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Note on LSC and wavelets

by

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Abstract: The use of wavelets have been much advocated for use in gravity field modelling. This note is an attempt to relate the wavelet philosophy - not the presently proposed implementations - to already used and well-founded methods of physical geodesy, especially Least-Squares Collocation (LSC).

1. Introduction. In the lecture notes from the recent school on the use of wavelets in geophysics, several very inspiring contributions were included. One of the ideas behind wavelets is to convert a function on R^n to a function on R^{n+1}. Subsequently the function is folded with a function which is both positive and negative.

In physical geodesy we have a methodology, which aims at creating an anomalous gravity potential function with mean value zero, by subtracting out the normal gravity potential. An even more “positive-negative” function is created when subtracting out a high degree and order reference field like EGM96.

The extra dimension we obtain by forming mean-values. Over equi-angular, equi-area blocks or spherical caps of continuously varying sizes. Obviously in all cases the third dimension is periodic, i.e. means of blocks larger than 180 deg. can not be formed. In the following I will try to put these concepts into a rigorous form.

2. One possible wavelet transforms on the sphere.

Let now A be a subset of n-dimensional real space. Then the wavelet transform W_g maps a function f(A) -> R into another function,

$$W_g(f) = \hat{f} : A \times R_+$$

using a wavelet g(A) -> R, and
mappings t(A) -> A, parameterized by t in A and
$D_a\, g(P) = g_a(P)$, parameterised by a in R_+ .

On the real line, t(u) = t - u, on the sphere t(P) = Q, a rotation around the origin which brings P to Q (using rotations $(t_1 , t_2) = t$).

We define

$$D_a g(t) = \frac{1}{a} g(\frac{t}{a})$$

on the real line and

$$D_a g(P) = \frac{1}{A} \int_a g(Q)\ dQ,$$

integration on a spherical cap with centere in P and radius a.

The transformation on the sphere is then

$$W_g(f) = \frac{1}{4\pi} \int\int_A f(P) \cdot g_a(t(P))\ dP = \sim f(t,a)$$

Now take g = f, then

$$K(P,Q,a) = f(t,a)$$

the auto-covariance function of f when a = 0, Q = t(P).

$$f(t,a) = \sum_{ij} a_{2_{ij}} \cdot \beta_i(a) \cdot Y_{ij}(P)\ Y_{ij}(Q),$$
$$\sum_{ij} a_{ij}^2 < \infty$$
$$\beta_i(a) = \int_a Y_{ij}(P)\ dP$$

The kernel is the reproducing kernel in a space where

$$f(P) = \sum_{ij} Y_{ij}^x(P) \cdot 1,$$
$$Y_{ij}^x = a_{ij} \cdot Y_{ij}, a_{ij} \neq 0$$

2. Conclusion.

We have here shown that the auto-covariance function used in LSC arise as the wavelet-transform, when a function is transformed using itself as the basic wavelet.