AN ESTIMATION OF THE HEIGHT SYSTEM BIAS PARAMETER N_o USING LEAST SQUARES COLLOCATION FROM OBSERVED GRAVITY AND GPS-LEVELLING DATA

MUHAMMAD SADIQ[1], CARL C. TSCHERNING[2] AND ZULFIQAR AHMAD[1]

1 Department of Earth Sciences, Quaid-i-Azam University Islamabad, Pakistan (sdq@geo.qau.edu.pk, fz97@hotmail.com)

2 Niels Bohr Institute, University of Copenhagen, 2100 Denmark (cct@gfy.ku.dk)

Received: October 24, 2008; Revised: March 12, 2009; Accepted: May 4, 2009

ABSTRACT

This paper deals with the analysis of gravity anomaly and precise levelling in conjunction with GPS-Levelling data for the computation of a gravimetric geoid and an estimate of the height system bias parameter N_o for the vertical datum in Pakistan by means of least squares collocation technique. The long term objective is to obtain a regional geoid (or quasi-geoid) modeling using a combination of local data with a high degree and order Earth gravity model (EGM) and to determine a bias (if there is one) with respect to a global mean sea surface. An application of collocation with the optimal covariance parameters has facilitated to achieve gravimetric height anomalies in a global geocentric datum. Residual terrain modeling (RTM) technique has been used in combination with the EGM96 for the reduction and smoothing of the gravity data. A value for the bias parameter N_o has been estimated with reference to the local GPS-Levelling datum that appears to be 0.705 m with 0.07 m mean square error. The gravimetric height anomalies were compared with height anomalies obtained from GPS-Levelling stations using least square collocation with and without bias adjustment. The bias adjustment minimizes the difference between the gravimetric height anomalies with respect to residual GPS-Levelling data and the standard deviation of the differences drops from 35 cm to 2.6 cm. The results of this study suggest that N_o adjustment may be a good alternative for the fitting of the final gravimetric geoid as is generally done when using FFT methods.

Key words: covariance function, height anomalies, geoid undulations, bias parameter, global sea level

1. INTRODUCTION

The use of GPS to determine orthometric or normal heights in a region to supplement existing levelling requires determination of a local geoid (or quasi-geoid) and estimation of the height bias between a gravimetric datum and the local levelling datum, since the global geoid or sea level will have its own zero level. The ocean surface does not coincide with a level surface (e.g. the geoid) of Earth's gravity field; the deviations are called Sea Surface Topography (SST, also denoted as Dynamic Ocean Topography). Instantaneous SST is affected by temporal variations of long term, seasonal and short term character, occurring at different scales. Averaging the ocean surface over time (at least over one year) or modeling ocean tides provides mean sea level (*MSL*) for the corresponding time interval. Even after reducing all time dependent parts, a quasi stationary SST would still remain (*Torge, 2001*). It is caused by nearly constant oceanographic and meteorological effects which generate ocean currents and ocean surface slopes. The root mean square (*RMS*) variation of the global sea level is 0.6 to 0.7 m and the maximum deviation from the geoid is about ± 1 m or more (*Lisitzin, 1974*).

In addition to this, all levelling datum have an inherent error, due to the fact that the zero levels have been fixed by a convention (adopting a specific zero level) and not through a physical measurement. This offset error can be described for a particular system by a bias parameter N_o if the local height datum has been fixed at only one tide-gauge. Now, when a purely gravimetric quasi-geoid is compared to a surface constructed from height anomalies derived from GPS-Levelling one will often note that the surfaces disagree (*Jiang and Duquenne, 1996*). These surfaces can be related through a bias or tilt. In this case the two surfaces may be brought to a close agreement by estimating and applying bias and tilt parameters.

Least squares collocation (LSC) has been used here to estimate a gravimetric height anomaly/geoid, a corresponding height reference surface and a bias parameter N_o. Least squares collocation filters the data which is used for the determination of an estimate of the anomalous potential $\tilde{T}$. We may then as done in e.g. geodetic network adjustment, inspect the residuals (*Pope, 1976*) by using the model for the computation of the estimates of the data used to determine the model and identify suspected outliers. The requirement for determination of a bias parameter from a collocation solution is (*Tscherning, 1994*) that reduced gravity anomaly and GPS-Levelling data with EGM96 and RTM effects removed are used as observations along with the specification of different parameters (here just one i.e. N_o) which must be determined from data.

The global geocentric datum is defined by adopting the GRS80/WGS84 gravity potential on the geoid as $W_0 = 62\,636\,856.88\ \mathrm{m^2/s^2}$ and a gravity-mass constant of $GM = 3.986\,004\,415 \times 10^{14}\ \mathrm{m^3/s^2}$ (*Burša, 1995*). A comparison of regional height anomalies ζ, calculated from EGM96 and RTM reduced free air anomalies with 19 GPS-Levelling (local) benchmarks computed as the difference between the ellipsoidal height h, the orthometric height H and corresponding geoid-quasigeoid separation term, shows an average offset of 68.9 cm with a total range of −23 cm to 125 cm. This represents an excessive error in comparison to the use of a geoid model based on EGM96 in the region for accurate surveying. However, since these measured orthometric heights (H) at the GPS-Levelling points are referred to a local tidal datum, these comparisons may be biased due to the effect of localized permanent dynamic oceanic topography and possible offset between global and local vertical datums.

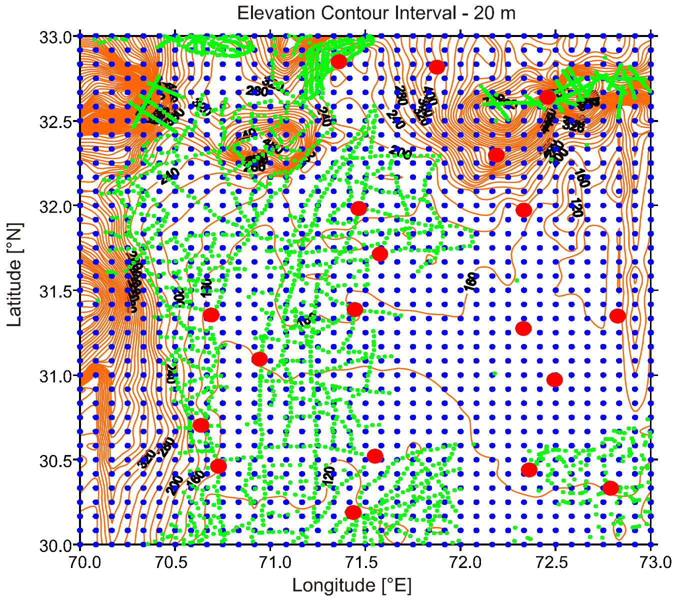

Fig. 1. Distribution of gravity, elevation and GPS-Levelling data in the study area. Green dots are observed gravity data points and red dots are GPS-Levelling control points, whereas red lines are elevation contours.

The data for the numerical investigation of this study was obtained from the Directorate General of Petroleum Concessions (DGPC) gravity database for Pakistan. Observed gravity data in the study area was available with varying distribution, mostly along the roads and valleys (Figs. 1 and 2). A total of 2 352 observed gravity data points were accepted in addition to the corresponding elevation data after careful outlier detection. The measured elevations are result of precise levelling measurements and are the strict form of orthometric heights collected as a part of gravity surveys.

2. COLLOCATION WITH PARAMETERS: A BRIEF NOTE

A linear collocation model is described by e.g. *Moritz (1980)*

$$\boldsymbol{Y} = \boldsymbol{B}T \,, \tag{1}$$

where $\boldsymbol{Y}$ is a vector of observations, $\boldsymbol{B}$ is a vector of linear functionals L_i associated with data $\boldsymbol{Y}$ and T the anomalous potential, $T = W - U$. W is the gravity potential and U the normal potential of WGS84 ellipsoid. The determination of anomalous potential T is normally made from a set of linear functionals e.g. free air anomalies or height anomalies. A generalization of this linear model may be expressed as

$$y_i = L_i\left(T\right) + e_i + \boldsymbol{A}_i^T \boldsymbol{X} \,, \tag{2}$$

Fig. 2. Study area bounded by the rectangle. Two tide gauge stations at Karachi and Gawadar are main tide stations used for GPS-Levelling network with Karachi being the reference Tide station.

where L_i is any linear functional of the anomalous potential, e_i is the error, $\boldsymbol{A}_i$ a vector of partial derivatives and $\boldsymbol{X}$ the vector of parameters (*Moritz, 1980*). If the unknown parameter is equal to N_o then $\boldsymbol{A}_i = 1$ for all values of i which are associated with height anomalies and 0.3 mgal/m for gravity anomalies if their associated altitude is derived from the local height datum.

If no parameters are present we obtain a least square collocation approximation to T by requiring the square of the norm plus the variance of the noise to be minimized. When parameters are present then there is an additional requirement that the square of the norm of the parameter vector is to be minimized.

The covariance between two quantities will be denoted by

$$\operatorname{cov}\left(L_i, L_j\right) = C_{ij} \,. \tag{3}$$

If one of the functionals is the evaluation of T in a point P, then covariance is written as

$$\operatorname{cov}\left(P, L_i\right) = C_{Pi} \,. \tag{4}$$

The square norm (or variance) of a functional L is denoted as

$$\sigma_L^2 = \operatorname{cov}\left(L, L\right) \tag{5}$$

and the variance-covariance of the noise as σ_{ij} and $\bar{\mathbf{C}} = \left\{ \operatorname{cov}\left(L_i, L_j\right) + \sigma_{ij} \right\}$ a $n \times n$ matrix, where n is the number of observations. Then estimates of T and of the parameters X (here just one N_o) are obtained as

$$T(P) = \left\{ C_{pi} \right\}^T \bar{\mathbf{C}}^{-1} \left(\boldsymbol{y} - \mathbf{A}^T \mathbf{X} \right) \tag{6}$$

and

$$\mathbf{X} = \left(\mathbf{A}^T \bar{\mathbf{C}}^{-1} \mathbf{A} + \mathbf{W} \right)^{-1} \left(\mathbf{A}^T \bar{\mathbf{C}}^{-1} \mathbf{Y} \right), \tag{7}$$

where **W** is the a-priori weight matrix for the parameters (generally the zero matrix). With

$$\mathbf{H} = \left\{ \operatorname{cov}\left(L_i, L_j \right) \right\}^T \bar{\mathbf{C}}^{-1} \tag{8}$$

the associated error estimates of an estimated quantity $L\left(\tilde{T}\right)$ are

$$m_L^2 = \sigma_L^2 - \mathbf{H} \left\{ \operatorname{cov}\left(L, L_i \right) \right\} + \mathbf{HA} m_x^2 \left(\mathbf{HA} \right)^T \tag{9}$$

and the mean square error of the parameter vector

$$m_x^2 = \left(\mathbf{A}^T \bar{\mathbf{C}}^{-1} \mathbf{A} + \mathbf{W} \right)^{-1} . \tag{10}$$

It should be kept in mind, that the determination of a datum-shift parameter requires data which covers a large area. However, if the area is not large, this will be reflected in large error estimates m_x^2. The properties of the general solution expressed by Eqs.(6) and (7) can be summarised as the following (*Moritz, 1980*):

- The result is independent of the number of the signal quantities to be estimated.
- Both observed and estimated quantities can be heterogeneous, provided that all required covariances are known.
- The method is invariant with respect to linear transformation of the data or of the results.
- The solution is optimal in the sense that it gives most accurate results obtainable on the basis of the given data.

3. DATA PROCESSING AND ANALYSIS OF RESULTS

The motivation for the use of collocation for gravity/geoid modeling is firstly that it may use a random distribution of gravity data and secondly, the varying error distribution due to data being collected and analysed at different times. LSC may also take into account data located at different altitudes through the use of a spatial covariance function. In the present study the LSC implementation with a rotationally invariant covariance functions was used (see Eq.(11)). The applied covariance model implies that the associated approximation to the anomalous gravity potential is harmonic down to the so-called Bjerhammar sphere, with radius R_B smaller than the mean Earth radius.

The data for the use in covariance function estimation and the onward collocation step is required to be smooth with small variance, cf. Eq.(9) and have a good statistical distribution in order to properly interpret the error-estimates. It is optimal, but not a condition for using LSC, that the mean of Δg is near to zero. To achieve the goal of smoothing and a small mean value, the ”Remove-Restore” technique (*Forsberg and Tscherning, 1981*) has been used.

Purely gravimetric calculations of the geoid heights are hampered by long wavelength systematic data errors normally due to the non-availability of data outside the considered region and by the inhomogeneous spatial resolution and errors of the gravity data. For this purpose a global geopotential models such as EGM96 (*Lemoine et al., 1997*) generally provides the long wavelength part of the gravity field. The local gravity data together with a high resolution digital elevation models provides the shorter wavelengths. This leads to a combination solution which can be applicable to a limited region. The terrain effects are calculated using the pure residual terrain modeling (RTM) technique (*Forsberg and Tscherning, 1997*). In this reduction technique topographic irregularities relative to a smooth mean elevation surface with resolution comparable to the resolution of the used EGM are computationally removed. For a mean elevation surface a reference elevation grid was generated from 1 × 1 km SRTM30 (*USGS, 2008*) data using the GRAVSOFT (*Tscherning et al., 1992*; *Tscherning, 1994*) programs SELECT and TCGRID. The generally recommended smoothing (*Forsberg and Tscherning, 1997*) was achieved by preparing a 2′ grid from the 30″ elevation grid which was again passed by a filter with mean factors of 3, 3 cells (to make 6′ grid) and average factors of 3, 3 (to make a 18′ grid) to obtain the reference height grid. In this way a reference grid of resolution of about 36 km was prepared for optimum RTM reduction with near zero mean value. This also corresponds to the resolution of the reference model EGM96. The parameters of reduced data are shown in Table. 1.

It can be seen from Table 1 that reduced free-air anomaly data has less gravity variance and thus results in the less residual geoid error variance as well.

Table. 1. Observed, RTM and residual gravity data used for geoid determination. FAA - free-air anomalies.

Physical Parameter	Observed Gravity Data			
	Min.	Max.	Mean	Std.Dev.
Elevation [m]	106.2	1502	265.78	206.97
Observed FAA at topography [mGal]	–167.54	47.948	–69.401	44.77
EGM96[2] FAA at topography [mGal]	–122.79	14.179	–68.38	37.19
Res. EGM FAA [mGal]	–57.777	88.099	–1.057	16.547
RTM effect on gravity [mGal]	–27.64	58.14	–1.05	8.94
RTM Res. FAA [mGal]	–51.93	50.02	0.00	12.82

3.1. Covariance Function Estimation

A wise determination of the covariance function is the key to the success for application of least square collocation. An often used approach is to compute empirical covariances. Subsequently, these values might be fitted to a pre-selected model covariance functions e.g. to the Tscherning-Rapp model (*Tscherning and Rapp, 1974*). The residual geoid undulations were determined by LSC, where the required auto and cross-covariance functions were computed by covariance propagation from the analytically modelled local covariance function represented as follows:

$$\operatorname{cov}\left(T_P, T_Q\right) = a \sum_{i=2}^{N} \left[\frac{a^2}{r_P r_Q}\right]^{i+1} \sigma_i^2 P_i\left(\cos\psi\right) + \sum_{i=N+1}^{\infty} \left[\frac{R_B^2}{r_P r_Q}\right]^{i+1} \frac{A}{(i-1)(i-2)(i+4)} P_i\left(\cos\psi\right), \tag{11}$$

where, P and Q are two points having a spherical distance ψ, and r_P, r_Q are the distances of two points from the origin, R_B is the radius of Bjerhammar sphere and σ_i is the error-degree-variance. The covariance parameters a (scale parameter), A and the Bjerhammar radius R_B are determined using a iterative non-linear adjustment, see e.g. *Knudsen (1987)*.

First the residual free air anomaly at the Earth surface was used as input to the program EMPCOV for empirical covariance function estimation (Fig. 3). After trials of different spacing for empirical covariance estimation in EMPCOV, an optimum spherical distance has been considered to be 2 arcmin for better correlation based on the distribution of data. It was found to be a quite good compromise for the covariance function of the

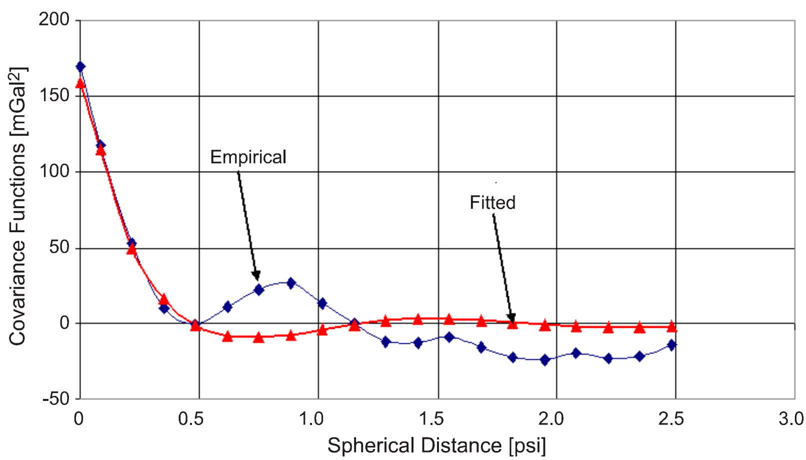

Fig. 3. Empirical and fitted covariance functions estimation using EGM and RTM reduced free-air anomalies.

entire area as it is evident from Fig. 3 and the prediction results of Section 3. The estimated covariance function is showing least correlation at distances greater than 2.5°, due to least effect of data from such distance. Therefore, covariance function was truncated beyond spherical distance i.e. 2.5° due to its least effect on prediction results.

The estimated covariance was fitted to model the covariance function i.e. basically to the *Tscherning and Rapp (1974)* model in Eq.(11). It was fitted to the empirical values using parameters which were determined by the least-squares method.

The value of $N = 220$ for the covariance model, Eq.(11) was found to agree better with the empirical data, than using the maximal degree of 360 of EGM96 for fitting the model covariance function. This is due to the absence of data in the higher frequency part of EGM96 in the area of Pakistan. The predicted geoid should have a maximum variance of 0.101 m^2 (standard deviation of 0.32 m) corresponding to the maximum variance of the free-air anomalies equal to 183.11 $mgal^2$. The covariance parameters for the reduced data are shown in Table. 2.

The estimates to be derived from the residual observed gravity data, the global gravity model and the terrain were determined using the GRAVSOFT (*Tscherning et al., 1994*) programs GEOCOL, EMPCOV, COVFIT, COVFFT, TCGRID, and TC respectively. The error associated with gravity anomalies was assumed to be 1.0 mgal. The related geometrical parameters of the height anomalies are given in Table. 3.

3.2. Bias Parameter Estimation and Adjustment

The resulting residual height anomalies (Fig. 4) as determined during the restore step, in principle refer to the global reference system i.e. global centre of mass, and the average zero-anomalous potential surface. An offset or height bias exist between the gravimetric and the GPS-Levelling height anomalies due to local mean dynamic ocean topography as described above and due to other errors in ellipsoidal height measurement or levelling. It must be determined for the consistency and applicability of the computed quasi-geoid in an area.

Table. 2. The fitted covariance function parameters for the observed data, R_E is the mean radius of the Earth.

Description of Dataset	$R_E - R_B$ [km]	a	Variance [$mgal^2$]
Observed Gravity Data	−1.661	0.6966	183.13

Table 3. Geometrical parameters (in meters) of the quasi-geoid from the observed data.

Parameters of the Estimated Quasi-Geoidal Heights	Gravimetric Height Anomalies from Observed Data [m]			
	Min.	Max.	Mean	Std.Dev.
RTM effect on height anomaly	−0.194	0.294	−0.0196	0.0727
RTM residual Height Anomaly	−0.821	0.832	−0.0391	0.327
Total Height anomaly	−49.933	−37.997	−46.326	2.11
Total geoid	−50.323	−37.976	−46.385	2.10

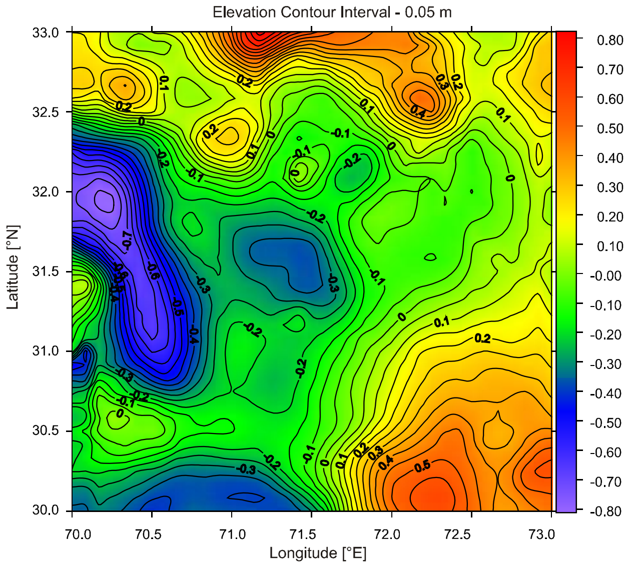

Fig. 4. Residual height anomalies determined from observed data.

In order to compute and adjust the height bias, least square collocation (Eqs.(6) and (7)) has been used. The gravimetric height anomalies were computed using surface gravity anomalies as per Molodensky's definition (*Heiskanen and Moritz, 1967*). The GPS-Levelling height anomalies were determined using geoid-quasigeoid separation term already computed for whole Pakistan area (*Sadiq et al., 2009*) based on theoretical formulations of *Sjöberg (1995)* and *Rapp (1997)*

$$N_P - \xi_P = \frac{\Delta g_B}{\bar{\gamma}} H + \frac{H^2}{2\bar{\gamma}} \frac{\partial \Delta g^F}{\partial H} + \frac{\partial \xi}{\partial r} H + \frac{\partial \xi}{\partial \gamma} \frac{\partial \gamma}{\partial h} H \ , \tag{13}$$

where N_P and ζ_P are the geoid and height anomaly at the Earth surface, ζ is height anomaly at the ellipsoidal surface, Δg_B and Δg^F are the Bouguer and free air anomalies, H is the orthometric height and $\bar{\gamma}$ is the average theoretical normal gravity along the ellipsoidal normal between the surface of the geocentric reference ellipsoid and the telluroid. The first two terms are based on observed gravity data and second two terms are relying on the gradient terms of height anomaly at the ellipsoidal surface.

These observations i.e. gravimetric and GPS-Levelling height anomalies were tied to the disturbing potential T and the vector of X parameters (only one bias in this case) by Eq.(2) in the least square collocation solution. The error in the measured quantities e.g. gravity anomalies was kept to 1.0 mgal and the corresponding GPS-Levelling height anomaly error fixed to 0.01 m.

A comparison with a pure gravimetric solution gave a 68.9 cm mean residual with 35.6 cm standard deviation w.r.t. GPS-Levelling data (Table. 5). This leads to a ~ + 69 cm

Table. 4. GPS-Levelling height anomalies before and after N_o adjustment.

Physical Parameter	Before N_o Adjustment [m]				After N_o Adjustment [m]			
	Min.	Max.	Mean	Std.Dev.	Min.	Max.	Mean	Std.Dev.
Height Anomaly	–49.951	–43.727	–46.539	1.773	–49.085	–43.304	–45.875	1.651

a-priori estimate for the average permanent ocean dynamic topography (PODT) in the area without considering the geographical distribution of the data. The computed bias parameter N_o from both gravity and GPS-Levelling data is estimated to be 70 cm with error estimate of 7 cm. This estimate is as mentioned above possibly biased by local levelling errors, and local mean sea level definitions. However, it is in fair agreement with the recent studies regarding the global mean sea level bias e.g. see *Andersen et al. (2006)*. A value of N_o was found to be 0.69 m with standard deviation of 0.05 m between mean sea surface of the DNSC06MSS and the PGM04 (*Pavlis et al., 2008*) geoid model in the area bounded by 18°−25°N and 63°−72°E in the Indian ocean. If the systematic errors and sea-level datum definition problems in the area were reduced, the *RMS* of 7 cm could be expected to be reduced significantly.

After the simultaneous use of reduced gravity anomalies and reduced GPS-Levelling data in bias adjustment, the difference to GPS-Levelling height anomalies range from –0.084 m to 0.033 m and has about zero value for mean of difference. These values however, should be interpreted as the improvement of the height anomaly differences. This bias was adjusted to the total gravimetric height anomalies through Eq.(2) to make them consistent for local application in GPS surveying. The results of the bias adjustment are shown in Table. 4, a fact which is evident from the difference of mean values (66.4 cm as absolute mean difference) before and after the bias adjustment. The residual of ~ 3.6 cm seems to remain unadjusted in the final GPS-Levelling height anomalies which lie within the 7cm error value determined by least square collocation as mentioned above.

It could be suggested based on this study that N_o adjustment could be considered as a good option for fitting of regional geoid or quasi-geoid with GPS-Levelling data in addition to having the capability of error estimation. There are other methods in this regard e.g. draping of the gravimetric geoid with GPS-Levelling using estimating geoid differences at grid nodes based on linear covariance function usually used in FFT based method and datum transformation based on parametric surface modeling (*Fotopoulos, 2005*). Some of these methods require data to be on regular grids and need interpolation and extrapolation in some cases as well. Least squares collocation however, has the flexibility with respect to the requirement of the data distribution (no gridding required, gravity and GPS-Levelling may be mixed) and give the results with error estimates as per Eq.(9).

3.3. Error Estimation of the Processed Data

It is important to know the error estimates of the data as well as its result. Using least squares collocation, we may compute the estimates of the data used as input as well as the resulting output error-estimates. These a-posteriori error-estimates will generally be smaller than the a-priori ("input") error estimates. The estimate of the error of the difference will then be the Gaussian sum of the a-priori error and the a-posteriori error. LSC has the property that for error-free data it will give zero error estimates. The typical character of collocation's error estimation dependence on data distribution and can be seen explicitly from Figs. 5 and 6, respectively. The covariance function parameters determined (Table. 2) were used when predicting the residual gravity anomalies in the area surrounded by 30°–33°N and 70°–73°E with standard common gravity data error of 1.0 mgal. The error of the predicted residual gravity anomalies has a range of 0.3 to 1.3 mgal. This limit also includes some large values in the central eastern and northern parts which are mainly due to the absence of gravity observations.

The error estimates of the predicted height anomalies, (Fig. 6) from the observed data show that additional data across the borders is required corresponding to a correlation length of the data ~ 25–30 km to limit the estimated error to less than 6.5–7.0 cm. The error distribution of the resulting height anomalies shows similar characteristics as the observed gravity data distribution and has less error in the area with uniform distribution. The LSC outputs the achievable accuracy which ranges from 0.04 m to 0.21 m (Fig. 6). There is missing data in the central eastern to north eastern and then south western parts as shown in Fig. 1, and this results in higher error estimates for computed height anomalies.

The computed geoid (or quasi-geoid) should have a check for accuracy. The absolute accuracy is checked by comparison to the GPS-Levelling bench marks within the study area. This is a reliable method to assess the accuracy of the calculated gravimetric geoid or the height anomalies. In the present study absolute test of accuracy was performed using the bias parameter N_0 adjustment as computed above using combined residual gravity and GPS-Levelling height anomalies.

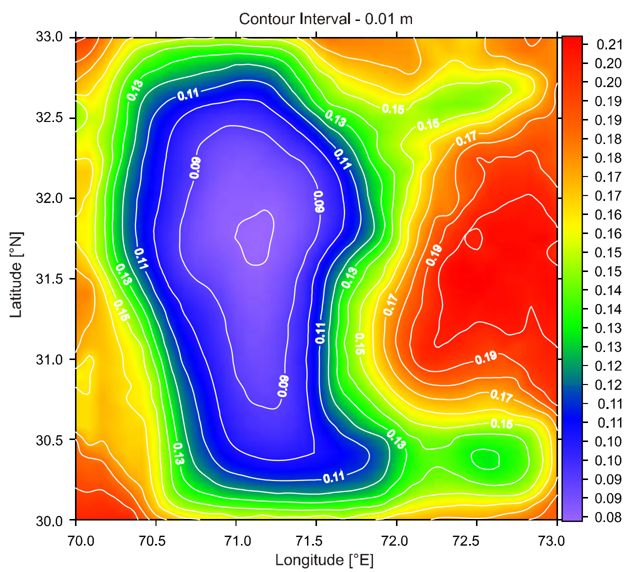

Fig. 5. Predicted errors of residual free air anomalies using collocation.

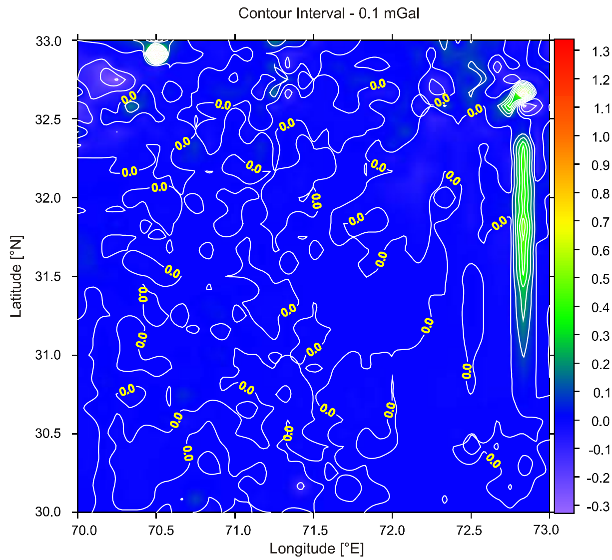

Fig. 6. Error estimates of the predicted height anomalies from observed data.

Table. 5. Statistics of the differences between the gravimetric quasi-geoid undulation and GPS-Levelling at 19 control points (in meters) before and after making the N_O adjustment.

Physical Parameter	Height Anomaly Difference of GPS-Levelling and Gravity Data before N_O Adjustment	Height Anomaly Difference of GPS-Levelling and Gravity Data after N_O Adjustment	Difference of Geoid of GPS-Levelling and Gravity after N_O Adjustment
Min.	–0.123	–0.084	–0.079
Max.	1.188	0.033	0.041
Mean	0.689	–0.0088	0.000022
St.Dev.	0.356	0.026	0.0228

The calibration of the computed gravimetric height anomalies-geoid was performed afterwards by employing N_O adjustment and results are shown in Table 5. The results of the adjustment show a good agreement of GPS-Levelling and gravimetric height anomalies-geoid in the study area.

4. CONCLUSION AND RECOMMENDATION

An attempt has been made to estimate the height bias parameter N_O using gravity data reduced to a geocentric datum by employing the least squares collocation method. The Remove-Restore technique was used to obtain a best possible covariance model with a low variance of the residual gravity anomalies. The subtraction of EGM96 gave the expected results of decreasing the variance and the mean value significantly. The mean value was again reduced to near zero after RTM reduction while using an optimal topographic grid of 30″, 5′and 18′ grid intervals and having resolution suitable for the maximum possible topographic reduction. The constant bias N_o between the global gravimetric datum and the local mean sea level has been estimated to be 70 cm with an error estimate of 7 cm. The estimates of the bias N_o of the local vertical mean sea levelling datum set at Karachi and a gravimetrically determined global datum are consistent with recent studies on global sea level in the Indian Ocean. The difference of the GPS-Levelling data with respect to a pure gravimetric geoid and the one determined by combining gravity and GPS-Levelling in a least square collocation solution are reduced from 35 cm to 2.3 cm in standard deviation with minimum and maximum range from −7.9 cm to −4.1 cm after N_o correction. Results of the study also suggest that N_o computation and adjustment could be a reliable option for fitting the gravimetric geoids with GPS-Levelling data. The bias parameter N_o and the associated (quasi)-geoid makes it suitable for use by the local surveying community by combining gravity data and height anomalies. In a similar manner, bias parameter estimation may be helpful for crustal movement and plate tectonic study by using a GPS array. It could also be used to detect subsequently the gross errors of gravity data and GPS-Levelling.

Though, an estimates of bias parameter has been calculated, which matches well to global values, however, a large area is required along with error free or at least data with known a-priori error estimates. The validity of the a-priori error estimates as well as the occurrence of outliers may be investigated by comparing the a-posteriori determined differences between observed and predicted values with the error estimate of the differences obtained using least squares collocation. An additional study is required to identify the height errors, datum biases and geoid errors. This will be possible only if an accurate local model of permanent ocean dynamic topography (PODT) with the precise geoid model of the whole region is available. Without PODT model, an unknown bias (or biases) could be present and it would be difficult to identify whether its source is oceanographic, due to crustal movements or measurement errors.

Acknowledgements: Higher Education Commission of Pakistan is highly acknowledged for sponsoring this study under the indigenous Ph.D. program at the Department of Earth Sciences Quaid-i-Azam University (QAU) Islamabad, Pakistan. The Directorate General of Petroleum Concessions (DGPC), Ministry of Petroleum and Natural Resources, Islamabad, Pakistan and Survey of Pakistan are also acknowledged for providing the Gravity, Elevation and GPS-Levelling data of the study area to the Department of Earth Sciences, QAU Islamabad, Pakistan. The authors also appreciate the valuable suggestions and comments from the associate editor and three anonymous reviewers.

References

Andersen O.B., Vest A.L. and Knudsen P., 2006. The KMS04 multi-mission mean sea surface. In: Knudsen P., Johannessen J., Gruber T., Stammer S. and van Dam T. (Eds.), *GOCINA: Improving Modelling of Ocean Transport and Climate Prediction in the North Atlantic Region Using GOCE Gravimetry. Cahiers du Centre European de Geodynamique et de Seimologie*, **25**, European Center for Geodynamics and Seismology, ISBN: 2-9599804-2-5 (http://gocinascience.spacecenter.dk/publications/4_1_kmss04-lux.pdf).

Burša M., 1995. Primary and derived parameters of common relevance of astronomy, geodesy, and geodynamics. *Earth Moon Planets*, **69**, 51−63

Forsberg R. and Tscherning C.C., 1981. The use of height data in gravity field approximation by collocation. *J. Geophys. Res.*, **86(B9)**, 7843−7854.

Forsberg R. and Tscherning C.C., 1997. Topographic effects in gravity field modelling or BVP. In: Sansó F. and Rummel R. (Ed.): *Geodetic Boundary Value Problems in View of the Centimeter Geoid*. *Lecture Notes in Earth Sciences*, **65**, Springer-Verlag, Berlin, Heidelberg, 241−272.

Fotopoulus G., 2005. Calibration of geoid error models via a combined adjustment of ellipsoidal, orthometric and gravimetric geoid height data. *J. Geodesy*, **79**, 111−123

Heiskanen W.A. and Moritz H., 1967. *Physical Geodesy*. W.H. Freeman, San Francisco.

Jiang Z. and Duquenne H., 1996. On the combined adjustment of a gravimetrically determined geoid and GPS levelling stations. *J. Geodesy*, **70**, 505−514.

Knudsen P., 1987. Estimation and modelling of the local empirical covariance function using gravity and satellite altimeter data. *Bulletin Geodesique*, **61**, 145−160.

Lemoine F.G., Smith D.E., Kunz L., Smith R., Pavlis E.C., Pavlis N.K., Klosko S.M., Chinn D.S., Torrence M.H., Williamson R.G., Cox C.M., Rachlin K.E., Wang Y.M., Kenyon S.C., Salman R., Trimmer R., Rapp R.H. and Nerem R.S., 1997. The development of the NASA, GSFC and NIMA joint geopotential model. In: Segawa J., Fujimoto H. and Okubo S. (Eds), *Gravity, Geoid, and Marine Geodesy*. International Association of Geodesy Symposia, **117**, Springer-Verlag, Berlin, Heidelberg, 461−470.

Lisitzin E., 1974. *Sea Level Changes*. Elsevier, Amsterdam, The Netherlands.

Moritz H., 1980. *Advanced Physical Geodesy*. H. Wichmann Verlag, Karlsruhe, Germany.

Pavlis N.K., Holmes S.A., Kenyon S.C. and Factor J.K., 2008. An Earth Gravitational Model to Degree 2160, EGM2008. http://www.dgfi.badw.de/typo3_mt/fileadmin/2kolloquium_muc/2008-10-08/Bosch/EGM2008.pdf

Pope A.J., 1976. *The Statistics of Residuals and the Detection of Outliers*. NOAA Technical Report NOS 652 NGS1 30, NOS/NOAA, Rockville, MD.

Rapp R.H., 1997. Use of potential coefficient models for geoid undulation determinations using a spherical harmonic representation of the height anomaly/geoid undulation difference. *J. Geodesy*, **71**, 282–289.

Sadiq M., Ahmad Z. and Akhter G., 2009. A study on the evaluation of the geoid-quasigeoid separation term over Pakistan with solution of 1st and 2nd order height terms. *Earth Planet and Space*, (in print).

Sjöberg L.E., 1995. On the quasigeoid to geoid separation. *Manuscripta Geodaetica*, **20**, 182−192.

Torge W., 2001. *Geodesy, 3rd Edition*. Walter de Gruyter, Berlin, New York.

Tscherning C.C., 1994. Geoid determination by least-squares collocation using GRAVSOFT. *Lecture Notes, 1st International School on the Determination and Use of the Geoid*, Milano, Oct., 1994, pp. 135−164. International Geoid Service, Milano, Italy.

Tscherning C.C. and Rapp R.H., 1974. *Closed Covariance Expressions for Gravity Anomalies, Geoid Undulations, and Deflections of the Vertical implied by Anomaly Degree Variance Models*. Report No. 208, Department of Geodetic Science, The Ohio State University, Columbus.

Tscherning, C.C., Forsberg R. and Knudsen P., 1992. The GRAVSOFT package for geoid determination. Proceedings of the 1st Continental Workshop on the Geoid in Europe, Prague. Research Institute of Geodesy, Topography and Cartography, Prague, Czech Republic (http://www.gfy.ku.dk/~cct/gravsoft.txt).

USGS, 2008. Digital Elevation Model DATA Center, SIOUX Fall, SD 57198-0001, http://topex.ucsd.edu/WWW_html/srtm30_plus.html