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Use of collocation for deflection prediction for the U. S. Area

1. When using collocation for the determination of an approximation to the anomalous potential, the main problem is the fact that as many linear equations as we have observations will arise. If we in some manner are able to break down the computation in smaller units, this problem is solved.

One way to break down the computation is to compute more and more detailed approximations. A first approximation is then given by a set of potential coefficients. A series of second approximations, each valid in a separate area may then be computed e.g., from mean gravity anomalies. Each "regional" approximation could for example be valid in a 5^o area. In order to take into account the effect of the gravity in the vicinity, a 5^o-10^o band around this area could be used. This would mean that a system of equations with between $15^2=225$ and $25^2=625$ unknowns must be solved. The size of the band will depend on whether there locally will be available astro-geodetic deflections which can eliminate the effect of "missing" data. Approximately 40 such blocks will be needed in order to cover the U. S.

The inner 5^o area can be further subdivided in 1^o blocks, where point gravity anomalies can be used. If the deflections in the area have been reduced by the attraction of the isostatically compensated terrain in all areas with a strongly varying topography, we can expect the overall variation of the deflections to be ± 3". If we require the predictions to have a standard deviation of ±1", this will require a spacing of the gravity anomalies of approximately 7'. In order to take into account the gravity field variations which occur around the area, each block will have to be surrounded by a 20' band. The extend of the whole area will then be 100'. Hence, 20 x 20 = 400 point gravity anomalies will be needed. The actual number will in many cases be considerably smaller (200) because of lack of data. (If much more data is available a further subdivision can be used.)

2. A local variation of the deflections of the vertical of ± 3" correspond to a variation of the gravity anomalies of ±20 mgal. In all blocks where this variation is larger, the effect of the terrain must be removed to an extend so that this variation is below ±20 mgal. In such a block the effect of the terrain must be removed using one specific representation of the terrain. This means that the reduced gravity anomalies may be different for each 1° block in case the model of the terrain can not be used for the whole 5° area. However, the use of a representation of the terrain, valid for a whole 5° area will cause some problems, because we then will have to consider a change in the used set of potential coefficients.

3. In order to determine where it is necessary to take into account the terrain, the gravity data must first be used for the computation of the variation of the point anomalies. In each area point gravity

anomalies must then be extracted so that they are spaced as uniformly as possible in a grid. The side length will depend on the variation. In areas where the variation is larger than 400 $mgal^2$ (even after the effect of the terrain has been subtracted) the spacing must be $< 7'$. When it is smaller, a larger spacing can be used, (100 $mgal^2$ requires a spacing of 10'

For the selected areas the effect of the terrain on the point anomalies must now be computed. The values should be stored together with the gravity anomalies. It may be worthwhile to compute the effect on the deflections simultaneously.

4. We can now visualize a partitioned data set consisting of 40 sets of mean gravity anomalies associated with the 5° blocks and 1000 sets of point gravity anomalies, contingently with their terrain "effects." The collocation method can then be used for the computation of these 40 and 1000, respectively, regional and local approximations to the anomalous potential. It is then possible to predict available deflections of the vertical and thereby get a check of the precision and of a contingent bias. If the precision is satisfactory, these deflections can be added in order to improve the solution. (This will not require much extra time, because the reduced normal equations can be used again.) If the result is not satisfactory, a number of additional anomalies must be selected and used to improve the solution. The implemention of this may be facilitated by the restart file method discussed by John Isner.

5. In the final step the deflections of the vertical of the 200,000 points in the U. S. can be computed. The advantage is now, that (1) the predicted deflections will agree with the computed in all points where astronomical latitude or longitude has been observed, (2) that predictions of deflections, gravity and geoid underlations can be computed in any new points - also in points where gravity is needed for computations in connection with leveling (3) error estimates can be computed for all quantities (4) new observations may be added if a higher precision is need.

Some problems have not yet been solved. (1) Can one covariance function/ reproducing Kernel be used everywhere, and which one should then be used, (2) There does not exist a FORTRAN-program system developed to a level where the above discussed procedure can be fast and easily executed.

6. An argument against the above proposed procedure has been that the highest precision can not be obtained because not all available gravity data points are taken into account. They may, however, be taken into account by the stepwise collocation method, simply by adding more steps. On the other hand, the method permits the use of all kinds of data - so in areas with scarce gravity information the collocation method will give the best (most precise) results.

7. For certain military and civil inertial navigation purposes, a fast flexible method is needed for computing not only deflections but also

the gravity (disturbance). The collocation method has here definite advantages and the precision have to be traded off against core-storage requirements and computational speed.

8. Recommendations

Continue the development of the integral formulae technique so that deflections can be computed in the Eastern U. S. in time for the upstart of the NAD-adjustment of these areas. Prepare data so that collocation also can be used.

In Mountains consider both the integral formula technique (with Molodensky corrections, gravity gradients etc.) and the collocation technique.

A final product should consist of deflections and standard deviations in all NAD - points - and a mathematical/numerical approximation to the anomalous potential which can be used in all areas of the U. S. Cooperation with DMAAC is recommended.

Literature

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