Generalizing the Harmonic Reduction Procedure in Residual Topographic Modeling

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Abstract

In gravity field modeling measurements are usually located on or above the terrain. However, when using the residual topographic modeling (RTM) method, measurements may end up inside the masses after adding the mean topography. These values do not correspond to values evaluated using a harmonic function. A so-called harmonic correction has been applied to gravity anomalies to solve this problem. However, for height anomalies no correction has been applied. To generalize the correction to e.g. height anomalies we interprete that the vertical gravity gradient inside the masses multiplied by height equals the correction. In principle the procedure is applicable to all gravity field functionals. We have tested this generalization of the procedure which consist in determining equivalent quantities in points Q on the mean surface if this surface is *in free air*. The procedure has as data the reduced values in P inside the masses but considered as being located at the mean surface. Numerical tests with height anomaly data from New Mexico and Norway as control data show that for gravity anomalies the general procedure is better than using the original harmonic correction procedure.

Keywords

Residual topographic modeling • Generalized harmonic reduction • Harmonic correction

1 Introduction

When using RTM for gravity field modeling (Forsberg and Tscherning 1981; Forsberg 1984), measurements may be located inside the masses added using a mean topography. They do therefore not anymore correspond to values evaluated using a harmonic function. For gravity anomaly data a so-called harmonic correction, $-4\pi G\rho \cdot \Delta h$ (Forsberg 1984), is applied, while no correction has been applied on for example height anomalies. G is the gravitational constant, ρ the mass density and Δh the difference between the altitude of

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N. Sneeuw et al. (eds.), *VII Hotine-Marussi Symposium on Mathematical Geodesy*, International Association of Geodesy Symposia 137, DOI 10.1007/978-3-642-22078-4_35,

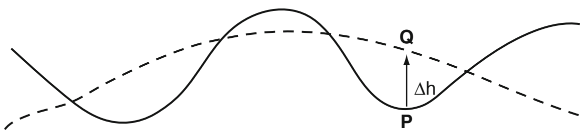

Fig. 35.1 The *dashed curve* shows the mean surface, the *solid curve* the actual topography

the point (Q) on the mean surface and the observation point (P), see Fig. 35.1. However, the correction is equal to the vertical gravity gradient inside the masses multiplied by Δh, if the horizontal gravity gradients are equal to zero. This interpretation makes it possible to generalize the correction to e.g. height anomalies.

We have implemented and tested an alternative procedure which is applicable to all types of gravity field observables. The basic idea is to continue the observations to the reference surface used in RTM, an idea which was already discussed in Elhabiby et al. (2009). However in order to continue the observation the vertical gradient has to be known. In fact the harmonic correction is such a vertical gravity gradient, if the horizontal gradients are considered equal to zero. This is due to Poisson's equation, Torge (2001), eq. (3.31). From this we may compute the harmonic correction for height-anomalies, ζ_c, using that the difference between the anomalous potential (T) in P and Q is equal to the integral of the gravity disturbance inside the masses from P to Q. Hence

$$\zeta_c = \frac{-4\pi G\rho\Delta h^2}{\gamma} \tag{35.1}$$

where γ is normal gravity.

Further development of this procedure requires that we are able to compute the gravity gradients inside the (homogeneous) mass, however, in this paper we have assumed that the horizontal gravity gradients are equal to zero.

We have tested the procedure by predicting height anomalies from gravity anomalies in the New Mexico White Sands test area and in an area in Western Norway with very deep fjords (Dahl and Forsberg 1999).

2 Generalized Harmonic Correction

The gravity potential of the Earth, W, may be split in two parts, $W = U + T$. Here U is an Earth Gravity Model (EGM) which includes the centrifugal potential, so that T becomes a harmonic function outside the masses. The EGM includes the contribution from the topography from wavelengths corresponding nearly to the maximal degree and order of the EGM, i.e. 360 for EGM96 (Lemoine et al. 1998) and 2,160 for EGM08 (Pavlis et al. 2008). Consequently point measurements (and sometimes airborne measurements) may be located inside the masses we have artificially placed outside the reference ellipsoid. For gravity anomalies, a so-called harmonic correction was introduced by Forsberg (1984) in order to solve this problem. We will here use the alternative procedure described above which for gravity anomalies turns out to give nearly the same result (when height anomalies are predicted) as when the harmonic correction is applied.

The basic equation is, with P an arbitrary point outside the *real* masses, T the anomalous potential, with subscript *rtm* for the contribution from the residual topography and *res* the remaining part:

$$T(P) = T_{rtm}(P) + T_{res}(P) \tag{35.2}$$

If the point P is inside the *artificial* masses, the generalized method is to use the observation at Q, see Fig. 35.1. For T and all other gravity functionals this means:

$$T(Q) = T_{rtm}(P) + T_{res}(Q) \tag{35.3}$$

where a harmonic correction had been applied in P. The harmonic correction has the function that it moves/continues the quantity from P to Q inside the masses, disregarding horizontal gradients. In order to determine an estimate of $T_{res}(Q)$ we use a Taylor expansion:

$$\tilde{T}_{res}(Q) = T_{res}(P) + \frac{\partial T}{\partial h}|_P \cdot \Delta h \tag{35.4}$$

Where we compute the vertical derivative using Least Squares Collocation (LSC) implemented in the GRAVSOFT (Forsberg and Tscherning 2008) program GEOCOL (Tscherning 1974).

The above equations may be written down for different functionals, L, such as the gravity disturbance δg, the gravity anomaly Δg or the height anomaly ζ. Equation 35.4 rewritten for gravity anomalies

$$\Delta\tilde{g}_{res}(Q) = \Delta g_{fa}(P) - \Delta g_{egm}(P) - \Delta g_{rtm}(P) - T_{zz} \cdot \Delta h, \quad (35.5)$$

and for height anomaly

$$\tilde{\zeta}_{res}(Q) = \zeta(P) - \zeta_{egm}(P) - \zeta_{rtm}(P) - \frac{\partial T}{\partial r} \cdot \Delta h, \quad (35.6)$$

where subscript fa is free-air anomaly, egm is the gravity/height anomaly signal from the EGM96, rtm is the terrain effect including the harmonic correction, T_{zz} is the vertical gravity gradient, $T_{zz} \cdot \Delta h$ and $\frac{\partial T}{\partial r} \cdot \Delta h$ is the first term of the Taylor expansion when moving from P to Q.

Note that data which are outside the mean surface are not at all changed in this procedure.

3 Computational Procedure

The GRAVSOFT program TC, used to compute the RTM effects, has been modified so that it optionally will produce a file where the height data-column in the output data-record contains the mean-surface height of the point Q if the point is inside the masses. This delivers a file with $L(T_{res}(P)) \approx L(T_{res}(Q))$ values associated with the point Q.

The values are then subtracted from the data (from which the effect of a global model already has been removed) and we obtain residual values. The effect of the new harmonic correction of height anomalies is for the Norwegian data seen as a further smoothing of the residual height anomalies from 0.44 to 0.34 m, see Table 35.4. With these data the vertical derivative may be computed using LSC (Moritz 1980).

4 Numerical Tests

We have selected our two test areas, New Mexico and Sognefjord, based on availability of control data, gravity data, and roughness of the topography.

4.1 New Mexico

The first test area is located in New Mexico, USA, covering an area from the Mexico border (El Paso) in south to Albuquerque in north, and stretching about

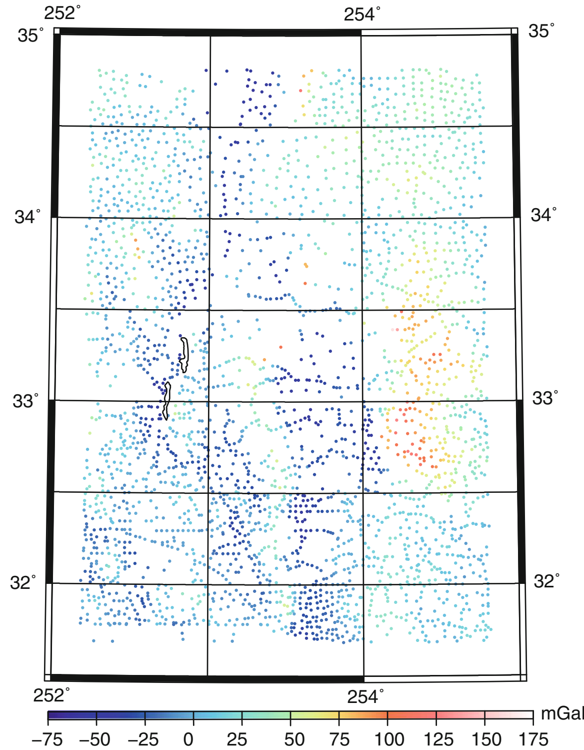

Fig. 35.2 Distribution of the 2,920 free-air gravity anomalies in the New Mexico test area

3.0° east–west. The New Mexico test area is located in an area starting at 1,000 m above sea level and has mountain areas ranging up to 3,350 m.

Within the geographical area 31.7° to 34.8° N, 252.2° to 254.8° E a total of 2,920 free-air gravity anomaly measurements were selected, see Fig. 35.2 for distribution and Table 35.1 for statistics. 1,979 points were inside the masses and their values were changed from line 3 to 4 in Table 35.1, while 941 points were unaffected.

As control data we have selected 20 height anomaly points (GPS/leveling) in New Mexico. Since the US height system uses orthometric height the GPS/leveling points were transformed to height anomaly data using the quasigeoid minus geoid

Table 35.1 Statistics of 2,920 gravity points in New Mexico test area. All values in mGal

	Mean	Std. dev	Min	Max
Δg_{fa}	9.182	30.405	−58.700	162.500
$-\Delta g_{egm}$	−2.932	21.283	−74.792	126.430
$-\Delta g_{rtm}$	0.282	13.153	−41.020	45.739
$-T_{zz} \cdot \Delta h$	0.308	13.035	−40.697	45.739

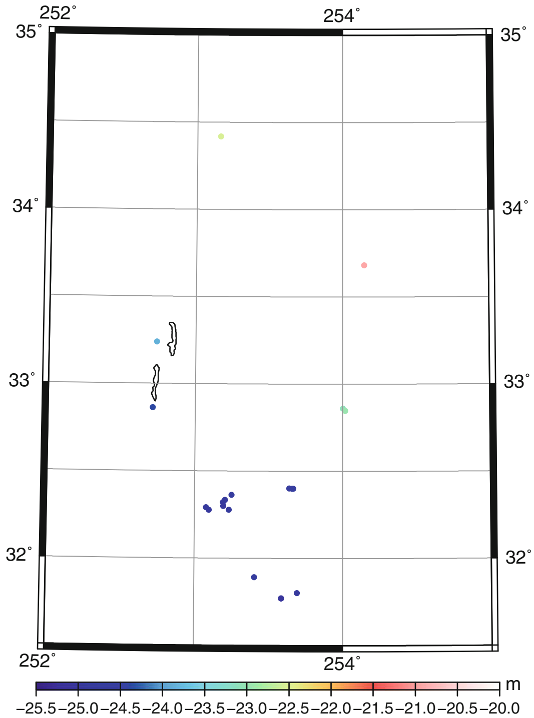

Fig. 35.3 Distribution of the control data in the New Mexico test area

Table 35.2 Statistics of the 20 control points in New Mexico test area. All values in meter

	Mean	Std. dev	Min	Max
ζ_{gps}	−24.268	1.083	−25.059	−20.917
$-\zeta_{egm}$	0.040	0.159	−0.330	0.305
$-\zeta_{hc}$	0.167	0.135	−0.107	0.396
$-\zeta^{Q}_{\frac{\partial T}{\partial r}\cdot\Delta h}$	0.166	0.122	−0.075	0.367
ζ_{gps}-ζ_{egm}-$\zeta_{tc_{wohc}}$	−0.897	0.159	−1.268	−0.632

separation formula (see e.g. Dahl and Forsberg (1999)). The data distribution is illustrated in Fig. 35.3 and statistics are given in Table 35.2. They are distributed in height from 1,130 to 1,662 m above sea level. Eighteen GPS/leveling points were inside the masses and their values where affected by the new procedure, while two points were unaffected.

4.2 Sognefjord

The Sognefjord test area was mainly selected due to its topography. The topography ranges from sea level to 2468.73 m.

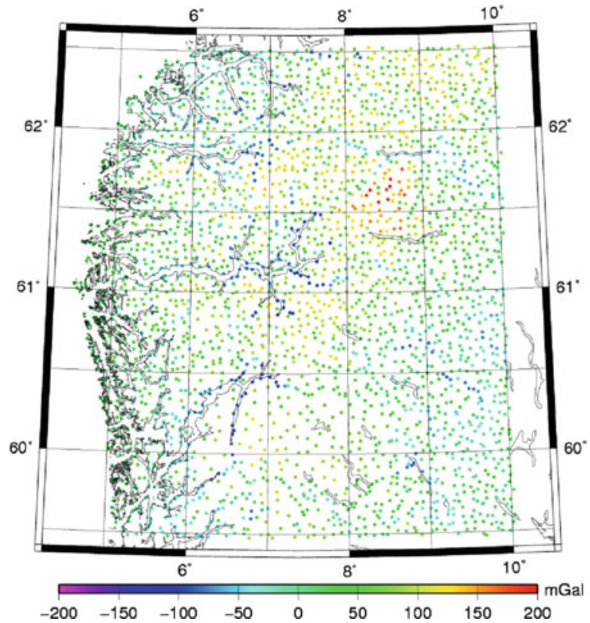

Fig. 35.4 Distribution of the free-air gravity anomalies in the Sognefjord test area

Table 35.3 Statistics of 2,877 gravity points. All values in mGal

	Mean	Std. dev	Min	Max
Δg_{fa}	29.926	53.978	−113.750	205.580
$-\Delta g_{egm}$	−1.176	46.547	−169.219	131.595
$-\Delta g_{rtm}$	1.070	12.060	−36.120	45.000
$-T_{zz}\cdot\Delta h$	1.147	11.823	−35.075	42.250

In the Sognefjord test area we have selected land only gravity data within the region 59.5° to 62.5° N, 4.5° to 10.0° E. A total of 2,877 free-air gravity anomaly measurements were selected, see Fig. 35.4 for distribution and Table 35.3 for statistics. 1,369 points were inside the masses and their values were changed from line 3 to 4 in Table 35.3, while 1,508 points were unaffected, by the generalized procedure.

As control data we have selected 131 height anomaly data (GPS/leveling) around the Sognefjord. The data distribution is illustrated in Fig. 35.5 and statistics are given in Table 35.4. They are distributed in height from 1 to 1424.7 m above sea level. 116 GPS/leveling points were inside the masses and their values were altered by the generalized procedure, while 15 points were unaffected.

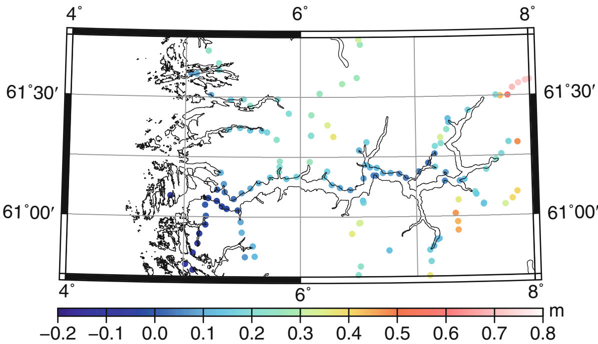

Fig. 35.5 Illustrates the difference between height anomalies estimated from gravity data and control data in Sognefjord. It refers point Q, see line 3 in Table 35.5. This figure also illustrates the distribution of the GPS/leveling data in the Sognefjord test area

Table 35.4 Statistics of the 131 control points in Sognefjord test area. All values in meter

	Mean	Std. dev	Min	Max
ζ_{gps}	46.270	0.416	45.372	47.604
$-\zeta_{egm}$	0.841	0.445	0.019	1.841
$-\zeta_{hc}$	0.796	0.347	0.163	1.370
$-\zeta^{Q}_{\frac{\partial T}{\partial r}\cdot\Delta h}$	0.736	0.296	0.200	1.254
ζ_{gps}-ζ_{egm}-$\zeta_{tc_{wohc}}$	0.857	0.387	0.155	1.441

5 Results and Conclusions

The generalized method gives an improved smoothing of the gravity and height anomaly data compared to the original harmonic correction method. In Tables 35.1 and 35.3 the gravity anomalies in line 4 are slightly smoother than line 3, which represents the original method. The original harmonic correction of the control data is given in line 5 of Tables 35.2 and 35.4 while the other lines indicates the new procedure. As the tables indicate the new method give smoother data than the original harmonic correction method.

A significant improvement is obtained when including harmonic correction for the height anomalies, compare line 1 and 2 in Table 35.5. An improvement of fit of 2.6 cm and 0.4 cm in the Sognefjord and New Mexico area, respectively.

By moving the point from P to Q in the Sognefjord test area improves the fit even further to 15.7 cm, see line 3 in Table 35.5 and Fig. 35.5, while in the New Mexico test area the result is slightly worse than using harmonic correction in point P, but better than not using. This may be due to the relatively low number of control points and their poor distribution or that we already are at the error level of the data set.

The numerical tests with height anomaly data from New Mexico, USA and Sognefjord, Norway as control data shows that for gravity anomalies the general procedure gives better to similar results compared to using the original harmonic correction procedure.

Table 35.5 Statistics of height anomalies estimated from gravity data compared with GPS/leveling control points. Subscript hc is harmonic correction, while $nohc$ is no harmonic correction. All values in meter

Δg	ζ_{GPS}	Mean	Std. dev	Max	Min
Sognefjord					
P_{hc}	P_{nohc}	0.287	0.197	0.702	−0.140
P_{hc}	P_{hc}	0.226	0.171	0.714	−0.120
Q	Q	0.175	0.157	0.712	−0.124
New Mexico					
P_{hc}	P_{nohc}	−0.891	0.052	−0.780	−0.974
P_{hc}	P_{hc}	0.172	0.048	0.276	0.111
Q	Q	0.171	0.050	0.282	0.109

Acknowledgements We would like to thank the reviewers for their comments and suggestions on this manuscript.

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