ON A STRATEGY FOR THE USE OF GOCE GRADIOMETER DATA FOR THE DEVELOPMENT OF A GEOPOTENTIAL MODEL BY LSC

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ABSTRACT

For the development of a spherical harmonic expansion of the Earth's gravity field using point data such as gravity gradient components from the GOCE mission, data with a global coverage are needed. For a satisfactory representation of the gravity field in an area using point data of a relevant quantity, the number of the data needed depends on the variance of the gravity field in this area. Therefore, for global coverage, using a proper data distribution it is possible to achieve a good approximation of the gravity field without increasing unnecessarily the number of data, using more data in areas with rough gravity field and fewer data in areas where the gravity field is smoother. This is essential for methods such as the method of Least Squares Collocation (LSC) where a system of linear equations is to be solved having a number of unknowns equal to the number of input data. The aim of this paper is to suggest a strategy for the collection of point data on real GOCE orbit, leading to optimal LSC determination of spherical harmonic coefficients, with the minimum number of data points. Numerical experiments for the determination of harmonic coefficients were carried out, using noise-free or noisy simulated data with different distributions. The suggested strategy is based on the comparison between computed and true coefficients, the collocation error estimate of the coefficients and the comparison of the original data with data generated by the computed coefficients.

Key words: spherical harmonic coefficients; least squares collocation; equal-area blocks; correlated noise.

1. INTRODUCTION

One of the main objectives of the GOCE mission is to provide a global model of the Earth's gravity field with accuracy better than 1 mGal for the gravity and below 1 cm for the geoid, with a spatial resolution of 100 km. To meet these requirements, spherical harmonic coefficients are to be determined to at least degree and order 200.

Different methods can be used for this purpose. Among them, LSC is a flexible method for the computation of the harmonic coefficients as well as of their errors and error covariance functions from GOCE gravity gradiometer data, without any formal requirement about their distribution (Tscherning 2001). Therefore no gridding of the data is necessary, since the data can be used on their original positions on the real GOCE orbit. Furthermore, the individual standard deviations of the observations, as well as systematic errors remaining after the preprocessing of the data may be taken into account. The drawback of this method is related to the current ability of the computers in handling the very large (full) systems of linear equations produced by the equally large amount of data used. For this reason a strategy aiming at the selection of the minimum number of data which could result in the best determination of the spherical harmonic coefficients might be very interesting for the application of LSC.

For this purpose numerical experiments were carried out, using simulated data sets of second order derivatives of the disturbing potential. The simulated data sets were either produced using the EGM96 geopotential model to degree 360 as noise-free gravity gradient components at mean satellite altitude (250 km) or were selected from a realistic end-to-end simulated data set provided by ESA. The SGG values of this dataset were computed on a realistic orbit with noise added, based on the a-priori Fourier spectrum noise characteristics of GOCE. Consequently, the noise was (partly) removed using Wiener orbital filtering (Reguzzoni & Tselfes 2006). In the next, this data set will be referred as "ESA/SGG".

Different distributions of the data were examined, based on equiangular or equal-area grids. The equal-area grid was based on the requirement to divide the surface of the (spherical) Earth in nearly equal area blocks, without the precondition that the longitude extend should be equal to an integer number of degrees. In the following the nodes of these 2° or 1° equal-area blocks will be considered to represent a "grid". In all the following experiments only the second order radial derivative T_{rr}. was used in order to simplify the computations. Furthermore, it include more information (better signal to noise ratio) compared to other components. This was found in local applications for the recovery of the gravity field using airborne gravity gradiometer data (e.g., Arabelos & Tziavos 1992) as well as in regional applications using simulated satellite grav-

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ity gradiometer data (e.g., Arabelos & Tscherning 1995).

The assessment of the results of the numerical experiments was based on the comparison between computed and true spherical harmonic coefficients, and on the collocation error estimates.

2. EXPERIMENTS USING EGM96 GENERATED DATA

For the EGM96 (Lemoine et al. 1998) generated data the following procedure was applied: The data were reduced to EGM96 up to degree 24 in order to statistically homogenize the data. A global (homogeneous) covariance function was used, based on the formula (Tscherning & Rapp 1974)

$$cov(\Delta g_P, \Delta g_Q) = \sum_{l=2}^{k} \sigma_l(\Delta g, \Delta g) s^{l+2} P_l(t) + \sum_{l=k+1}^{\infty} c_l(\Delta g, \Delta g) s^{l+2} P_l(t) \quad (1)$$

where Δg is the gravity anomaly, $P_l(t)$ the Legendre polynomials of degree l, $t = \cos\psi$, $s = R^2/(r_P \cdot r_Q)$, ψ the spherical distance between the points P and Q, R the radius of the so-called Bjerhammar sphere, r_P, r_Q the distance of the points P and Q from the origin, the error degree variances of the geopotential model used for the reduction of the data and the anomaly degree variances estimated empirically, using the degree variance model by Tscherning & Rapp (1974)

$$c_l = \frac{A(l-1)}{(l-2)(l+B)}. \quad (2)$$

The following values were used for the parameters of the equations (1) and (2): $R = R_E - 1.561\text{km}$, with R_E the radius of the Earth, error degree variances from degree 2 to 24 from EGM96 with scale factor equal to 1.03, $B = 4$ and variance of the point gravity anomalies on the surface of the Earth $A = 540.9\text{mgal}^2$. A common standard deviation of observations equal to 0.01 E was adopted for the data. From this covariance function the necessary covariance functions for our experiments were derived using the covariance propagation law. It should be pointed out that the use of a global (homogeneous) covariance function calls for a further homogenization of the input data (Migliaccio et al. 2004). Such a homogenization was not used in this paper.

2.1. Distribution of the data on the nodes of a grid

In a first attempt spherical harmonic coefficients from degree 25 to 40 were computed from T_{rr}. The data points were distributed on the nodes of the $2°$ equal-area grid (10,448 nodes). As a quality criterion of the computed coefficients the standard deviation per degree of the differences between computed and "true" EGM96 coefficients was used, relative to the mean collocation error estimation per degree.

Although using data on the nodes of the 2^o equal area grid, coefficients up to higher degree and order might be computed, the degree 40 was chosen in this phase to save computational time.

In Table 1 the standard deviation per degree of the differences between computed and true (EGM96) coefficients and the mean collocation error estimation per degree are shown (columns 2-3). In the last column the degree standard deviation of the EGM96 coefficients is shown for comparisons.

To account for the effect of the lack of data at the polar caps affecting mainly the zonal harmonics the previous experiment was repeated using the previous T_{rr} data set, after removing the data with $\varphi > 86°$ and $\varphi < -86°$. With the remaining 10,428 data points the results were very similar to the corresponding to complete grid (see Table 1, columns 4-5).

The distribution of the data points on a equiangular $2° \times 2°$ was examined by a similar experiment. A number of 15,300 T_{rr} values were computed on a $2° \times 2°$ grid, neglecting data with $\varphi > 84°$ and $\varphi < -84°$. The computation of coefficients was extended up to degree 90 and the results are shown in Fig. 1.

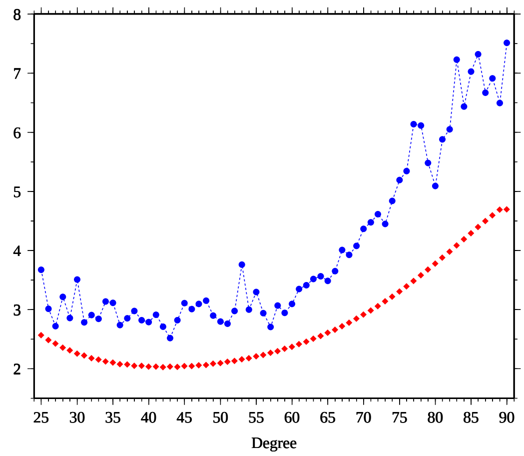

Figure 1. Standard deviation per degree of the differences between computed and true coefficients from degree 25 to 90 (blue line) and mean collocation error estimation for the same degrees. 15,300 T_{rr} values on a $2° \times 2°$ equiangular grid were used in this experiment. All numbers are multiplied by 10^9.

Comparing the results of Table 1(columns 4-5) with that of Fig. 1 up to degree 40, it is easy to conclude that the distribution on the equal-area grid gives an improvement compared to the distribution on the equiangular grid, be-

Table 1. Statistics of the differences between computed and true EGM96 harmonic coefficients resulted from the radial component T_{rr} with data in the polar caps (10,448 data points) and without (10,428). All values are multiplied by 10^9.

	With data in the polar caps		Without data in the polar caps		
Degree	Standard deviation	Mean collocation error estimation	Standard deviation	Mean collocation error estimation	Degree standard deviation of EGM96
25	3.693	2.789	3.735	2.819	134.690
26	3.285	2.698	3.322	2.726	124.569
27	2.800	2.617	2.830	2.647	115.547
28	3.251	2.545	3.220	2.573	107.470
29	2.699	2.481	2.705	2.511	100.212
30	3.389	2.425	3.398	2.452	93.664
31	2.656	2.375	2.654	2.404	87.738
32	2.961	2.330	2.951	2.358	82.356
33	3.029	2.292	3.019	2.321	77.454
34	3.359	2.258	3.364	2.285	72.977
35	3.231	2.228	3.210	2.257	68.876
36	2.689	2.203	2.653	2.231	65.111
37	3.201	2.182	3.183	2.211	61.646
38	2.860	2.164	2.856	2.192	58.450
39	2.928	2.150	2.922	2.179	55.496
40	2.654	2.139	2.677	2.167	52.759

cause with much fewer data results in equal or even better quality of the computed coefficients.

In a next computation, spherical harmonic coefficients were determined using again EGM96 generated T_{rr} data set on the nodes of a 1° equal-area grid. A number of 41,522 data points were used to compute spherical harmonic coefficients from degree 25 to degree 150. The results in terms of the differences between the computed and true EGM96 coefficients, relative to the mean collocation error estimation are shown in Fig. 2. Note the convergence of the two curves for increasing degree.

Furthermore, the predicted coefficients were used to compute T_{rr} values at the positions of the file ESA/SGG, consisting of 509,222 data points. The same computation was repeated using the EGM96 model. The results of these computations in terms of the statistics of the differences between ESA/SGG and T_{rr} from EGM96 and between ESA/SGG and T_{rr} from the predicted coefficients, are shown in Table 2. It is remarkable that both, the mean value and the standard deviation of the differences between ESA-SSG and T_{rr} from the predicted coefficients are slightly better than the corresponding between ESA/SGG and T_{rr} from EGM96.

From the experiments described above it might be concluded that using T_{rr} data with uncorrelated noise, distributed on the nodes of an equal-area grid, the determination of spherical harmonic coefficients is possible with an accuracy which in terms of the standard deviation of the differences between computed and true coefficients remains within the collocation error estimates.

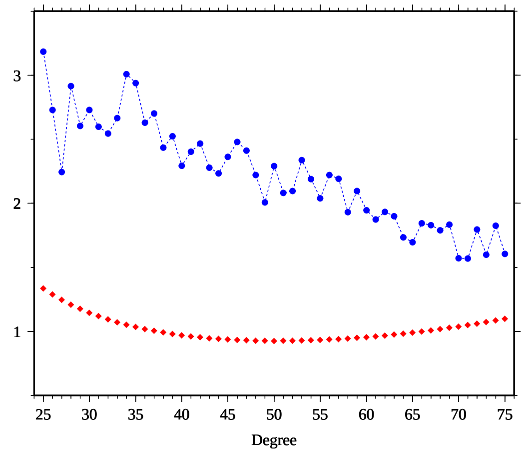

Figure 2. Standard deviation per degree of the differences between computed and true coefficients from degree 25 to 75 (blue line) and mean collocation error estimate for the same degrees. The standard deviation of the differences computed-true at degree 150 is 3.369. All numbers are multiplied by 10^9.

2.2. Distribution of data on the real orbit, close to the nodes of a grid

The distribution of the GOCE data is different of that used in the previous experiments. In order to examine the effect of the real GOCE data distribution on the accuracy of the computed coefficients, two new data sets were generated in the following way. Using EGM96 to degree 360, totally 509,222 T_{rr} values were computed on the 5 second points of the GOCE orbit provided by ESA. The two

Table 2. Statistics of the differences between ESA/SGG and T_{rr} from EGM96 and from the coefficients computed to degree 150, using the 41,522 data points. The number of observations is 509,222. Unit is E.

		ESA/SGG - T_{rr} from EGM96		ESA/SGG - T_{rr} from computed coefficients to degree 150	
	Observations	Predictions	Difference	Predictions	Difference
Mean value	-0.0049	-0.0048	-0.0001	-0.0048	-0.0001
Standard deviation	0.2354	0.2330	0.0162	0.2371	0.0158
Maximum value	1.7847	1.8306	0.0900	1.9179	0.1078
Minimum value	-1.5811	-1.5639	-0.1340	-1.6594	-0.1340

data sets mentioned previously were selected from these values, selecting data points as closest to the nodes of the 2° equal-area grid. For the first data set a maximum distance of 30 km between grid and selected point was allowed, for the second one this distance was increased to 50 km. In this way the first set consists of 8,500 and the second one of 9,686 values. Using the homogeneous covariance function of the previous experiments spherical harmonic coefficients were computed from degree 25 to 40. The results of these computations in terms of the standard deviation of the differences computed-true and of the collocation error estimation are shown in Table 3. In the same table the corresponding results (Table 1, columns 2-3) of the data set consisting of the 10,448 points (complete grid) are shown for comparison.

The results from the complete grid are of course better (by about a factor of 3) compared to the corresponding from the data set of the 8,500 points but a considerable improvement by 50% was achieved, when the number of data points was increased from 8,500 to 9,686 points, although the new added 1,186 data points are at larger distances (from 30 to 50 km) from the nodes of the 2° equal-area grid.

The conclusion derived from the results of the experiments of section 2.1 could be repeated also in this section. The quality of the computed coefficients is better when the data are distributed on a grid. Between equal-area or equiangular grids the first is beneficial because using considerably less number of data better quality of the computed coefficients was achieved. In the case of the equal-area grid, the transition from the grid nodes to the actual positions of the data on a real GOCE orbit results generally in a 50% decrease of the quality of the coefficients. However, the standard deviation of the differences between computed and true coefficients remains very close to their mean estimated errors.

3. EXPERIMENTS WITH DATA PROVIDED BY ESA

As it was described previously the data set referred to here as ESA/SGG of realistic 5 second end-to-end simulated data with correlated noise was available by the "POLIMI" GOCE-HPF group (Reguzzoni, personal communication) after a Wiener filtering of the noise. The data were already reduced to EGM96 to degree 50 and contribution above degree 150 was removed. The covariance function used for the prediction of spherical harmonic coefficients was based on the formulas (1) and (2) with anomaly degree variances from degree 51 to 150 from EGM96, with scale factor equal to 0.2689, and $R = R_E - 1.061\text{km}$, $A = 285.0\text{mgal}^2$. A common standard deviation of the observations equal to 0.0134 E was adopted for the data. The correlated errors affecting the data were represented by a finite error covariance function (Sansó, & Schuh 1987) with noise variance equal to $(0.01337\text{E})^2 = 0.0001787\text{E}^2$ and correlation distance equal to 7 degrees.

Spherical harmonic coefficients were computed from degree 51 to 90 and compared with EGM96 coefficients.

The numerical experiments were carried out collecting data in different ways. In all cases the data were retaining their original positions on the GOCE real orbit. In first experiment data collected as close as possible (up to 50 km) to the nodes of the 2° equal-area grid (9,686 point T_{rr}). The results in terms of the standard deviation of the differences between computed and true coefficients relative to mean collocation error estimation are shown in Fig. 3.

The same experiment was repeated using data collected as close as possible to a $2^\circ \times 2^\circ$ equiangular grid. Allowing a maximum distance of 50km for the collection the number of data points is 14,131. The results of this experiment are shown in Fig. 4.

Comparing to the previous experiment with the equal-area grid (Fig. 3), the results are very similar, although in the case of the equiangular grid 46% more data points were used.

Up to now, it was numerically shown that using T_{rr} data with white or correlated noise, a distribution based on an equal-area grid results in considerably better quality of the predicted coefficients with the same amount of data points.

As it was discussed in section 1, the aim of the paper was the computation of spherical harmonic coefficients with the best possible quality, using the minimum number of data points. Therefore, the data set must consist of the

Table 3. Standard deviation of the differences computed - true (A) and the corresponding mean collocation error (B) of the harmonic coefficients determined using: 10,448 points on the nodes of a 2° equal-area grid (columns 2 and 3 of Table 1), 8,500 ponts on realistic GOCE orbit (columns 4-5) and 9,686 on realistic GOCE orbit (columns 6-7). All numbers are multiplied by 10^9.

	10,448 points on 2° equal-area grid		8,500 points on GOCE real orbit (collection radius 30km)		9,686 points on GOCE real orbit (collection radius 50km)	
Degree	A	B	A	B	A	B
25	3.693	2.789	11.216	7.655	5.151	4.710
26	3.285	2.698	10.657	7.379	4.906	4.544
27	2.800	2.617	13.612	7.576	5.970	4.451
28	3.251	2.545	11.530	7.330	5.664	4.316
29	2.699	2.481	11.618	7.230	5.705	4.251
30	3.389	2.425	10.447	7.015	5.603	4.137
31	2.656	2.375	9.556	6.926	4.915	4.106
32	2.961	2.330	8.851	6.752	4.883	4.011
33	3.029	2.292	9.163	6.657	5.217	3.983
34	3.359	2.258	9.915	6.499	5.352	3.899
35	3.231	2.228	9.444	6.452	4.324	3.886
36	2.689	2.203	8.387	6.320	4.258	3.815
37	3.201	2.182	7.846	6.296	4.215	3.818
38	2.860	2.164	8.022	6.200	4.286	3.764
39	2.928	2.150	8.994	6.174	4.441	3.772
40	2.654	2.139	8.180	6.096	4.075	3.728

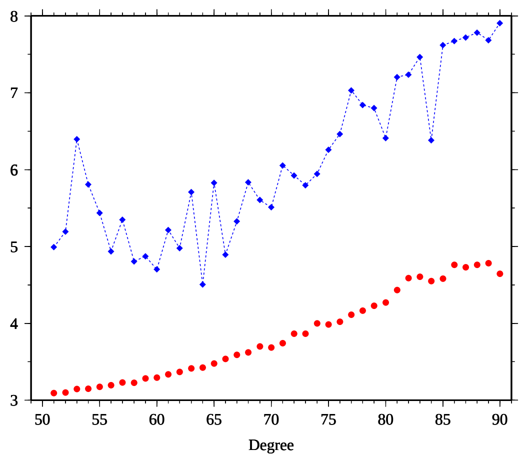

Figure 3. Standard deviation per degree of the differences between computed and true coefficients from degree 51 to 90 (blue) and mean collocation error estimates for the corresponding degrees (red). The coefficients were computed using 9,696 T_{rr} values selected as closest to the nodes of a 2° equal-area grid. All numbers are multiplied by 10^9.

most representative values of the gravity field.

For this reason the idea to collect data using the criterion that each of them should be the point with the maximum absolute value of T_{rr} ($\max|T_{rr}|$) within the block was examined in the two next experiments. More specifically, in the first experiment the data were collected as one point per 2° equal-area block with the $\max|T_{rr}|$. In the second experiment the data were collected as in the first one, but in blocks with variance of T_{rr} greater than $0.01\mathrm{E}^2$ ($\mathrm{var}(T_{rr}) > 0.01\mathrm{E}^2$), instead of the point with $\max|T_{rr}|$, the 4 data points closest to the middle of the four 1° sub-blocks were selected, increasing in this way the number of input data from 10,218 to 15,340. The philosophy behind this selection was already explained at the beginning of the paper.

The limit of $0.01\mathrm{E}^2$ in variance was chosen arbitrarily to be about one order of magnitude larger than the common standard deviation of the observations ($0.0134\mathrm{E}$). From the amount of 10,482 equal-area blocks only 1,793 have $\mathrm{var}(T_{rr}) > 0.01\mathrm{E}^2$.

The results of these experiments are shown in Fig. 5. The red line represents the standard deviation between computed and true coefficients resulted from the first experiment (10,218 data points) and the green line the corresponding standard deviation resulted from the second experiment (15,340 data points).

Comparing the results of Fig. 3 (blue line) with that of Fig. 5 (red line), it is shown that up to degree 79 the standard deviation of the differences (computed-true) in Fig. 3 is better than the corresponding of Fig. 5 but from degree 80 up to 90, the standard deviation of Fig. 5 is better than those of Fig. 3. Generally, the results in terms of the standard deviation of the differences (computed-true) are improving with increasing degree in the case of collecting data with the criterion $\max|T_{rr}|$. Note that almost same number of data points was used in both experiments (9,686 versus 10,218).

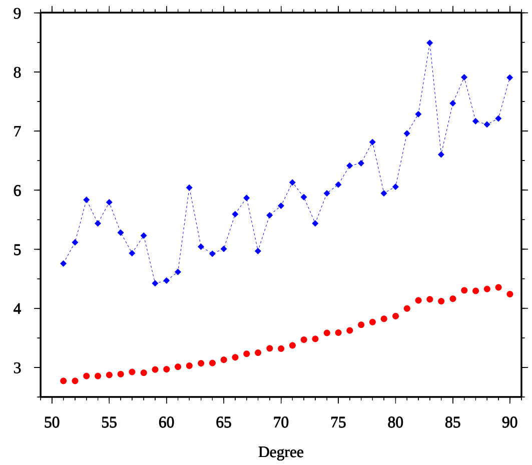

Figure 4. Standard deviation per degree of the differences between computed and true coefficients from degree 51 to 90 (blue) and mean collocation error estimation for the corresponding degrees (red). The coefficients were computed using 14,131 T_{rr} values selected as closest to the nodes of a $2° \times 2°$ equiangular grid. All numbers are multiplied by 10^9.

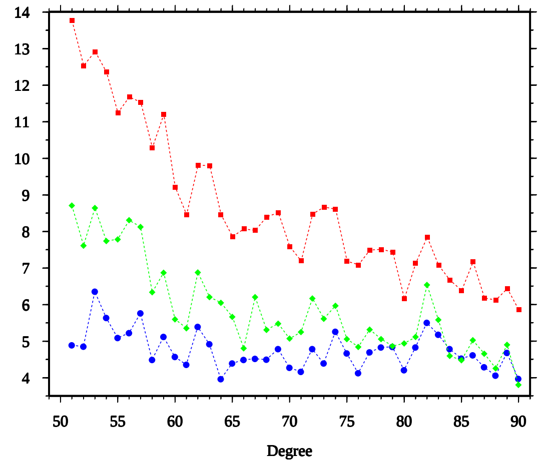

Figure 5. Standard deviation per degree of differences between computed and true coefficients from degree 51 to 90. Red: The coefficients were computed using 10,218 T_{rr} values selected using the $\max |T_{rr}|$ criterion. Green: The coefficients were computed using 15,340 T_{rr} values selected using the $\max |T_{rr}|$ and the $\mathrm{var}(T_{rr}) > 0.01\mathrm{E}^2$ criteria. Blue: The coefficients were computed using 16,601 T_{rr} values selected as closest to the nodes of $2°$ equal-area grid and the $\mathrm{var}(T_{rr}) > 0.01\mathrm{E}^2$ criterion. All numbers are multiplied by 10^9.

As it was expected the results of the second experiment with the 15,340 points (Fig. 5, green line) are considerably better than those of the first experiment with the 10,218 points (Fig. 5, red line). But the results of Fig. 3 (blue line) up to degree 64, are still better than those of the Fig. 5, (green line), in spite of the large difference of data points used in the two experiments (9,686 versus 15,340). A plausible interpretation is that for the lower degrees it is important to have a regular distribution of the data, while for the higher degrees the density of the data is critical especially in blocks with a rough gravity field.

This was numerically verified by a new experiment (3), with data selected closest to the nodes of the $2°$ equal area grid for blocks with $\mathrm{var}(T_{rr}) \leq 0.01\mathrm{E}^2$ plus data selected closest to the centers of the four $1°$ sub-blocks, in which each $2°$ block fulfilling the criterion $\mathrm{var}(T_{rr}) > 0.01\mathrm{E}^2$, was divided. An amount of 16,601 data points was used in this experiment. The results are shown in Fig. 5 (blue line). In this case, where both preconditions are fulfilled (evenly distribution of the data, and sufficient density of the data in blocks with rough gravity field), the standard deviation per degree between computed and true coefficients is more homogeneous than in all other experiments of section 3.

The determined coefficients in the experiment (3) were used to compute T_{rr} values at the positions of the ESA/SGG, consisting of 509,222 data points. The same computation was repeated using EGM96 model. The results of these computations in terms of the statistics of the differences between (a) ESA/SGG and EGM96 as well as between (b) ESA/SGG and the coefficients of experiment (3) are shown in Table 4. The distribution of the differences between (a) and (b) are shown in Fig. 6 (upper) and (lower) respectively.

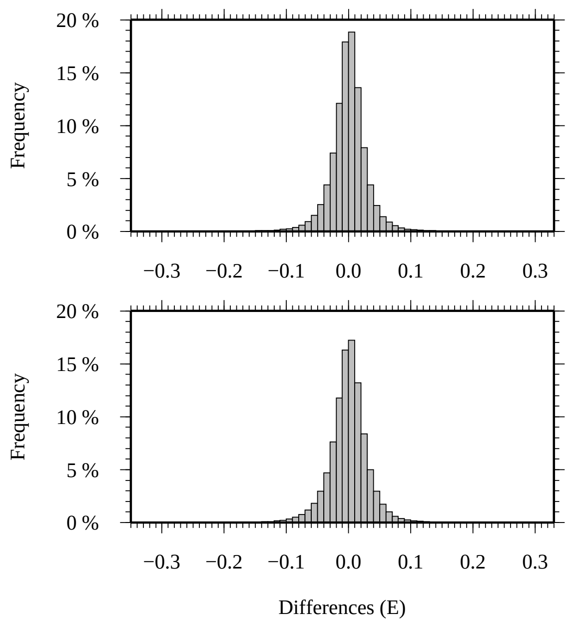

Figure 6. Distribution of the differences between T_{rr} from ESA/SGG and T_{rr} computed from EGM96 (upper) and T_{rr} computed from the coefficients determined from the experiment 3 (lower)

Table 4. Comparison between ESA/SGG and T_{rr} computed from EGM96 and from the model resulted from the experiment 4 to degree 90. The number of observations is 509,222. Unit is E.

		ESA/SGG - T_{rr} from EGM96		ESA/SGG - T_{rr} from computed coefficients to degree 90	
	Observations	Predictions	Difference	Predictions	Difference
Mean value	-0.0049	-0.0048	-0.0001	-0.0048	-0.0001
Standard deviation	0.2356	0.2319	0.0309	0.2358	0.0320
Maximum value	1.7847	1.9462	0.3280	2.0038	0.3082
Minimum value	-1.5811	-1.5165	-0.3489	-0.0048	-0.0001

There is a small difference between (a) and (b) in terms of the standard deviation of the residuals equal to 0.0011E. It could be considered as negligible, taking into account the standard deviation (0.01E) of the observations as well as the characteristics of their correlated noise, described at the beginning of section 3. Furthermore, the distribution of the differences is very similar, as it is shown in Fig. 6, with a higher concentration (of about 1900 differences) around zero in the case of Table 4(A).

4. CONCLUSION

Numerical experiments for the determination of a geopotential model by LSC using simulated GOCE data indicate that a distribution of the data based on an equal-area grid is superior compared to a distribution based on an equiangular grid, because using considerably fewer data points it results in better quality of the coefficients.

Since a global covariance function is to be used, a homogenization of the data is necessary.

In order to take advantage of the possibility of LSC to use the data in their original positions on the real GOCE orbit, it is essential to have a distribution providing a homogeneous quality of the computed coefficients. The numerical experiments indicate that for the lower degrees an evenly distribution of the data closest to the nodes of the equal-area grid is necessary, while for the higher degrees the amount of the data in blocks with rough gravity field is critical.

This issue does not support selection of data points with other criteria, such as to have the maximum absolute value within the equal-area block, because the distribution of such points could be not even.

The excellent agreement between EGM96 and the coefficients computed according to the suggested strategy was reflected also in the agreement of the residuals of T_{rr} values from the noisy ESA/SGG, reduced to EGM96 and to predicted coefficients up to degree 90.

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