Calibration of GOCE gravity gradient data using smooth ground gravity.

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Abstract. Terrestrial gravity anomalies selected from three extended continental areas having a smooth gravity field have been used to determine the appropriate size of the area for gravity data collection as well as the data-sampling required for an external calibration of the GOCE gravity gradient data. The gravity data and a global gravity field model (EGM96) were used as input to least-squares collocation (LSC) which was used to calculate the error of predicted gravity gradient components at points on a realistic orbit of a simulated data set. The mean error showed that gravity gradient components could be predicted with an error of 2-3 mE (i.e. much below the expected error in the measurement band-width) in the case of an optimal size of the collection area and of the optimum resolution of the data. These optimal conditions e.g. for a smooth area inside Australia, correspond to an $10^0 \times 12^0$ area extend and a 5 arc minute data-sampling.

Using a combination of EGM96, ground gravity and satellite gravity gradients the error of bias, tilt and scale factor parameters for the 3 gradient component with the smallest expected error on orbits crossing the 3 areas were estimated using different error covariance functions. The results showed a strong dependence on the correlation distance of the error-covariance functions. However, the error-estimates show that a satisfactory estimation of the parameters is possible for error-correlation distance smaller than 1 degree.

Keywords. Gravity gradients, calibration, GOCE

1 Introduction

Let us denote the gravity potential of the Earth (without centrifugal force) by *V*. Gravity gradients are then the 6 second order derivatives of *V* in a suitable local Cartesian coordinate system with axes (x, y, z). If a reference potential is subtracted the so-called anomalous potential (*T*) is obtained and values denoted T_{xx}, T_{xy}, T_{yy} etc are obtained.

The GOCE satellite (ESA, 1999) will carry a 3 axis gradiometer which will measure all components of the gravity gradients. The gradients are affected by systematic and random errors, resulting in an error power-density spectrum (PSD), and associated error-covariance functions with correlation distances longer than 10 .

The GOCE gravity gradients have to be calibrated in order to reduce the influence of errors outside the measurement band-width (systematic errors). Such errors may be due to bias, drift and scale-factor uncertainties. We will show that such errors may be detected (and later corrected for) when ground gravity data can be used to compute the gradients with an error smaller than expected noise in the measurement band-width (around 10 mE) and if the error and error correlation distance of the gravity gradients are small (0.5 – 1.0 degrees, or 7 – 15 s if considered as time-wise).

Many methods have been proposed for the estimation and reduction of systematic errors, see e.g. (Bouman et al., 2004, Wolf and Denker, 2005). The methods in general take advantage of a-priori data either in the form of ground data or using gradients computed from series of spherical harmonic coefficients like EGM96 (Lemoine et al., 1998).

In (Arabelos and Tscherning, 1998) it was proposed to use as spherical harmonic gravity model in combination with ground data from areas with a very smooth gravity field. Tests were carried out using an area in Canada and we have here carried out similar tests using the more precise information about the orbit of GOCE and the error characteristics of the gradiometer which has become available. Also the numerical magnitude of the (full) gradients must be used in the equation determining scale factors, so other areas with different latitude-extend are needed. This has lead us to consider areas in Scandinavia and in Australia. We will in the following describe how the method of least-squares collocation (LSC), (Moritz, 1978), may be used to estimate bias, drift and a scale-factor such parameters using ground data combined with the data to be calibrated.

2 LSC and parameter estimation

An observed gravity gradient y_i associated with the anomalous potential T through a linear functional L_i may be expressed through the equation

$$y_i = L_i(T) + e_i + A_i^T \cdot X$$

with

- Parameters X equal to bias, tilt and contingently scale factor for each observation and each track or part of a track.
- A_i = 1 if bias only present
- A_i = time-difference if tilt/drift is present
- A_i = L_i (V) (full potential) if scale-factor present

(Observed ground data or spherical harmonic coefficients may also be expressed using the same basic equation.)

In the following we will primarily deal with the gradient components with the smallest (expected) error. They are the second order derivatives T_{xx}, T_{yy} and T_{zz} in the gradiometer reference frame, which is approximately aligned with the along-track direction (x), the cross-track direction (y) and approximately radial (z).

Then an estimate of T and of the parameters X are obtained as

$$\tilde{T}(P) = \{C_{Pi}\}^T \overline{C}^{-1}(y - A^T X)$$

$$\tilde{X} = (A^T \overline{C}^{-1} A + W)^{-1}(A^T \overline{C}^{-1} y)$$

where W is the a-priori weight matrix for the parameters (generally the zero matrix).
C_{Pi} is the covariance between the i-th observation and the value of T in a point P and
$\overline{C}$ the variance covariance of the observations with the error-covariances D added.

$$\overline{C} = C + D$$
$$C_{ij} = COV(L_i, L_j) = L_i(L_j(COV(T(P), T(Q))$$

The observation vector y may contain any combination of ground and satellite data.

Predicted values are obtained by applying the associated linear functional (L) on the estimate of T. Error estimates may also be computed. The mean square error of the parameter vector becomes

$$m_X^2 = (A^T \overline{C}^{-1} A + W)^{-1}$$

and using

$$H = \{COV(L, L_i)\}^T \overline{C}^{-1}$$

the mean square error of an estimated quantity $L(\tilde{T})$ (such as T_{zz}) will be

$$m_L^2 = \sigma_L^2 - H\{\mathrm{cov}(L, L_i)\}H\ Am_X^2\ (HA)^T$$

Hence least-squares collocation with parameters may be used to combine ground gravity and GOCE gravity gradients and to determine related physical parameters and their error-estimates. The above equations have been implemented in the GRAVSOFT (Tscherning et al., 1992, Tscherning, 2003) program GEOCOL, which has been used in the calculations presented here.

3 The covariance functions

The (signal) covariance function is expressed as a Legendre series in two parts, equivalent to using a reference field with appropriate error-degree-variances, σ_{ei}, P_i are Legendre polynomials, ψ the spherical distance between P and Q and r, r' are the radial distances of P, Q respectively from the origin. A is a constant and N=360 since we are using EGM96 to this degree.

$$COV(T(P), T(Q) = COV(\psi, r, r') =$$
$$\sum_{i=2}^{N} \sigma_i^e \left(\frac{R^2}{rr'}\right)^{i+1} P_i(\cos\psi)$$
$$+ \sum_{i=N+1}^{\infty} \frac{A}{(i-1)(i-2)(i+4)} \left(\frac{R^2}{rr'}\right)^{i+1} P_i(\cos\psi)$$

The noise covariances should be derived from the error power spectral density (PSD), after initial calibration steps and it is unknown at this time. We have modelled it using a noise standard deviation of 0.010 E for T_{xx} and 0.012 E for T_{yy}, T_{zz} and varying noise correlation (50%) distances, see Figure 3. The (x, y, z) frame refers to the gradiometer reference frame introduced above. The error of T_{xx} will probably be the smallest, since the drag-compensation is along the x-axis.

4 Selection of calibration areas

The error of estimated quantities is proportional to the gravity field signal standard-deviation if the noise does not dominate. However the gravity field is not statistically homogeneous. Consequently we should select areas with low signal standard deviation in order to get the best estimates. Such areas are areas with smooth topography, and we have selected the Canadian Plains, South Scandinavia and the central part of Australia. The areas have also been selected because data is available with a gravity error below 2 mgal, height or height datum induced systematic error are estimated to be below 0.3 mgal and they cover different latitude intervals.

We also have available spherical harmonic coefficients, which may be used to represent the gravity field outside the selected areas. The EGM96 model has been used in a remove-restore procedure, which is equivalent to using the coefficients (without error-correlations) as observations in LSC. The subtraction also lowers the signal standard deviation in the area to values around 100 $mgal^2$.

The empirically estimated covariance function of such differences is shown together with its analytic representation from one of the areas in Figure 1.

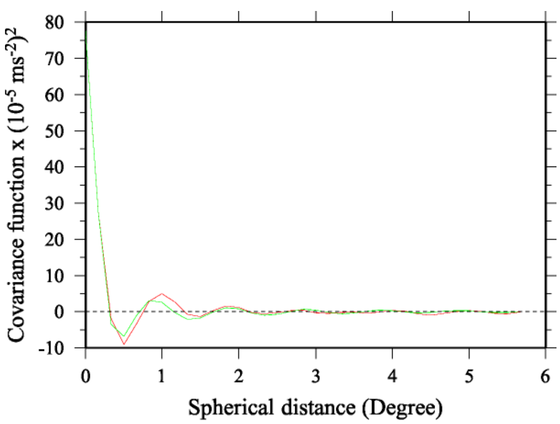

Figure 1 Empirical (red) and analytic (green) covariance of EGM96 reduced gravity anomalies. Note the very low signal variance of 80 $mgal^2$.

5 Data selection

One limitation in the use of LSC is that a number of equations equal to the number of observations have to be solved. However not all data are needed in order to have a satisfactory estimate of the parameters. We have studied the needed size of the area and the density of the data by systematically varying these parameters. As a criteria we have used that the estimation error should be considerably smaller than (25 %) the noise in the measurement band-width, which is expected to around 10 mE. Based on Table 1 and 2, we see that an area size of 10 x 12 and a 5 arcminute spacing fulfil this requirement.

Table 1. Values of the error of the second order derivative, T_{zz} predicted at satellite altitude from gravity anomalies as a function of area-size:

Size (decimal degree)	Error estimate (mE)
5.0 × 6.0	4.3
7.0 × 8.0	2.9
9.0 × 10.0	2.3
10.0 × 12.0	2.2

Table 2. Dependence of the Error of T_{zz} on sampling interval.

Interval Arcmin	Error estimate (mE)
5.0	2.5
7.5	2.6
10.0	2.9
15.0	3.7
20.0	4.6

More details are given in (Arabelos et al., 2005).

6 Simulations combining ground gravity and gravity gradient data

In simulations, where ground gravity was combined with gravity gradients at satellite altitude, we used ESA delivered orbit positions and noise-free quarternions defining the attitude, i.e. the rotations between the instrument (x, y, z) system and an East-North-Radially Up system. Error-estimates of bias, tilt and scale-factors were computed combining ground gravity and satellite data:

The simulation results will depend on how many tracks cross the area. An example of number of 5 s sampled data on crossing tracks is shown in Figure 2.

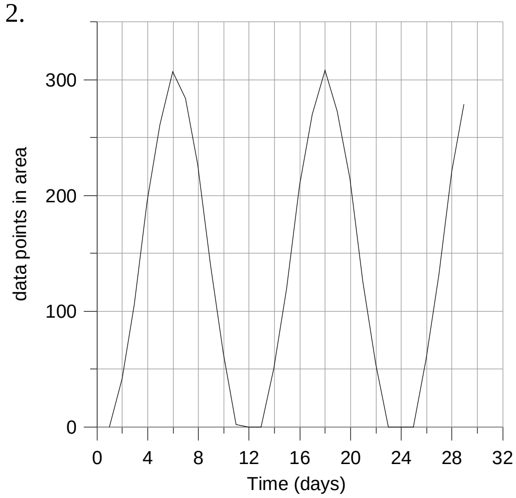

Figure 2 Typical number of ground-tracks per day

The error of estimated parameters will depend strongly on the noise correlation, as seen in Figures 3 and 4.

Using an error-correlation distance of 0.5 deg., the results in Table 3 were obtained.

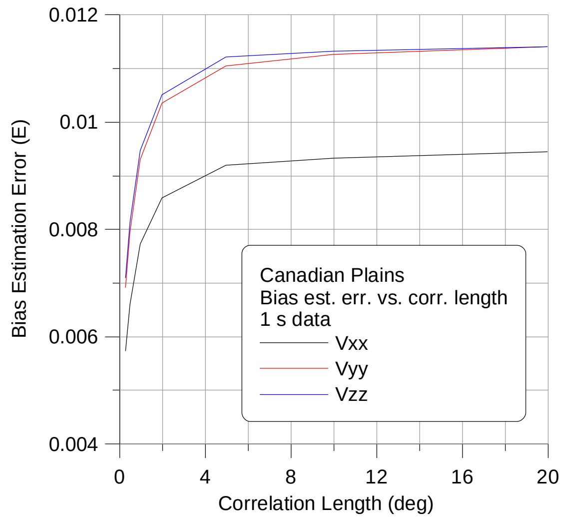

Figure 3 Error estimate of bias-parameters as a function of error correlation distance.

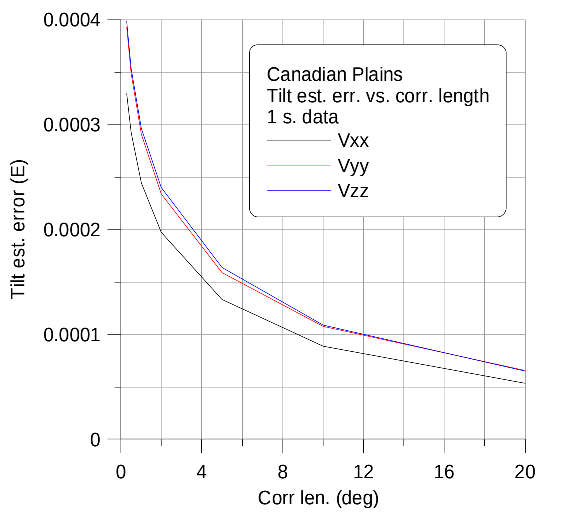

Figure 3 Error estimate of tilt-parameters as a function of error correlation distance.

7 Conclusion

Collocation with parameters may be used to estimate bias, tilt and scale factors with an error depending on the error variance and correlation distance of the error-covariance function. With an error-correlation distance below 1.0 deg., biases can be estimated for all 3 components. Considering that it will last more than 100 s to overfly the areas, it is also possible to estimate the tilt or drift. For longer correlation distances the estimate of the tilt improves. This can be understood as that the long error-correlation is caused by the tilt.

Table 3. Results of the estimation of systematic parameters

Component	Bias (EU)	Tilt (EU/s)	Scale factor
	Canadian plains		
T_{xx}	0.0064	0.000250	0.000003
T_{yy}	0.0078	0.000298	0.000004
T_{zz}	0.0080	0.000303	0.000002
	Central Australia		
T_{xx}	0.0042	0.000138	0.000003
T_{yy}	0.0052	0.000167	0.000003
T_{zz}	0.0052	0.000167	0.000002
	Scandinavia		
T_{xx}	0.0029	0.000143	0.000003
T_{yy}	0.0037	0.000175	0.000004
T_{zz}	0.0035	0.000170	0.000002

The computation of error-correlations showed, however, that bias and scale factor estimates were 100 % correlated.

The absolute value of the gravity gradients is of the order of 3000 E. So it is also possible to estimate a scale factor. However, it will be impossible to distinguish between a scale factor and a bias.

A global calibration is planned to be carried out by partners in the so-called High Level Processing Facility (HPF), cf. Bouman et al. 2004. Consequently the above described procedure may be used to verify this calibration if reasonably small variances and correlation distances are obtained using the corrected data obtained as a result of the global calibration.

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