Error Characteristics of Dynamic Topography Models Derived from Altimetry and GOCE Gravimetry

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Abstract. The impact of the GOCE satellite mission on the recovery of the gravity field is analysed for two simulated cases. In the first case the GOCE Level 2 product is used where the gravity field is approximated by spherical harmonic coefficients up to degree and order 200. In the second case synthetic GOCE Level 1B data are used directly in a gravity field determination using least squares collocation. In case two the full spectrum geoid error was improved from 31 cm to 15 cm and the resolution was doubled.

To get reliable errors associated with the mean dynamic topography (MDT) a reliable model for the spectral characteristics of the MDT is needed. Such a model was derived reflecting empirically derived properties such as MDT variance and correlation length. Combining the MDT characteristics with the estimated geoid errors in the spectral domain resulted in a-posteori error estimates. In the two cases the MDT errors were improved from 20 cm to 6 cm and 5 cm respectively. For the geostrophic surface current components the errors were improved from 23 cm/s to 18 cm/s and 16 cm/s.

1 Background

The GOCE (Gravity and Ocean Circulation Experiment) satellite mission by the European Space Agency is planned to improve the knowledge about the Earth gravity field and to improve the modelling of the ocean circulation. The central quantity associated with the ocean circulation and transport is the mean dynamic topography (MDT), which is the difference between the mean sea surface (MSS) and the geoid. The MDT provides the absolute reference surface for the ocean circulation.

In the EU project GOCINA (Geoid and Ocean Circulation In the North Atlantic) the use of GOCE data is prepared by developing methodology for accurate mean dynamic topography modelling in the region between Greenland and the UK (Knudsen et al., 2004). The improved determination of the mean circulation will advance the understanding of the role of the ocean mass and heat transport in climate change. The GOCINA project will support the GOCE mission in a distinct case, namely to educate and prepare the community in using GOCE data for oceanography including sea level and climate research as well as operational prediction.

In this study, the impact of GOCE on the mapping of the gravity field is studied. Furthermore, its impact on the estimation of the MDT is analysed.

2 GOCE simulations

The simulations of GOCE impact on the gravity field recovery are done using the full spectra of the signals and the errors. Hence, both commission and omission errors are taken into account when, e.g., a spherical harmonic expansion truncated at a certain harmonic degree and order is considered. This is important because the omission error usually is larger that the commission error.

Using spherical harmonic functions signal and error covariances associated with the gravity field between points P and Q may be expressed as a sum of Legendre's polynomials multiplied by degree variances. That is

$$K(\mathrm{P,Q}) = \sum_{\mathrm{i}=2}^{\infty} \sigma_{\mathrm{i}}^{\mathrm{TT}} \mathrm{P_i}(\cos\psi) \qquad (1)$$

where σ_i^{TT} are degree variances associated with the anomalous gravity potential field and ψ is the spherical distance between P and Q. Hence, eq.(1) only depends on the distance between P and Q and neither on their locations nor on their azimuth (i.e. a homogeneous and isotropic kernel). Expressions associated with geoid heights and gravity anomalies are obtained by applying the respective functionals on K(P,Q), e.g. $C_{NN}=L_N(L_N(K(P,Q)))$ (more on collocation by Sansò, 1986, Tscherning, 1986). That is

$$C_{NN} = \sum_{i=2}^{\infty} \left(\frac{1}{\gamma}\right)^2 \sigma_i^{TT}\, \mathrm{P_i}(\cos\psi) \tag{2}$$

$$C_{\Delta g \Delta g} = \sum_{i=2}^{\infty} \left(\frac{i-1}{R}\right)^2 \sigma_i^{TT}\, \mathrm{P_i}(\cos\psi) \tag{3}$$

$$C_{N\Delta g} = \sum_{i=2}^{\infty} \left(\frac{i-1}{\gamma R}\right) \sigma_i^{TT}\, \mathrm{P_i}(\cos\psi) \tag{4}$$

The determination of the degree variances is essential to obtain reliable and useful signal and error covariance functions. For the gravity field it has been accepted that the degree variances tend to zero somewhat faster than i^{-3} and that the Tscherning-Rapp model (Tscherning & Rapp, 1974) may be used as a reliable model. This expression has the advantage that the kernel can be evaluated using a closed expression instead of the infinite sum. When a spherical harmonic expansion of the gravity field up degree and order N has been used as a reference model and, hereby, been subtracted from the quantities, then the associated error degree variances should enter the expression, eq. (1), up to harmonic degree N. That is

$$\sigma_i^{TT} = \begin{cases} \varepsilon_i & i = 2,\ldots,N \\ \dfrac{A}{(i-1)(i-2)(i+4)} \left(\dfrac{R_B^2}{R^2}\right)^{i+1} & i = N+1,\ldots \end{cases} \tag{5}$$

where A = 1544850 m^4/s^4, $R_B = R - 6.823$ km were found in an adjustment so that agreement with empirical covariance values calculated from marine gravity data was obtained. This procedure is described in Knudsen (1987*a*). The error degree variances, ε_i, are associated with the errors of the reference model.

2.1 GOCE Level 2 Product

The standard Level 2 product coming from GOCE is a spherical harmonic expansion to degree and order 200 that can fulfil the aim of the satellite mission, which is to model the geoid at a resolution of 100 km with an accuracy of 1-2 cm. Based on mission parameters and extensive simulations it has been demonstrated that GOCE will meet those requirements (e.g. Visser et al., private communication). An important outcome of the simulations is a set of error degree variances that may be included as commission errors in other simulations of the GOCE performance.

The unmodelled part of the gravity field remains unknown and will be considered as the omission error. This omission error will be modelled using the Tscherning-Rapp model from degree 201 and up. Hence, the full error of the GOCE level 2 harmonic expansion as an approximation of the gravity field consists of both the commission and the omission errors. The degree variances are shown in Figure 1. The error covariance function is shown in Figure 2. It has a variance of $(0.31\ \mathrm{m})^2$ and a correlation length (which is the distance where the covariance is 50 % of the variance) of about 0.3°.

2.2 GOCE Collocation Product

As part of the simulations of the GOCE performance alternative methods such as least squares collocation have been tested (Tscherning, 2004). In this study a test was carried out using simulated GOCE Level 1B observations in the GOCINA region. Here least squares collocation was used to estimate the geoid in the region using the following expression

$$x = C_x^T \left(\mathrm{C} + \mathrm{D}\right)^{-1} \mathrm{y} \tag{6}$$

where C and D are covariance matrices associated with the signal and the errors of the observations y. x is the estimated quantity.

Then error covariances were estimated using

$$\hat{c}_{x'x} = c_{x'x} - C_x^T \left(\mathrm{C} + \mathrm{D}\right)^{-1} \mathrm{C}_{x'} \tag{7}$$

where $c_{xx'}$ is the *a-priori* (signal) covariance between x and x' (see e.g. Moritz, 1980).

The results show that the estimated errors range from 10 cm to 15 cm. Note that this is full spectrum errors demonstrating that the this approach will give a significant improvement of the geoid. The estimated error covariances show that the errors are associated with scales shorter that half a degree. Hence, the resolution appears to have been doubled.

For the subsequent simulations of the GOCE performance in terms of modelling a MDT a degree variance model was obtained by extending the GOCE part of the previous degree variance model to harmonic degree 360 (also shown in Figure 1). This model give an error covariance function (also shown in Figure 2), which has a variance of $(0.15\ \mathrm{m})^2$ and a correlation length of about 0.1°. Hence, compared to the standard GOCE Level 2 product, a significant improvements may be obtained by using the GOCE data directly in a determination of the gravity field using least squares collocation.

3 Modelling the Signal Characteristics of the Mean Dynamic Topography

To get reliable results of simulations and tests carried out using least squares methods it is important that both the signal and the error characteristics have been taken into account. In least squares collocation that means that the covariance function models should agree with empirically determined characteristics such as the variance and correlation length. In analysis of errors formally estimated using eq.(7), it is very important that those quantities are reliable. That is also the case when MDT errors are analysed. Hence, a model describing the magnitude and the spectral characteristics of the MDT is needed.

A kernel function associated with the MDT, may be expressed in a similar manner as the gravity fields as

$$C_{\zeta\zeta} = \sum_{i=1}^{\infty} \sigma_i^{\zeta\zeta}\, \mathrm{P_i}(\cos\psi) \tag{8}$$

where the degree variance in this expression are associated with the MDT, naturally.

The degree variance model was constructed using 3rd degree Butterworth filters combined with an exponential factor (e.g. Knudsen, 1991). Hence, the spectrum of the MDT is assumed to have similar properties as the geoid spectrum; same type of smoothness and infinite. That is

$$\sigma_i^{ss} = b \cdot \left(\frac{k_2^3}{k_2^3 + i^3} - \frac{k_1^3}{k_1^3 + i^3} \right) \cdot s^{i+1} \tag{9}$$

where b, k_1, k_2, and s are determined so that the variance and the correlation length agree with empirically derived characteristics.

Since geostrophic surface currents are associated with the slope of the MDT, it may be shown how covariance functions associated with the geostrophic surface currents can be obtained. If accelerations and friction terms are neglected and horizontal pressure gradients in the atmosphere are absent, then the components of the surface currents are obtained from the MDT by

$$u = \frac{-\gamma}{f\,R}\frac{\partial\zeta}{\partial\phi},\quad v = \frac{\gamma}{f\,R\cos\phi}\frac{\partial\zeta}{\partial\lambda} \tag{10}$$

where f=2$\omega_e$sinφ is the Coriolis force coefficient.

Expressions associated with the geostrophic surface currents and the MDT depend on the azimuth between the two points P and Q, α_{PQ}. That is (Knudsen, 1991)

$$C_{uu} = \frac{\gamma^2}{f_P f_Q}\left(-\cos\alpha_{PQ}\cos\alpha_{QP}\, C_{ll} - \sin\alpha_{PQ}\sin\alpha_{QP}\, C_{qq}\right) \tag{11}$$

$$C_{vv} = \frac{\gamma^2}{f_P f_Q}\left(-\sin\alpha_{PQ}\sin\alpha_{QP}\, C_{ll} - \cos\alpha_{PQ}\cos\alpha_{QP}\, C_{qq}\right) \tag{12}$$

$$C_{u\zeta} = \frac{-\gamma}{f_P}\cos\alpha_{PQ}\, C_{l\zeta} \tag{13}$$

$$C_{v\zeta} = \frac{\gamma}{f_P}\sin\alpha_{PQ}\, C_{l\zeta} \tag{14}$$

where

$$C_{ll} = \tfrac{1}{R^2}\left(\cos\psi\, C'_{\zeta\zeta} - \sin^2\psi\, C''_{\zeta\zeta}\right) \tag{15}$$

$$C_{qq} = \tfrac{1}{R^2}\, C'_{\zeta\zeta} \tag{16}$$

and

$$C_{l\zeta} = \tfrac{-1}{R}\sin\psi\, C'_{\zeta\zeta} \tag{17}$$

With those expressions it is possible to estimate geostrophic surface currents using collocation. They also give a very important constraint on the modelling of the degree variance model.

In Knudsen (1993) the parameters in the degree variance mode, eq.(9), were fitted iteratively to the empirical covariance values. This resulted in the model where b = 6.3 10^{-4} m^2, k_1 = 1, k_2 = 90, s = $((R\text{-}5000.0)^2/R^2)^2$. The variance and correlation length are $(0.20\ m)^2$ and 1.3° respectively. The variance and correlation length of the current components are $(0.16\ m/s)^2$ and 0.22° respectively.

To study the properties of the degree variance model in more detail characteristic parameters associated with the MDT and its associated geostrophic surface current components were derived. This was done for block averages of varying block sizes, because those numbers may be compared with output parameters from ocean circulation models with different grid sizes and resolutions.

The MDT signal covariance properties were computed rigorously using the series of Legendre's polynomials to which the smoothing operators associated with the running averages have been applied. That is

$$C_{\zeta\zeta} = \sum_{i=1}^{\infty} \beta_i^2(s)\, \sigma_i^{\zeta\zeta}\, \mathrm{P_i}(\cos\psi) \qquad (18)$$

where the beta factors are the so-called Pellinen operators that depend on the side length, s, of the cells.

The covariance functions associated with the MDT and with the geostrophic surface current components (represented by the C_{ll}) and averaged in cells of ½, 1, and 2 degree were computed. The resulting variances and correlation lengths are summarized in Table 1. It is important to emphasize that those numbers are statistical expected values representing a region as the GOCINA region. They are not representative for the strong Western boundary currents as the Gulf Stream.

Table 1. Standard deviations in cm and cm/s and correlation lengths in degrees of MDT and geostrophic surface current components (u,v) as point values and averaged in cells.

	MDT		**(u,v)**	
	St.dev.	C.length	St.dev.	C.length
Points	20	1.3°	16	0.22°
½° x ½°	19	1.4°	13	0.38°
1° x 1°	18	1.6°	10	0.56°
2° x 2°	16	2.1°	6	0.85°

4 Modelling A-posteori Mean Dynamic Topography Error Characteristics

Combining the MDT signal degree variances and the geoid error degree variances it may be provide information about the a-posteori errors of an MDT that has been estimated using the geoid and a mean sea surface computed from satellite altimetry. Using least squares to estimate the MDT by degree its error degree variance is expressed as

$$\frac{1}{\hat{\sigma}_i^{\zeta\zeta}} = \frac{1}{\sigma_i^{\zeta\zeta}} + \frac{1}{\sigma_i^{NN}} \qquad (19)$$

where the errors of the mean sea surface have been ignored since they are very small compared to the geoid errors.

For both GOCE simulations the a-posteori MDT error degree variances were computed. Subsequently, error covariance functions for the MDT and the surface current components were computed and their variances were found. The results are summarized in Table 2.

By comparing the numbers in Table 1 with the numbers in Table 2 it is obvious that the GOCE satellite mission will have a large impact on the estimation of the MDT. With the Level 2 product the error of point values is brought down substantially from 20 to 6 cm. The current components are associated with shorter wavelengths and moderately improved from 16 to 13 cm/s. For 1 x 1 degree averages however, the current components is improved from 10 to 4 cm/s.

The solution obtained using least squares collocation improved the recovery of the geoid substantially. The impact on the estimation of the MDT is not that pronounced, since most of the signal contents in the MDT are more long wavelength character. However, the MDT is improved at point values and ½ x ½ degree averages by about 20 %. Here the improved wavelengths has a larger impact of the current components that compared to the signal standard deviations have been improved twice as much almost.

Table 2. A-posteori errors in cm and cm/s of MDT and geostrophic surface current components (u,v) as point values and averaged in cells as estimated using the two GOCE simulations; to harmonic degree 200 and 360 respectively.

	MDT		**(u,v)**	
	200	360	200	360
Points	6	5	12	11
½° x ½°	5	4	8	6
1° x 1°	3	3	4	3
2° x 2°	2	2	1	1

5 Perspectives

The impact of the GOCE satellite mission on the recovery of the gravity field has been analysed for two simulated cases. In the first case the GOCE Level 2 product is used where the gravity field is approximated by spherical harmonic coefficients up to degree and order 200. In the second case synthetic Level 1B GOCE data are used directly in a gravity field determination using least squares collocation. In case two the full spectrum geoid error was improved from 31 cm to 15 cm and the resolution was doubled. The results are important for the future users of GOCE that need the extra accuracy.

Then the impact of the improved geoid on the estimation of the MDT is analysed. To get reliable errors associated with the MDT a reliable model for the spectral characteristics of the MDT is needed. Such a model was derived reflecting empirically derived properties such as MDT variance and correlation length. This model, naturally, is purely empirical and more information needs to be collected to verify the reliability of the model characteristics.

Combining the MDT characteristics with the estimated geoid errors in the spectral domain resulted in a-posteori error estimates. In the two cases the MDT errors were improved from 20 cm to 6 cm and 5 cm respectively. For the geostrophic surface current components the errors were improved from 23 cm/s to 18 cm/s and 16 cm/s. Those results depend of the MDT a-priori degree variance model and will not be reliable unless that model is reliable. So the results may change accordingly. However, since this a-priori model actually do reflect empirically derived characteristics, they may not change that much.

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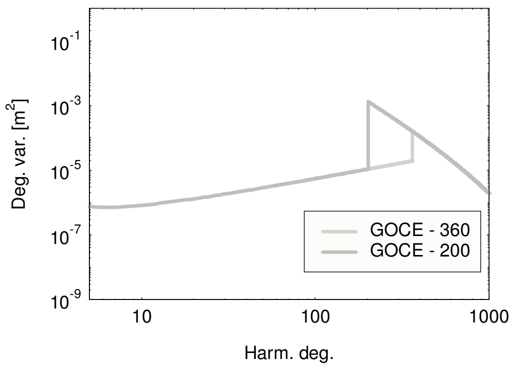

Figure 1. Geoid error degree variances associated with.GOCE Level 2 harmonic expansion to degree 200 and to degree 360 simulating the collocation solution

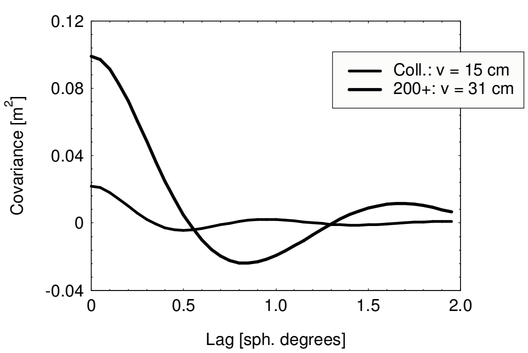

Figure 2. Geoid error covariance functions associated with.GOCE Level 2 harmonic expansion to degree 200 and to degree 360 simulating the collocation solution

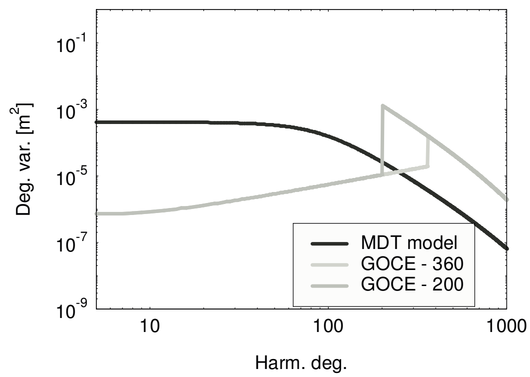

Figure 3. MDT signal degree variance model shown together with the geoid error degree variances associated with.GOCE Level 2 harmonic expansion to degree 200 and to degree 360 simulating the collocation solution

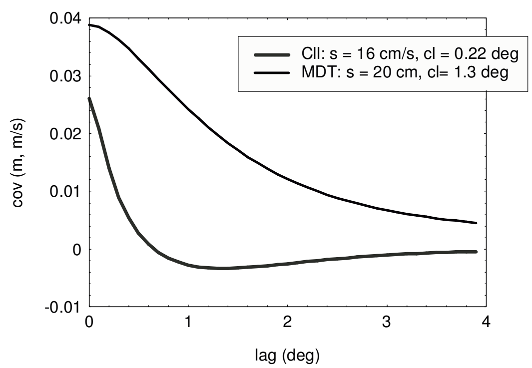

Figure 4. Covariance functions of the MDT and the geostrophic surface current components based of the MDT degree variance model shown in Figure 3.