Great Britain's GPS Height Corrector Surface

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BIOGRAPHIES

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Professor Carl Christian Tscherning, University of Copenhagen, has a long and distinguished career in geodesy. He became a professor in 1988. Before 1988 he was external lecturer in physical geodesy at the University of Copenhagen, State Geodesist and Chief of Office at the former Geodetic Institute (today's National Survey and Cadastre, Denmark). Internationally, he held many high ranking positions within the International Association of Geodesy. He is currently Secretary General of the IAG. He was also Editor of Bulletin Geodesique 1986-1995 and Manuscripta Geodaetica 1992-1995.

ABSTRACT

Great Britain uses a traditional heighting datum (Ordnance Datum Newlyn) based on a series of tide gauge measurements made from 1915 to 1921. This datum is realised through the measured heights of a series of fundamental benchmarks established along geologically stable lines across the country. The establishment of the heighting network suffered from various systematic errors and, having been tied to the sea level measurements at a single point in space and a single epoch in time, does not model height above a physical surface such as the geoid. The reference surface is in fact numerical rather than physical deriving from the

published height values of the monuments from which it is realised. However, as GPS has matured as a precision surveying and positioning technology it is used increasingly to establish both position and elevation. As most heighting data in Great Britain is still referenced to the traditional datum Britain's national mapping agency, the Ordnance Survey, needed a way to exploit the precision and availability of GPS, whilst at the same time enabling users to determine height either above the WGS84 reference ellipsoid (GRS80) or with respect to the Newlyn datum, which is nominally orthometric, despite its contamination with systematic biases.

This paper describes the solution to this problem. The process leading to the solution consisted of the computation of a rigorous physical reference surface (a gravimetric geoid model) that was subsequently warped to fit the differences between the datum surface (Newlyn) and the WGS84 ellipsoid. This 'corrector surface' is then used to convert GPS derived ellipsoidal heights into the Newlyn datum. The fitting process, its quality control and quality assessment are described. The post-fit residuals of this surface to the fundamental benchmark network had a standard deviation of 4mm, with maximum and minimum height residuals of +18 and -11mm respectively. The quality of these results is largely attributed to the excellence of the geoid model and the GPS data collected at the benchmark locations.

This means that, given high precision GPS positioning, users in the UK can now determine their height with respect to the country's fundamental benchmark network with a precision of 1-2 cm. This gives an unprecedented level of heighting consistency across a country of approximate extents 950 km (north-south) and 550 km (east-west).

The derived corrector surface has also been adopted as the national standard for transforming height between Newlyn and the European Terrestrial Reference Frame (ETRF). The study was carried out under contract to the Ordnance Surveys of Great Britain, the Republic of Ireland and Northern Ireland.

INTRODUCTION

GPS relative positioning in the UK is supported by the Ordnance Survey (the UK's national mapping agency) active and passive GPS station network. The positions of the network stations are computed rigorously with respect to the European Terrestrial Reference Frame (ETRF) based on the GRS80 ellipsoid with an estimated precision of circa 5 mm (one sigma). These coordinates are publicly available free of charge, as is RINEX data at 15 second intervals from the active station network via the Internet, with a latency of around 1 hour. The Ordnance Survey also has a national real time kinematic system under development. Given such ease of access to the reference frame and the advances in modelling of atmospheric effects on GPS phase measurements, and improvements in both the predicted and post-processed GPS orbits, it is now feasible to measure the ellipsoidal height of the country's topography at the level of a 3-4 centimetres, both reliably and economically. With long observation periods and suitable processing software this uncertainty can be halved.

However, the UK's existing heighting system is based on a traditional datum linked to a tide gauge on the country's south-west coast, known as the Newlyn datum. Engineering projects, utilities, regional authorities and the national mapping are all tied into Newlyn – their means of access to this datum has invariably been through using the country's network of benchmarks. Despite the extremely high cost of maintaining a traditional benchmark network, the stakeholders in the system require ongoing access to the datum. GPS offered a straightforward method for measuring height without the requirement to maintain an extensive network of physical monuments, but to meet the needs of the user community the Ordnance Survey had to reconcile the difference between the datums. The solution to that problem is what is described in this paper.

PROBLEMS WITH THE EXISTING SYSTEM

In principle, national heighting networks use an equipotential gravitational surface known as the geoid to define their datums. As an intuitive model the geoid can be considered to be a surface that coincides with mean sea level in the open oceans, and it can be imagined that this surface is projected beneath the continental land masses. More formally the geoid is that surface over which the gravitational potential (W_0) is $62636856.85 m^2 s^{-2}$ +/- $1.085 m^2 s^{-2}$ [1]. The geoid can be modelled geometrically by quantifying its height above or below a reference ellipsoid. These heights are termed the 'geoid-ellipsoid separation' or 'geoid undulation' (*N*). See figure 1.

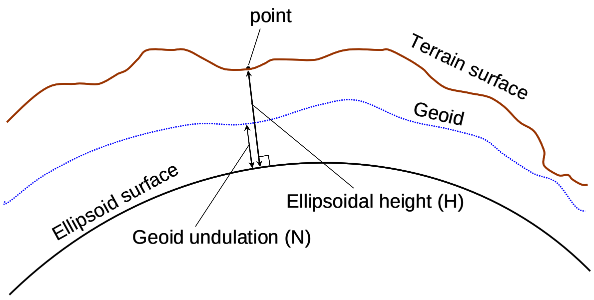


Figure 1: The relationships between the geoid, an ellipsoid and the terrain surface.

Traditionally using the geoid as a reference surface has been attempted by taking measurements of sea level at a point on the coast of a country over time (although some countries have used multiple tide gauges, but this has generally led to inconsistencies), and then taking the average of these values as a realisation of the geoid at that one point. This was the procedure adopted for the UK, and hourly measurements were recorded over the time period 1st May 1915 to 30th April 1921 [2] at a tide gauge in Newlyn, Cornwall, in the south-west part of England. Access to the Newlyn datum was then enabled by taking levelling observations across the country and combining this data with a reference gravity model to compute the height of the terrain (and more specifically the height of the benchmarks) above the geoid. In practice the resulting heights seldom refer to the geoid, for several reasons.

In the first case, mean sea level at the tide gauge is not necessarily coincident with the surface of the geoid. This can be because of local current and wind forcing of the water, thermal, frictional or salinity variations resulting in sea surface topography above (or below) the geoid surface. See figure 2. In the second case, as was the intention in the UK, the determination of orthometric heights requires knowledge of the gravity field along the levelling lines for the height computation, which was not available. A reference gravity model was used instead. Finally unmodelled instrumental biases could have a serious effect in a network derived from millions of individual measurements. Evidence for the non-conformity of the heighting network to the underlying, assumed physical model arose fairly early on – a second tide gauge at Dunbar in Scotland yielded a mean sea level that disagreed with the levelling from the network at the level of 250mm [3]. This lead to the suspicion that the network was at the very least tilted with respect to the geoid, although the reality, as will be shown later in the paper, was a little more complex. One final practical consideration is that of change – sea level has risen at Newlyn (as inferred from the tide gauge measurements) by circa 153mm over the course of the twentieth century [4] – whilst the benchmark values have remained fixed by necessity to satisfy the needs of the user community.

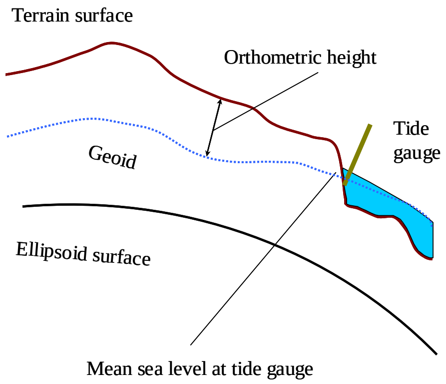


Figure 2: Mean sea level and the geoid are not necessarily coincident at any one particular tide gauge.

Hence, to conclude this section, although the UK's national heighting datum is, by design, *a physical system* based on gravitational potential, it is, in practice, *a numerical system* – that is, the shape of its underlying, implicit reference surface is defined by the values of the published heights for its benchmarks, through which users gain access to the datum. Despite all these problems the Newlyn datum is used successfully by a very large UK community for engineering, mapping and construction – it is simply not possible to discontinue access to the datum. Given the gradual degradation of the benchmark network and the cost of maintaining it, some alternative had to be found, and GPS was identified as the way around the problem [2]. The task at hand is to compute a continuous mathematical reference surface above the GRS80 ellipsoid (the underlying reference surface used to define height determined from GPS) that models the national heighting datum as inferred from the discrete points at which benchmark values are known. See figure 3.

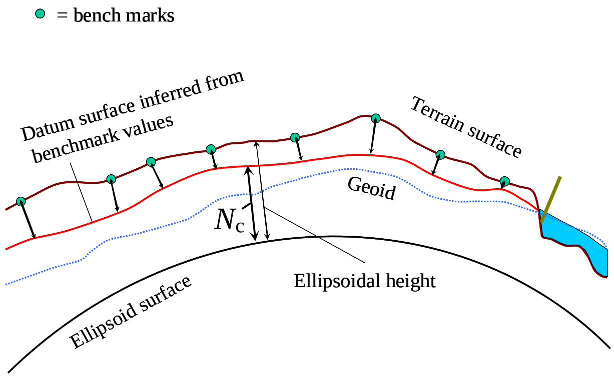


Figure 3: A datum surface that is defined implicitly by the values of benchmarks in the heighting network

METHODOLOGY

Referring to figure 3 it can be seen that if we know the ellipsoidal height (H) of a point on the terrain as measured by a GPS receiver, and we want to convert this to a height in the national datum (h_{nd}), then we need to know the height of the national datum above the ellipsoid at that point. That is:

$$h_{nd} = H - N_c$$

In this equation N_c models a 'corrector surface' which allows us to toggle between the national datum and ellipsoidal heights. It should be stressed that this value is *not* the geoid-ellipsoid separation, but does function in a similar way.

Whilst the corrector surface implied by the known benchmark values is not physically coincident with the geoid its physical shape is likely to be similar. Hence, the method adopted to model the corrector surface in a continuous sense was to first compute a high quality model of the geoid for the UK, and then to fit this to a selected subset of values of N_c, determined by measuring ellipsoidal heights at benchmark locations.

SOURCE DATA

For the purposes of the geoid computation, the long wavelength component of the gravity field was obtained using the EGM96 geopotential model complete to degree and order 360 [5]. In addition a total of 142826 land gravity measurements and 15900 marine gravity measurements were used, as well as a further 14719 values taken from the KMS99 model. For such a study this is probably the densest coverage of gravity data used to date, featuring a mean gravity measurement spacing of one point per 1.5 km by 1.5 km square. See figure 4. The geoid computation method required a terrain model to assist in modelling the anomalous potential, and this data was supplied by the Ordnance Survey at a post spacing of 50m.

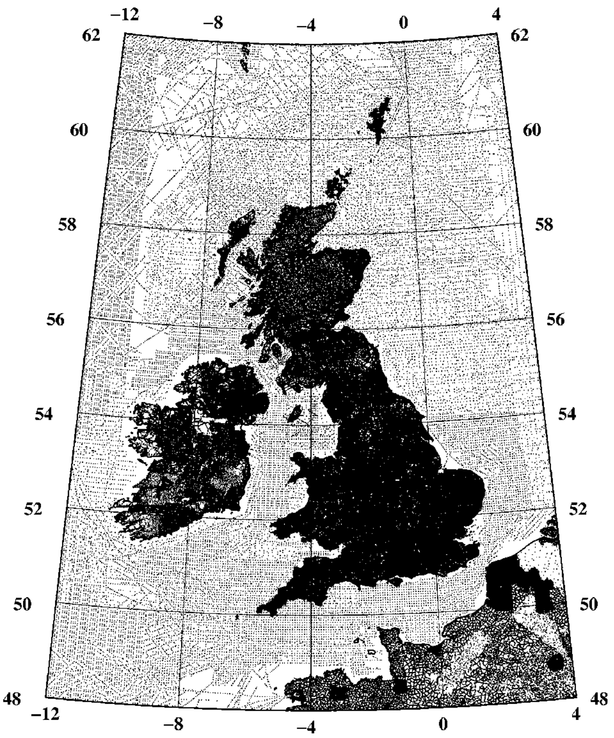


Figure 4. Distribution and density of gravity data used in the project

The realisation of the Newlyn datum used the UK fundamental benchmark (FBM) network (see figure 5). These 180 points were chosen because they were originally sited deliberately on geologically stable ground, and under these circumstances are unlikely to have suffered local subsidence – which would distort the shape of the inferred corrector surface, this being a result of a change in the actual relative height between points in the network as a consequence of the subsidence conflicting with the inferred difference in height given by the published height values. Ellipsoidal heights were obtained for the FBMs from a network campaign tied to the European Terrestrial Reference Frame.

Figure 5: UK fundamental benchmark network

GEOID COMPUTATION

The geoid computation was carried out by Rene Forsberg and Gabriel Strykowski in Denmark. The computation method is covered in detail in reference [6]. In short, the anomalous gravitational potential (T) was split into three components:

$$T = T_1 + T_2 + T_3$$

Respectively these components model the contribution from EGM96, the residual terrain model (derived from prism integration of the smoothed digital elevation model), and finally the residual gravity anomaly.

In the study, the computation was carried out using the height anomaly ζ or quasi-geoid (that is the form of the geoid determined with respect to the terrain), using the same subdivision as the anomalous potential as follows:

$$\zeta = \zeta_{EGM\,96} + \zeta_{RTM} + \zeta_{res}$$

The relationship between the quasi-geoid and the anomalous potential being given by Bruns' equation [7]:

$$\zeta = \frac{T(\varphi, \lambda, H)}{\gamma(\varphi, \lambda, H)}$$

With ϕ and λ referring to the geographical latitude and longitude, and H is the orthometric height, whilst γ is the value of the reference gravity model.

The relationship between the quasi-geoid and the geoid is given by the approximation:

$$\zeta - N = \frac{\Delta g_B}{\gamma_0} H$$

where Δg_B is the Bouguer gravity anomaly.

From a practical viewpoint the computational process can be considered as one of reducing the observed gravity (height) anomalies for the effects of the modelled long wavelength gravity field and the residual terrain effects above this, and then interpolating the residual field using a modified Stokes' Integration, whilst screening for outliers. The Stokes integral was evaluated using Fast Fourier Transform techniques [8]. The resulting geoid undulations are illustrated in figure 6.

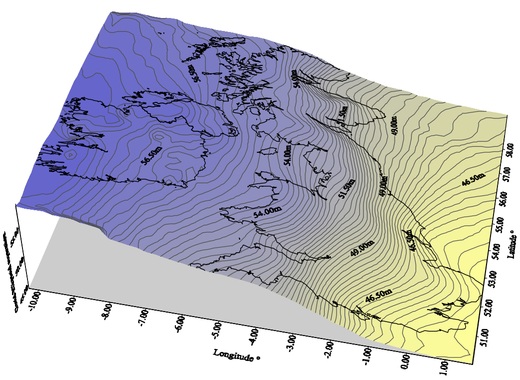


Figure 6. OSGM02 geoid with respect to the GRS80 ellipsoid

FITTING PROCESS AND RESULTS

In the first step the differences between the point values of the Newlyn Datum-ellipsoid separations were compared directly to the computed geoid. That is we compare:

$$N_c = H - h_{nd}$$

with the geoid undulation (N), derived from the computed model at the same point. We expected to see a smoothly varying, sloping surface. The results for all 180 points, termed the pre-fit residuals are shown in figure 7.

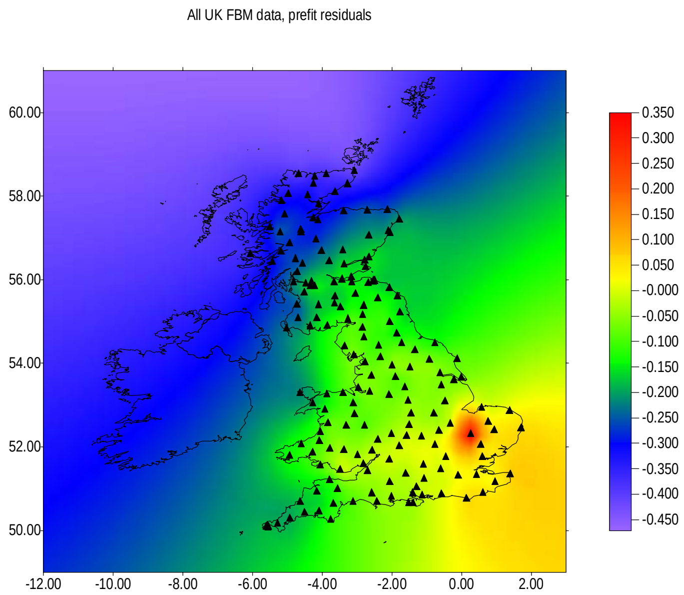


Figure 7. Differences in metres between the 180 point values of the Newlyn-ellipsoid separation and the geoid

One point (Wicken in East Anglia) in figure 7 appeared to buck the trend. In fact it turned out there was a gross error in the supplied data at this point. This was removed and the surface was recomputed, see figure 8. Although it is clear from the surface that the differences between the geoid and the levelling network amount to more than a simple north-south slope, it is also apparent that the spatial structure of the differences is not chaotic or riddled with discontinuities and abrupt changes – that is, given a suitable transformation the geoid could provide the required interpolator function.

In the next step a rigid (conformal) transformation of the geoid was computed using a four parameter Helmert model:

$$\varepsilon = a_1 \cos\varphi \cos\lambda + a_2 \cos\varphi \sin\lambda + a_3 \sin\varphi + a_4 R + \varepsilon'$$

In this function ε is the difference between the Newlyn-ellipsoid separations (N_c) and the geoid-ellipsoid separation, ϕ and λ are the latitude and longitude, and ε' is the residual after the transformation, which was to be modelled subsequently by least squares collocation.

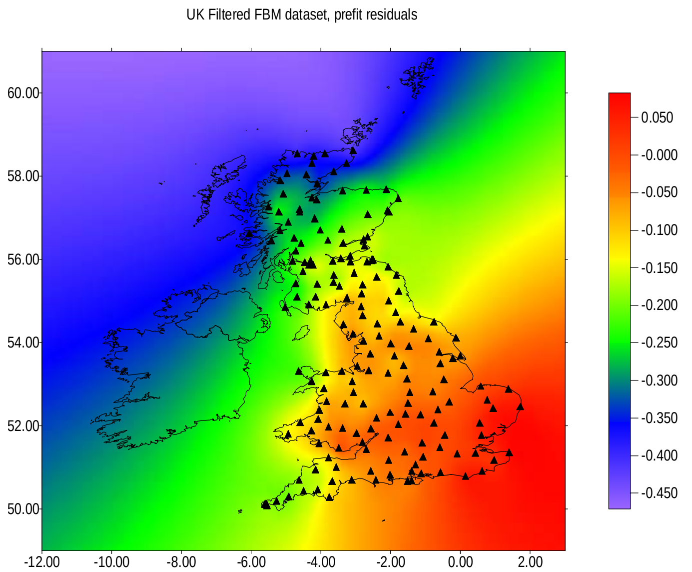


Figure 8. Differences in metres between the remaining 179 point values of the Newlyn-ellipsoid separation and the geoid

Prior to the conformal transformation the mean difference between the two surfaces was -0.129m +/-0.110m, accommodating a range of values -0.431m to +0.081m. After the application of the transformation (computed by least squares analysis) the differences were given by a mean of zero +/-0.032m with a range -0.089m to 0.092m.

In the final step, to carry out the least squares collocation modelling of the residuals, a covariance function was determined empirically from the data, using a second order Markov model:

$$C(s) = C_0(1 + \alpha s)e^{-\alpha s}$$

where $C(s)$ is the covariance as a function of the distance (s) between the observation points (that is, where we have levelling data, GPS observations and a value from the geoid). C_0 is the zero variance which, along with the constant α, is to be estimated from the data. The estimation of these parameters sets the model *correlation length*, which is the distance from a point at which the covariance falls to half the zero variance. In the study the correlation length was set to 50 km – in practical terms this has the net effect of stiffening the geoid such that over short wavelengths the geoid dominates the form of the corrector surface, whereas the long wavelength trend is derived from the GPS-levelling. It is worth stressing that setting a very short correlation length would give the impression in the fit that the model fitted the data exactly. The mean inter-station distance in the network was around 35 km, and hence the rigorously computed geoid has a strong influence in the mathematical form of the corrector surface.

The post-fit statistics gave a mean of zero +/-0.004m with maximum and minimum residuals of +0.018m and -0.011m respectively. The statistics of the fit at each stage in the process are given in table 1, and the post-fit surface residuals are illustrated in figure 9.

Stage	Mean	Std. Dev.	Min.	Max.
Pre-fit	-0.129	0.110	-0.431	0.081
De-trended	0.000	0.032	-0.089	0.092
Post-fit	0.000	0.004	-0.011	0.018

Table 1. Statistics (in metres) of fit of geoid to FBM network at various stages of calculation

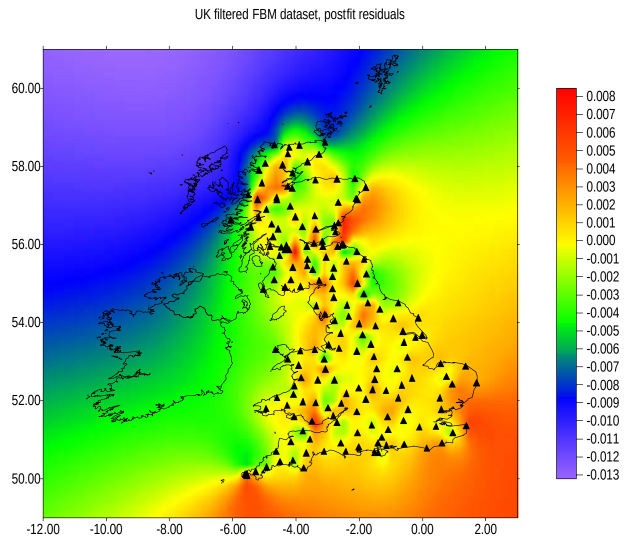
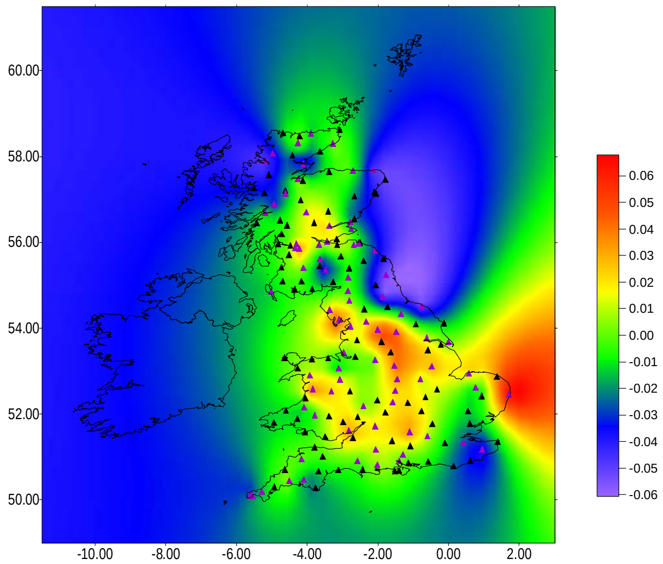


Figure 9. Post-fit residuals after modelling of the difference between the GPS-levelling and the de-trended geoid

The key output of this process is a gridded surface model that allows the user to input a latitude, longitude and ellipsoidal height, and to then compute the height of that point expressed in the Newlyn datum, as realized through the fundamental benchmark network. If the user were to carry out levelling from the FBM network to the same point the height obtained should agree at the level of 1-2 cm.

MODEL TESTING

The first test is less a test of the derived model, but more a fundamental test of the methodology. This consisted of carrying out the fit to a subset of 100 of the points in the network and then using the remaining 79 as GPS observations at which precise levelling information was also available. Naturally this greatly reduced the granularity of the network and increases the inter-station distance to well above the adopted correlation length. At the check points the mean residuals were -0.001m with an RMS scatter of +/-0.033m. The residual surface is illustrated in figure 10.



Purple triangles = points used as checks

Figure 10. Corrector surface fitted to subset of 100 data points (black triangles) and checked against remaining network stations (purple triangles)

The second test uses points in the network that are relatively close together (in comparison to the correlation length of 50 km). The difference in the underlying reference surface between these points inferred from the geoid surface is compared to the N_c differences inferred from the GPS-levelling. The test highlights the suitability of the geoid for acting as an interpolator. Were the geoid relatively flat and featureless it is unlikely this would amount to very much of a test, however, there is considerable variation in the geoid gradient across the country (refer to figure 6), and at one point in the test data the geoid dropped by 0.52m over just 15 km. At the thirteen pairs of points considered in the network (with a mean inter-station distance of 8.9 km) the mean difference between the two surface differences was 0.0004 m with an RMS scatter of 0.011m. This test illustrates clearly the fidelity of the model for interpolating between stations over distances shorter than the correlation length, where the geoid form dominates the model. The test is explained more fully in [9].

COMMENTARY

The success of the modelling can be attributed in part to the fine quality of the data – the FBM network was observed and computed rigorously resulting in a homogeneous system, the GPS data and network processing was also strongly quality controlled and computed by experienced scientists [10], and the gravity data set was probably the most dense ever employed in such a study.

Initial feedback from the user community has borne out the testing carried out by the authors. However, this will not necessarily be the case everywhere in the UK. The corrector surface models successfully the datum as realized by the fundamental benchmarks, and thus introduces the opportunity to maintain a high quality and homogeneous heighting system. Problems will arise in areas where there has been active subsidence in the last hundred years without update of the local benchmark values. This is certainly the case in mining areas where subsidence can easily be measured in metres. This problem will largely be one of inconsistency between the heights obtained from the new corrector surface, with the heights measured with respect to benchmarks that should have been either re-measured or destroyed.

A second limitation is that of the GPS data that is employed by the user community – the excellent level of consistency in the heighting that can be achieved using the corrector surface will only be maintained provided that systematic errors in GPS derived ellipsoidal heights are minimized. Whilst this is supported by the Ordnance Survey's excellent outreach campaign (http://www.gps.gov.uk) and national active and passive networks, users will still need relatively long occupation times or benign observing conditions to reduce effects due to multipath. A similar consideration applies to the modelling of receiver phase centre offsets and variations when differing receiver/antenna types are used.

Similar work was carried out in the study in Northern Ireland, the Republic of Ireland and in some of the outlying UK island chains (e.g. the Orkneys and the Shetland Islands), although in these cases lower order levelling had to be used in the computation resulting in poorer fit statistics driven by the lower precision in the levelling [9].

The modelled surface has since been implemented into a freely available software utility (see http://www.gps.gov.uk).

CONCLUSIONS

A new scientific geoid has been calculated for the United Kingdom and used to derive a ‘corrector surface’ allowing a user to convert ellipsoidal heights into heights expressed in the national datum derived from the tide gauge at Newlyn and realised through the country’s fundamental benchmark network. This corrector surface is consistent across the country at the level of 1-2 cm, enabling the realization of a repeatable and homogeneous heighting system using GPS data comparable in performance to first order levelling. That is, if the height of a point is determined by levelling from a good quality benchmark in the UK, and the same point’s height is also obtained by the combination of GPS and the UK corrector surface, the results will be within 1-2 cm of each other over most of the country.

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REFERENCES

[1] McCarthy, D. D., (1996), IERS Technical Note 21, Paris, International Earth Rotation Service.

[2] Christie, R.R., (1994), A new geodetic heighting strategy for Great Britain, *Survey Review*, 32. 252 (April 1994)

[3] Kelsey, J., (1972), Geodetic aspects concerning possible subsidence in southeastern England: Phil. Trans. R. Soc. Lond., v. 272, p. 141-149.

[4] Wöppelmann, G., J. Adam, W. Gurtner, B-G. Harsson, J. Ihde, M. Sacher and W. Schlüter, 2000, Status report on sea-level data collection and analysis within the EUVN Project: Proceedings of EUREF2000, Tromso, Norway.

[5] Lemoine, F.G., S.C. Kenyon, J.F. Factor, R.G. Trimmer, N.K. Pavlis, D.S. Chinn, C.M. Cox, S.M. Klosko, S.B. Luthcke, M.H. Torrence, Y.M. Wang, R.G. Williamson, E.C. Pavlis, R.H. Rapp and T.R. Olson: The Development of the Joint NASA GSFC and the National Imagery and Mapping Agency (NIMA) Geopotential Model EGM96. NASA/TP-1998-206861 report. Goddard Space Flight Center, Greenbelt, Maryland, 1998.

[6] Forsberg, R., G. Strykowski, J.C. Illife, M. Ziebart, P.A.Cross, C.C. Tscherning , P. Cruddace, K. Stewart, C. Bray and O. Finch, (2003), OSGM02: A new geoid model of the British Isles. Proceedings of the 3rd meeting of the International Gravity and Geoid Comission, GG2002, Aug. 26 - 30, 2002, Thessaloniki, I. Tziavos (ed.), Editions Ziti, pp. 132-137, 2003.

[7] Bruns, H. (1878). *Die Figur der Erde*. Publikation des Koniglichen Preussischen Geodetischen Institutes, Berlin.

[8] Forsberg, R. and M.Sideris (1993), Geoid computations by the multi-band spherical FFT approach, Manuscripta Geodaetica, 18, pp. 82-90.

[9] Iliffe, J. C., M. Ziebart, P. A. Cross, R. Forsberg, G. Strykowski, and C. C. Tscherning, (2003), "OSGM02: A new model for converting GPS-derived heights to local height datums in Great Britain and Ireland". Survey Review, Vol. 37, No. 290, pp 276 - 293, 2003.

[10] Penna, N.T., R.M.Bingley and A.H.Dodson, (2002), Single receiver heighting using the active stations of the National GPS Network of Great Britain, Survey Review, Vol. 36, No.283: 340-350.