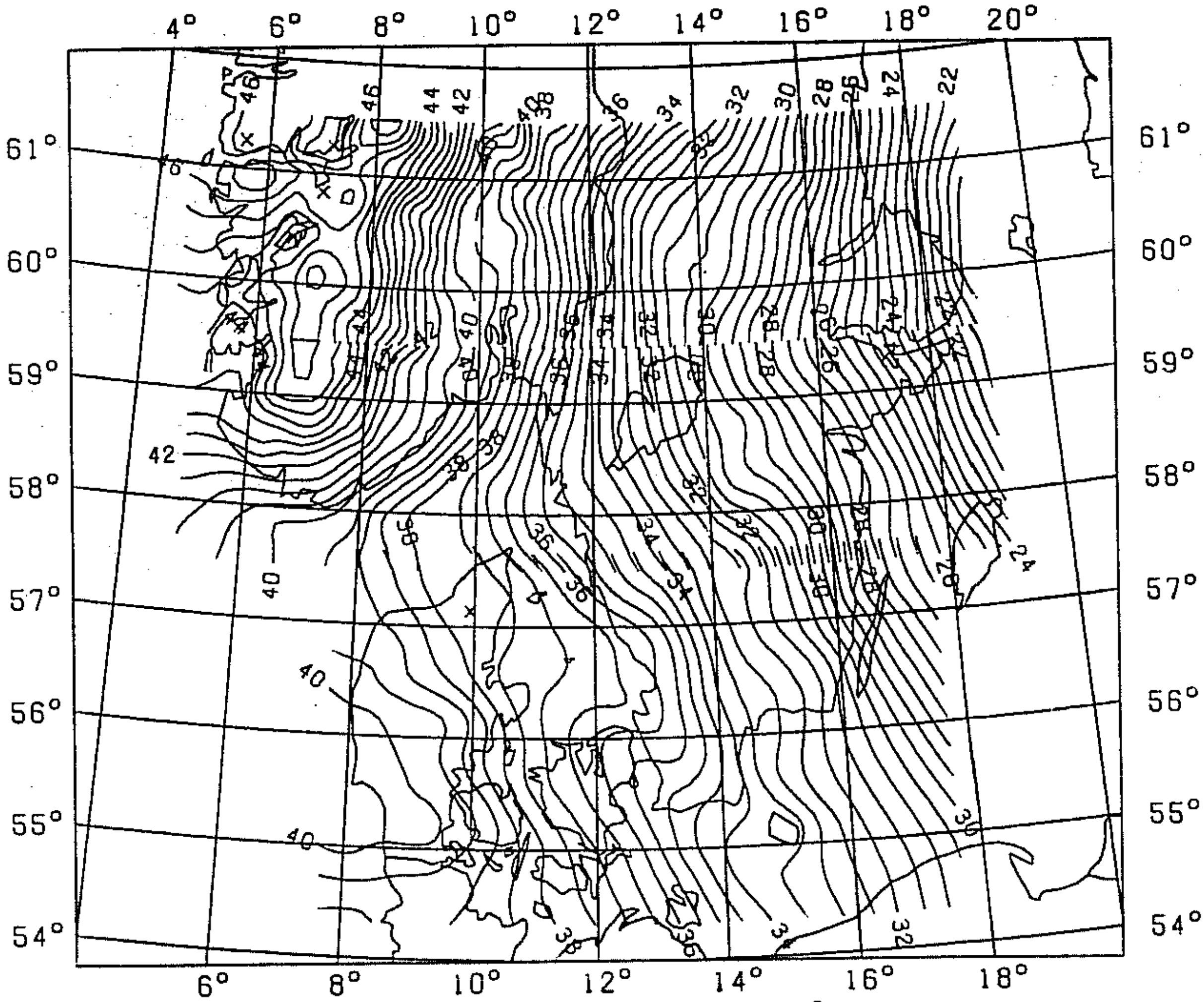

Figure 4. Geoid heights in GRS1980.

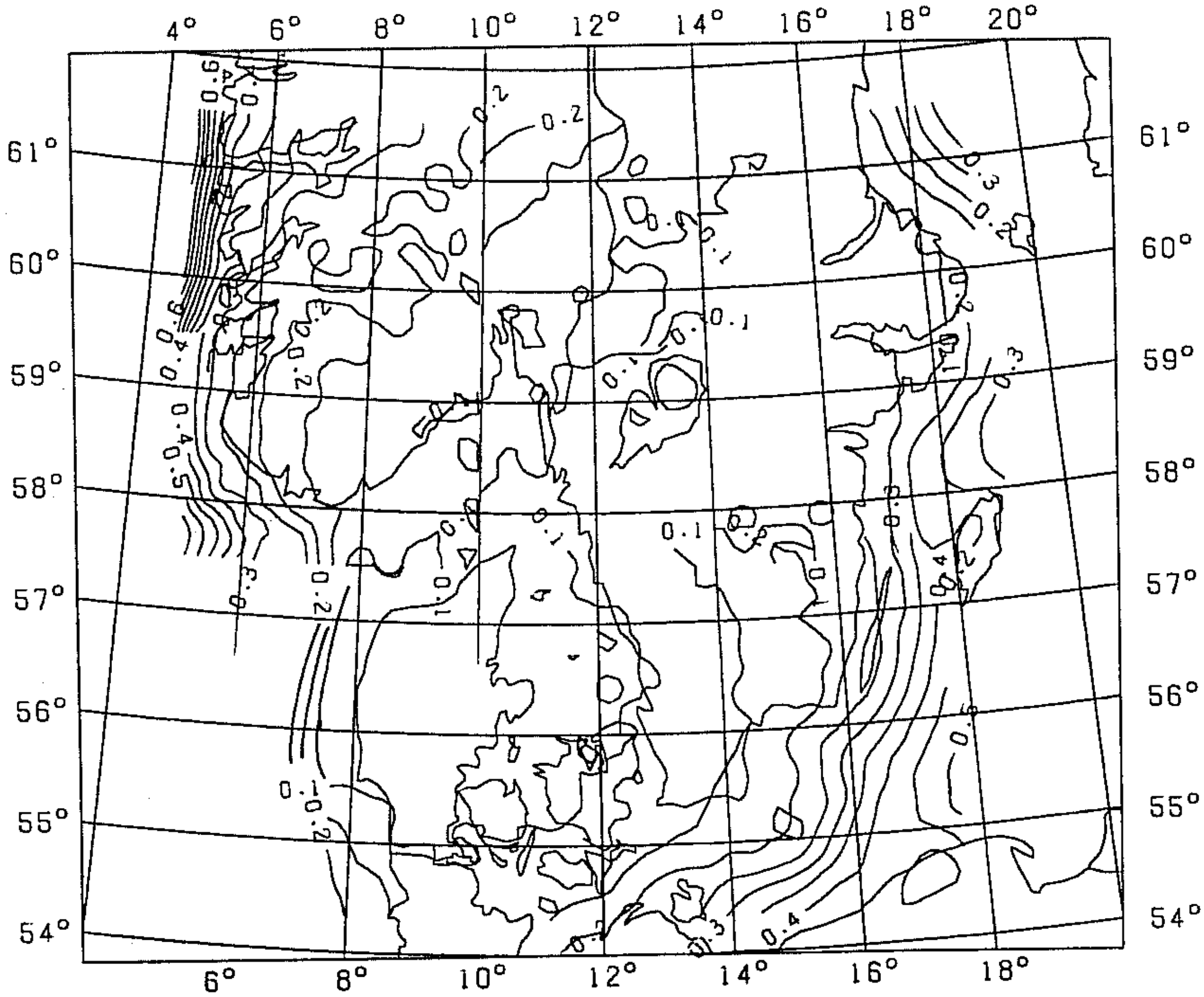

Figure 5. Estimated error, cont. 0.05m.