Estimating Errors and Error-Correlations In Geodetic Research

My Own Example(s)

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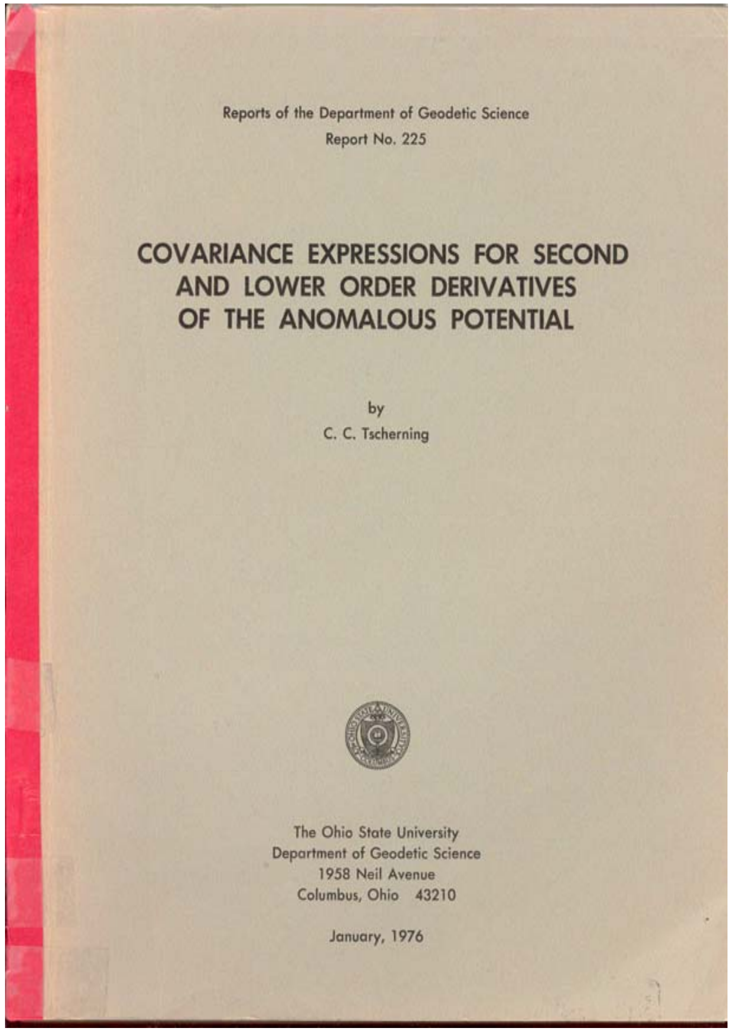
EGU General Asssebly 2008.

Geodesy – errors in equations



Reports of the Department of Geodetic Science
Report No. 225

COVARIANCE EXPRESSIONS FOR SECOND AND LOWER ORDER DERIVATIVES OF THE ANOMALOUS POTENTIAL

by
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January, 1976

Geodesy – errors in equations

Error detected by US Military Organisation

Classified: "For US Nationals only"

$$(71)\quad D_t^4F_1 = s^5[12/(L^6N^2)+6/(L^4N^3)+6/(L^5N^3)$$
$$+8/(L^4N^3)+12/(L^2N^4)+12/(L^3N^4)+6/(L^5N^3)+6/(L^3N^4)$$
$$+6/(L^4N^4)+6/(L^5N^3)+4/(L^3N^3)+4/(L^4N^3)+2/(L^3N^3)$$
$$+6/(LN^4)+6/(L^2N^4)+15/(L^7N)+3/(L^5N^2)+3/(L^6N^2)]$$
$$-\sum_{\ell=4}^{p-1} 1/(\ell+1)s^{\ell+1}D_t^4P_\ell(t)$$

$$= 3s^5[5/(L^7N)+5/(L^6N^2)+6/(L^4N^3)+4/(L^5N^3)+6/(L^2N^4)$$
$$+6/(L^3N^4)+2/(L^4N^4)+2/(L^3N^3)+2/(LN^4)+3/(L^5N^2)]$$
$$-\sum_{\ell=4}^{p-1} 1/(\ell+1)s^{\ell+1}D_t^4P_\ell(t)$$

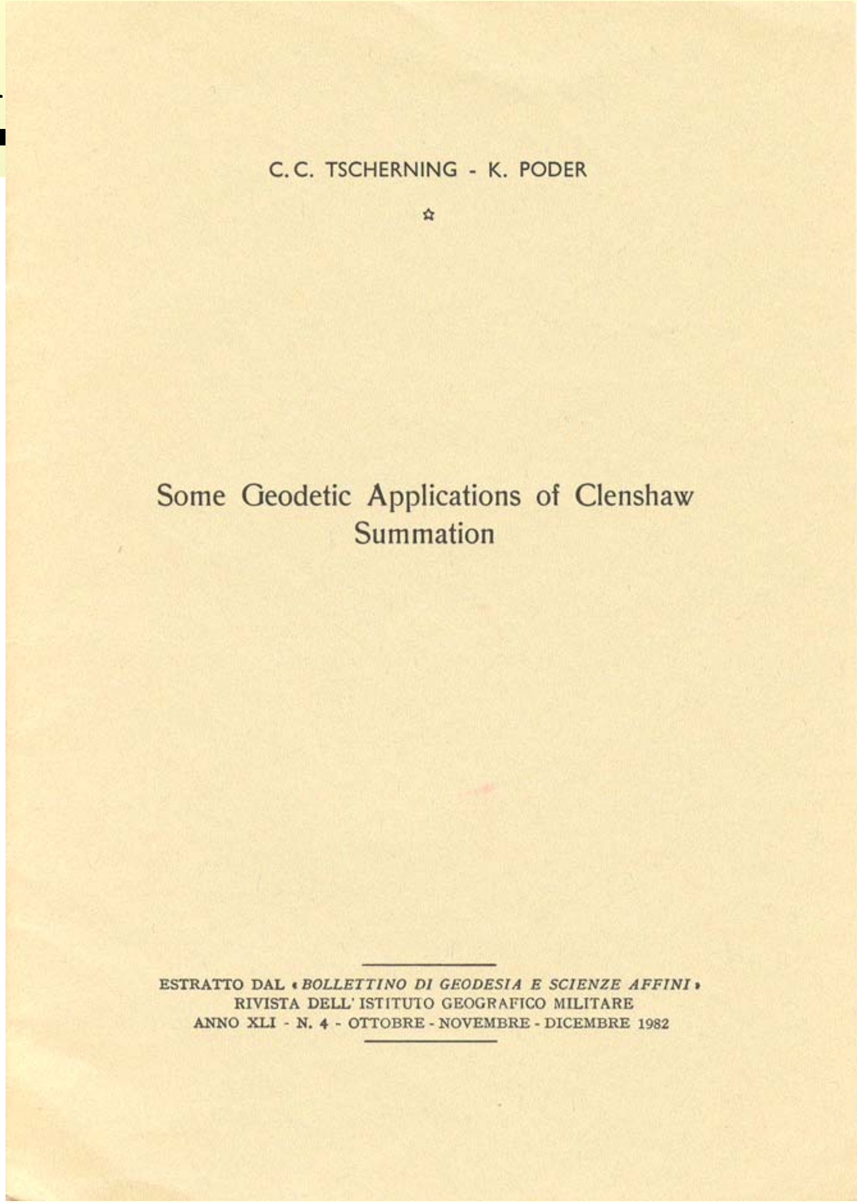
$$=3s^5[(5/L^6+((3+5/L)/L^4+((2+(6+4/L)/L)/L^2$$

L)/N

$$(2\ +(6+(6+2/L)/L)/(NL))/N)/N)/(NL)]$$
$$-\sum_{\ell=4}^{p-1} 1/(\ell+1)s^{\ell+1}D_t^4P_\ell(t).$$

Error in Publication

Contain descriptions of Algorithms and Software.



C. C. TSCHERNING - K. PODER

☆

Some Geodetic Applications of Clenshaw Summation

ESTRATTO DAL « *BOLLETTINO DI GEODESIA E SCIENZE AFFINI* »
RIVISTA DELL' ISTITUTO GEOGRAFICO MILITARE
ANNO XLI - N. 4 - OTTOBRE - NOVEMBRE - DICEMBRE 1982

Error in publication

Last page missing!

No readers have ever complained!

SOME GEODETIC APPLICATIONS OF CLENSHAW SUMMATION 375

```
comment appendix_2               * page 7    17 08 81, 12.20;

  comment length of sa;
  set_utm_const := utm_f;

  comment test of coord system;
  r := reg_lab(2) extract 12;
  if r = 3 <*itm*> or r = 4 <*utm*> then
  begin
    comment utm, long used as boolean;
    utm       := r = 4;
    sa.utm_f := if utm then (-1) else 0;
    comment normalized meridian quadrant
    see Kønig und Weise p.50 (96), p.19 (38b), p.5 (2);
    n        := reg_lab(7)/(2 - reg_lab(7));
    m        := n**2*(1/4 + n**2/64);
    w        := (long reg_lab(6))*(-n - reg_lab(8)
    -                          + m*(1 - reg_lab(8)))/(1 + n);
    sa.Q_n := long reg_lab(6) + w;

    comment central easting and longitude;
    sa.E0 := long reg_lab(9);
    sa.L0 := if utm then (((((long reg_lab(4) - 30)*2 - 1)
    -                     shift 41)//15) shift 4)
    -                 else long reg_lab(4);

    comment check-tol for transformation;
    sa.μ_tol_f := pi/2*1.2'-10*sa.Q_n;  <* 1.2 mm for earth*>
    sa.g_tol_f := conv_t_geo(exteng 40,
    -               false add (1 shift 6 + 6));  <*0.000040 .x*>

    comment coef of trig series;

    for r:=1 step 1 until 4 do
    begin
      comment ell. geo -> sph. geo., KW p186 - 187 (51) - (52);
      sa.bg_f(r):=case r of (
      n   *(-2        + n*(2/3    + n*(4/3         + n*(-82/45)))),
      n**2*(5/3       + n*(-16/15 + n*(-13/9))),
      n**3*(-26/15    + n*34/21),
      n**4*(1237/630));

      comment sph. geo - ell. geo., KW p190 - 191 (61) - (62);
      sa.yb_f(r):=case r of (
      n   *(2         + n*(-2/3   + n*(-2          + n*116/45))),
      n**2*(7/3       + n*(-8/5   + n*(-227/45))),
      n**3*(56/15     + n*(-136)/35),
      n**4*(4279/630));

      comment sph. N, E -> ell. N, E,  KW p196 (69);
      sa.gtu_f(r):=case r of (
      n   *(1/2       + n*(-2/3   + n*(5/16        + n*41/180))),
      n**2*(13/48     + n*(-3/5   + n*557/1440)),
      n**3*(61/240    + n*(-103/140)),
      n**4*(49561/161280));

      comment ell. N, E -> sph. N, E,  KW p194 (65);
      sa.utg_f(r):=case r of (
      n   *(-1/2      + n*(2/3    + n*(-37/96      + n*1/360))),
      n**2*(-1/48     + n*(-1/15  + n*437/1440)),
      n**3*(-17/480 + n*37/840),
      n**4*(-4397/161280));
    end;
  end else  <* error actions, not shown here*>;
end set_utm;
end;
```

Error explained away

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 92, NO. B8, PAGES 8157-8168, JULY 10, 1987

COMPUTATION OF THE GRAVITY VECTOR FROM TORSION BALANCE DATA IN SOUTHERN OHIO

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Abstract. The method of least squares   the ones planned for gradiometry, and the

Error explained away

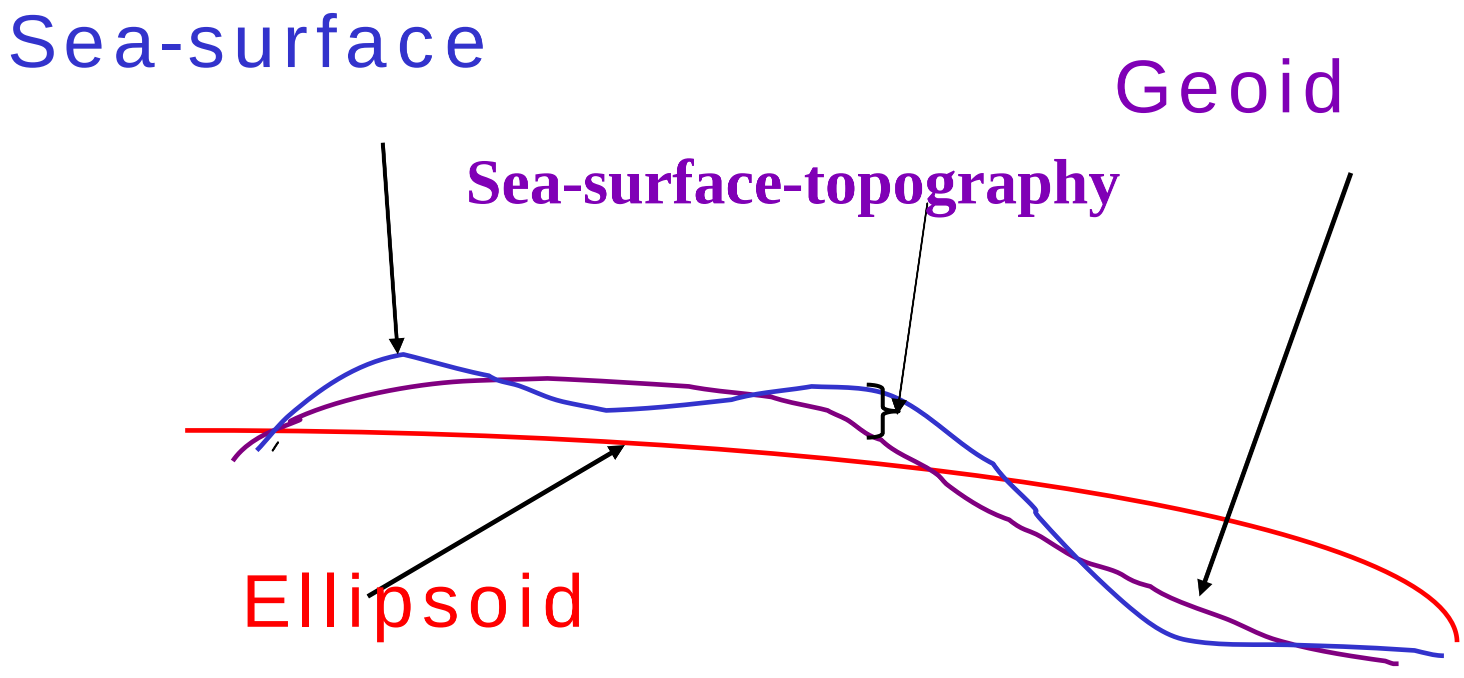
Correct explanation:

Error in FORTRAN-program - faktor $\sqrt{2}$ missing !

of observed minus computed values. This made us go back to the publication Badekas (1967), which contains detailed maps showing the individual data in vectorlike form. This revealed that neighboring values could be rather different, signifying a much larger error level than we had expected. Also the empirically estimated autocovariance functions indicated that a rather large noise was present. This can be seen from Figures 8-11, where the variance is much larger (around 100 EU^2) than the value found by smoothly extrapolating from the first two to three empirical covariance values to the value at spherical distance zero (the variance). A value of 100 EU^2 is pessimistic but not impossible. Also one could argue that the large estimated variance comes from the signal itself. However, this would mean that the power spectrum should not go smoothly to zero like the degree raised to the power of -3 as used in equation (10) but would jump to some higher value after a certain degree.

Changing the noise variance to 100 EU^2, we obtained consistent results. We

Underestimation of sea-surface topography



Sea Surface topography

Caused by

- **Tides**
- **Temperature**
- **Pressure**
- **Salinity**
- **Wind**

Sea-surface topography

If mean values are formed over long periods then I (we) expect the mean sea-surface topography to be very small.

Mean sea surface may then be used as the geoid.

Used in the Baltic and in the Caribian outside the delta of Mississippi to bend the geoid.

Sea surface topography

There are however a nearly constant difference from North to south in the temperature and salinity, which do not disappear when forming mean values.

40 cm height difference from the Bay of Bothnia to Skagerrak.

Sea surface topography

The Geoid should not have been bended !

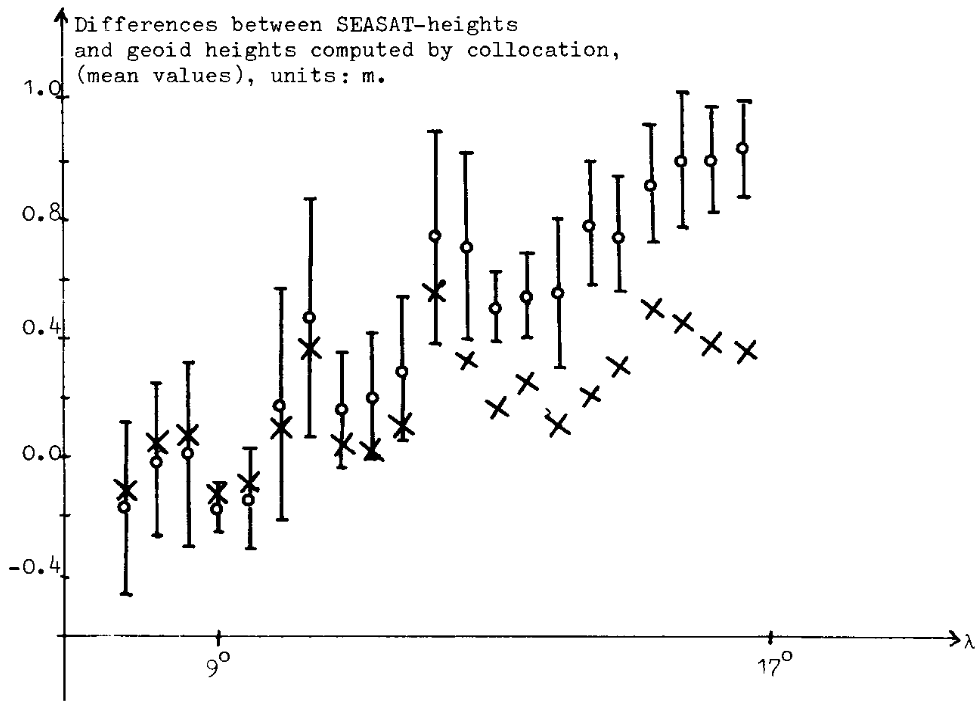
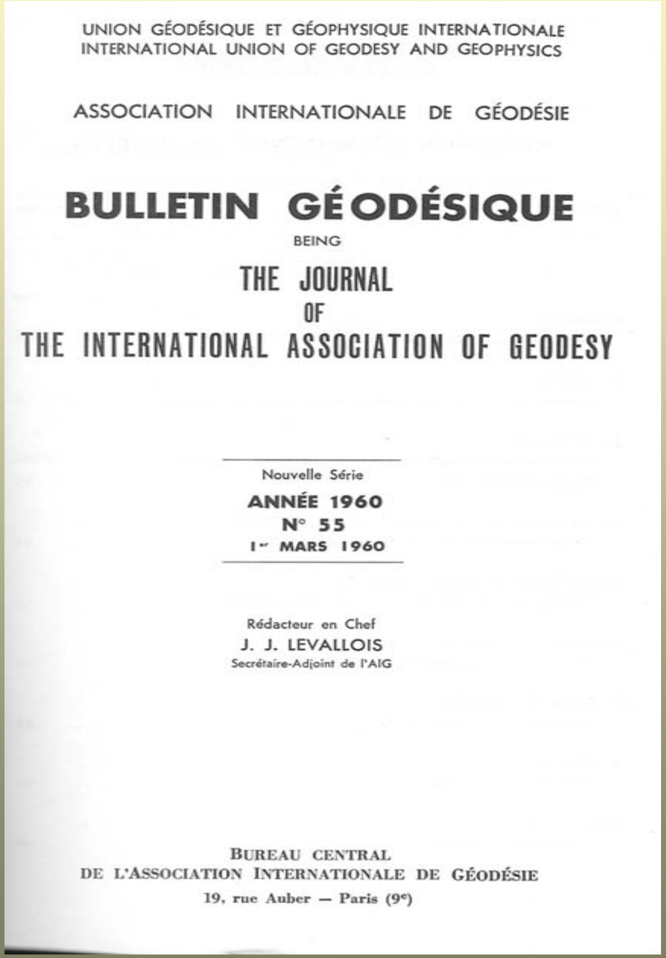


FIGURE 3. Mean values of differences between SEASAT sea surface heights and geoid heights determined using (○) gravity anomalies *and* deflections of the vertical and (×) only gravity anomalies. The differences have been samples in equidistant belts having a longitude extent of 0.25° (at $\varphi = 56°$) bounded by $54.5° \le \varphi \le 57.5°$. The standard deviation for the differences in each class is shown as a bar for the first kind of differences (○). The 0.5″ tilt of the computed astrogravimetric geoid is seen clearly.

Punishment from above

One volume of
Bulletin Géodésique
fell down from
a book-shelt and
hit me in the head.

Opened up at a Tabel



UNION GÉODÉSIQUE ET GÉOPHYSIQUE INTERNATIONALE
INTERNATIONAL UNION OF GEODESY AND GEOPHYSICS

ASSOCIATION INTERNATIONALE DE GÉODÉSIE

BULLETIN GÉODÉSIQUE
BEING
THE JOURNAL
OF
THE INTERNATIONAL ASSOCIATION OF GEODESY

Nouvelle Série
ANNÉE 1960
N° 55
1er MARS 1960

Rédacteur en Chef
J. J. LEVALLOIS
Secrétaire-Adjoint de l'AIG

BUREAU CENTRAL
DE L'ASSOCIATION INTERNATIONALE DE GÉODÉSIE
19, rue Auber — Paris (9e)

Paper by Bowden showed the right numbers

From Skagen to Åland Islands:

30 cm !

THE MEAN SLOPE OF THE SEA SURFACE

Table I

Section	Mean σ_t	$\Delta \sigma_t$	Mean h m	$\Delta \zeta$ cm	ζ cm
A. Lands End - Ushant	26.9				0
		-0.2	60	0.6	
B. Straits of Dover	26.7				0.6
		-1.4	35	2.4	
C. Off Esbjerg	25.3				3.0
		-1.2	30	1.8	
D. Skagen - Orust	24.1				4.8
		-5.3	30	8.0	
E. Kattegat	18.8				12.8
		-10.5	20	10.5	
F. Skanør-Møen -Dars Pt.	8.3				23.3
		-0.5	40	1.0	
G. Bornholm Is.	7.8				24.3
		-2.2	100	11.0	
H. Åland Is.	5.6				35.3

Typical errors

- Missing or misplaced text in publication
- Error in mathematcal expressions
- Errors in programs (software)
- Underestimation of orders of magnitude

Conclusion (I)

- If results do not agree with theori: probably errors in measurements or software.
- Misprints/mis-placing of text difficult to spot – wait a week until the final proof-reading.
- Check software by computing the same quantity using two different method or compute the same quantity using different algoritmns.
- Test software by hand using very fee variables.
- Errors in equations difficult to spot. Exploit theory to find errors – for example that a quantity must be zero.

Conclusion (I)

You learn most of your own errors – if they are detected by yourself or if other detect the error

Is it possible to learn something from the errors of others ?

I have in the last 45 years written more than 250 publications and reports.

Percent errors: 2 - 3 % - probably very typical !!

Error Estimates of Results

In Geodesy we always deliver error estimates and (if possible) error-correlations of estimated quantities.

But can we trust these estimates ?

Least-squares Collocation

$$\tilde{T}(P) = \{C_{Pi}\}^T \overline{C}^{-1}(y - A^T X)$$

$$y_i = L_i(T) + e_i + A_i^T \bullet X$$

$$\tilde{X} = (A^T \overline{C}^{-1} A + W)^{-1} (A^T \overline{C}^{-1} y)$$

LSC – error-correlations

$$H = \left\{ COV(L_k, L_i) \right\}^T \overline{C}^{-1}, \text{ MxN matrix}$$

$$\left\{ ec_{kl} \right\} = \left\{ \sigma_{kl} \right\} - H \left\{ \text{cov}(L_j, L_l) \right\} + HAM_X (HA)^T,$$

σ_{kl} is the a-priori correlation between the k and l predicted quantity = supposed to be zero for spherical harmonic coefficients

Isotropic covariance function

As a consequence will geoid and gravity covariance functions only depend on spherical distance and radial distances of points from Earth's center.

Hypothesis does not harm predictions if we have many data (Sansò et al. 2000).

Data Gridded, symmetric wrt. Equator.

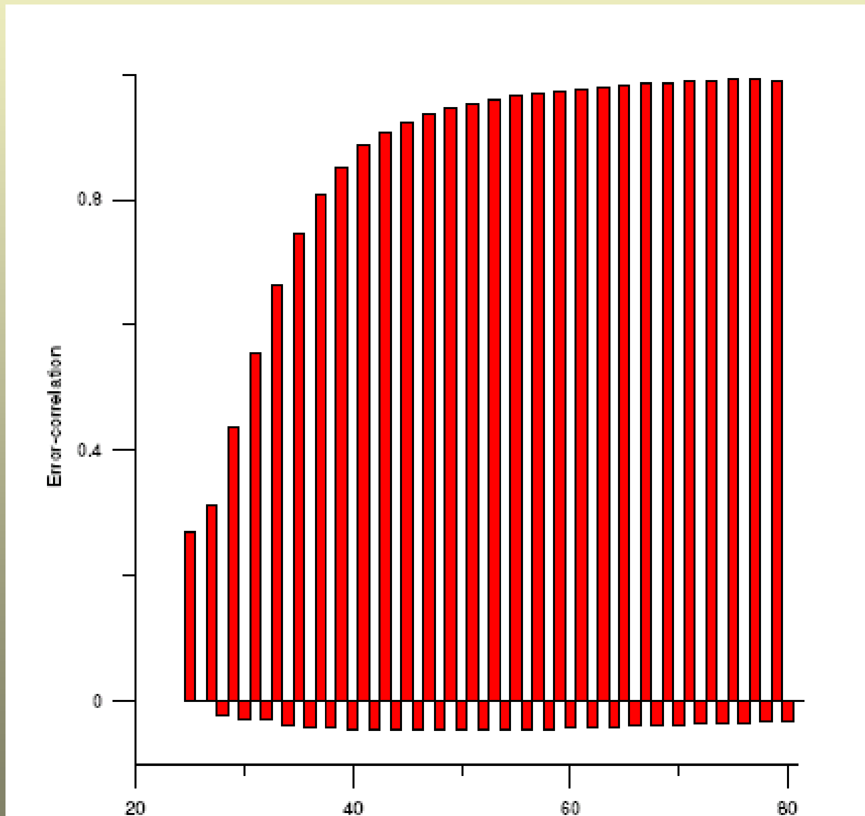
Uniform noise along each parallel:

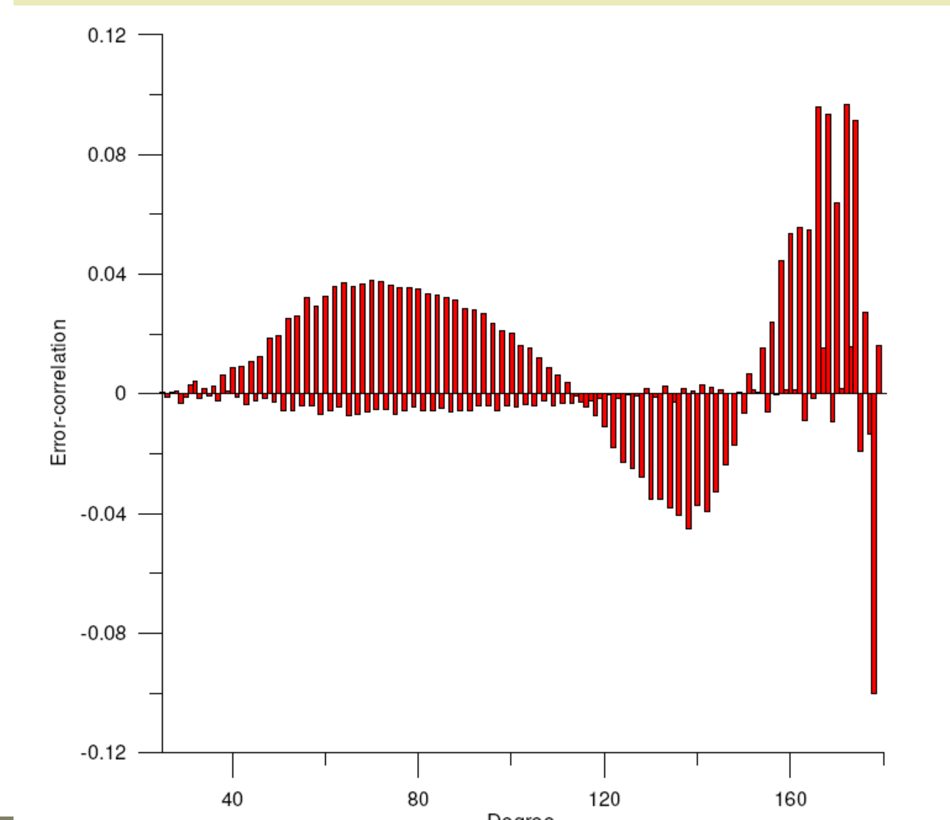
Estimated spherical harmonic coefficients have uncorrelated errors

if order different !

(Shown originally by O.Colombo for band-limited data)

Error-estimates sensitive





Conclusion (II)

When calculating error-estimates or correlations we have to be aware how they depend on the a-priory model adopted.

More research is needed in order to understand this for different estimation methods, including ordinary Least-Squares.