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Improvement of gravity prediction from satellite altimetry in coastal areas using data on land.

by

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Abstract: Gravity prediction from altimetry in a coastal area has sometimes caused difficulties due to noisy data, the shape of the coastline or the sea-surface topography. Using altimeter data along the south-west coast of Norway alone, and combined with data on land it has been demonstrated that gravity prediction may be improved especially in fjords or around islands. The improvement can be seen comparing error-estimates of the predicted gravity values determined from least-squares collocation (LSC). It was not possible to evaluate the results using observed data closer than 10 km from the coast because such marine data were not available.

Key-words: Gravity, satellite radar altimetry, least-squares collocation.

1. Introduction.

The computation of gravity anomalies using satellite radar altimetry is a very active area of research, see Hwang et al., 2003. However, the computation of gravity in coastal regions have caused problems, see e.g. Hwang and Hsu, 2003. In this paper we will investigate whether we may improve gravity prediction from altimetry in coastal zones using gravity on land using the method of least-squares collocation (LSC), (Moritz, 1980). This method permit the direct combination of different data types as well at the determination of contingent bias parameters, (Knudsen, 1987, Tscherning and Knudsen, 1986). The method is implemented in the GRAVSOFT (Tscherning et al., 1992) FORTRAN program GEOCOL (http://cct.gfy.ku.dk/geocol16.pdf) with supporting programs EMPCOV (http://cct.gfy.ku.dk/empcov.pdf) and COVFIT (http://cct.gfy.ku.dk/covfit16.pdf).

The method does not – as is the case for Fourier based methods - require the availability of gridded data covering an area bounded by meridians and parallels, (see Andersen et al., 2004). We shall demonstrate that LSC using data on land in two areas studied does not give result which are superior to those obtained by the Fourier based method in the areas away from the coast.

2. Data.

The area bounded by latitude 54^o and 60^o , longitude 3^o and 11^o was used due to the availability of good quality altimetry (Fig. 1) and gravity data (Fig. 2). The gravity data from the KMS (National Survey ad Cadastre, Denmark) database updated up to 1988 was used with 5210 data selected in a 0.05^o x 0.1^o grid, see Table 1.

Table 1. Characeristics of gravity data. Row 1 is the gravity anomalies, row 2 is gravity minus the EGM96 (Lemoine et al., 1998) contribution and row 3 is the associated error estimate. The maximal

error estimate of 99 mgal is in the database used to indicate that the error is unknown (and probably large). Units mgal. Total number of points: 5210, located within an area bounded by: 54.0012 deg. and 60.0248 deg. in latitude and 3.0008 deg. and 11.0450 deg. in longitude.

	Mean	Standard deviation	Minumum	Maximum
Observed	6.81	21.70	-90.24	123.50
Observed-EGM96	-1.71	15.53	-95.85	97.50
Error estimate	3.77	15.53	0.10	99.00

Altimetry from several missions were used, see Table 2 and 3. The altimeter records received from KMS also contained a field with the value of the minimum distance to the coast.

Table 2. Altimetry mission data used. GM: Geodetic Mission, ERM: Exact repeat mission.
ERS GM
ERS ERM
GFO ERM
TP ERM
TP Tandem mission ERM
Geosat GM.

Table 3. Characeristics of altimetry. Row 1 is the altimetry minus EGM96 contribution and Row 2 is the associated error estimate. Units m. Total points: 72139, located within the area bounded by 52.0006 deg. and 59.9997 deg. in latitude and -.9998 deg. and 10.9672 deg. in longitude.

	Mean	Standard deviation	Minumum	Maximum
Data-EGM96	0.00	0.12	-0.78	0.75
Error estimate	0.05	0.01	0.02	0.13

Note that the altimetry has zero mean for the total area. But regionally there are two related problems:
(1) the altimetry data-set could be biased, (contingently biased and tilted).
(2) the altimetry data very near to the coast may have larger errors than the data at open sea.

In order to solve the first problem, we might (and have) determine a bias parameter for each altimeter track and calculated the error estimate of these parameter. In the test area (North Sea) we have found a bias for each track, but it is for nearly all tracks not significantly different from zero, see Appendix 2. (It is however remarkable that the error-estimates of the track biases decreases slightly when land gravity is used, see Appendix 2.)

For the second problem we may, using LSC, combine the altimeter data with land gravity data. It is also possible to down-weight the altimeter data close to the coast, take into account the information about reduced data quality close to the coast. For this purpose the altimetry file also contained information about the distance to the nearest coast.

We have used this to scale the weights, (scale factor 1 at the open sea, rizing to 3 at the coast from a distance of 9 km (1 - 2 measurements), see below).

3. Numerical experiments.

Gravity may be predicted using the covariances between the observations and between the quantity to be predicted and the observations, (see e.g. Andersen et al., 1996). The observations may be altimetry or gravity anomalies. Here we have throughout subtracted the contribution from EGM96 from the data (and subsequently added the contribution back when needed).

$$\Delta\tilde{g} = \{\mathrm{cov}(\Delta g, obs_i\}^T \{\mathrm{cov}(obs_i, obs_j) + \sigma_{ij}\}^{-1} \{obs_j\}$$

$$\sigma_{ij} \textit{ the noise } \mathrm{var}\textit{iance} - \mathrm{cov}\textit{ariance } (\textit{here always diagonal})$$

The error estimate is

$$error(\Delta\tilde{g} - \Delta g)^2 = \mathrm{var}(\Delta g) - \{\mathrm{cov}(\Delta g, obs_i\}^T \{\mathrm{cov}(obs_i, obs_j) + \sigma_{ij}\}^{-1} \{\mathrm{cov}(\Delta g, obs_j\}$$

This means that the error variance of the difference of an observed quantity minus a predicted quantity will be equal to the sum of this quantity and the observation error variance. This should be kept in mind when inspecting Tables 4 – 6 below.

If parameters, like here track biases, have to be estimated similar, but slightly more complicated, equations are used, see Moritz, 1980.

The covariance was first estimated empirically. The empirical values were then used as input to the program COVFIT (Knudsen, 1987a) to determine an analytic expression for the covariance function. This analytic expression is the basis for using covariance propagation for the calculation of auto and cross-covariances between gravity and altimetry.

As mentioned above the North Sea was selected as a test area, because we also here have ship-gravity so that we might see if we improve the prediction of gravity. Unfortunately the quality of this data is not very good and only a very few data points are close to the coast.

The numerical experiments of predicting gravity were carried out separately in two areas (South and North) bounded by (1) latitude 57.3^o and 58.3^o, longitude 7^o and 9^o, (2) latitude 58.5^o and 59.5^o, longitude 4^o and 6.5^o respectively. The two areas also have similar statistical characteristics, and the same covariance function was used in the two areas. The gravity variance at sea-level is 248 $mgal^2$ and the corresponding altimetry variance is 0.04 m^2. The value for altimetry is larger that the values found for the total area, see Table 2.

The prediction of sea-gravity from altimetry (with or without track biase estimation) or from the combination of altimetry and gravity did not show any significant difference, see Table 4 and 5.

Table 4. Prediction results from Northern area. Number of data of sea: 224. 1568 altimetry observations and 68 land gravity observations. 153 trach bias parameters solved for.
Diff. is the difference between observed sea-gravity and predicted values. Error is the prediction error estimated by LSC. Units mgal.

Data used for prediction	Altimetry	Altimetry with parameters	Altimetry with land Gravity

	Obser.	Diff.	Error	Diff.	Error	Diff.	Error
Mean	-3.17	-1.00	10.66	-1.34	10.83	-0.78	10.77
St.dev.	12.67	6.27	12.02	6.24	12.01	6.28	12.01
Max	47.00	19.14	99.32	19.51	99.36	20.34	99.35
Min	-30.48	-26.53	7.64	-25.13	7.78	-24.18	7.75

Table 5. Prediction results from Southern area. Number of data at sea: 171. 1445 altimeter data, 169 land gravity data. 145 track parameters solved for. Units mgal.

Data used for prediction			Altimetry	Altimetry with parameters		Altimetry with land gravity	
	Obser.	Diff.	Error	Diff.	Error	Diff.	Error
Mean	-1.99	-3.02	10.17	-2.15	10.34	-3.99	10.32
St.dev.	13.94	5.49	1.30	5.83	1.29	5.85	1.31
Max	36.72	16.87	12.77	13.46	13.19	11.93	13.06
Min	-35.47	-14.31	6.22	-15.08	6.45	-19.52	6.39

Altimeter data up to 9 km from the coast were downweighted, using a linear scale-factor (on the associated data error estimates) dependent on the distance equal to 1 at 9 km and equal to 3 at zero distance. Again the results are not in any manner significantly different.

Table 6. Statistics of associated error-estimates for data (618) closer than 9 km from the coast and further away (71521). Values after down-weighting are also shown. Units m.

	Mean	Standard deviation	Minimum	Maximum
Close	0.051	0.016	0.023	0.133
Close, downweighted	0.087	0.046	0.031	0.383
Far away	0.048	0.010	0.020	0.130

The maximal error found in the area close to the coast and away from the coast, 0.133 m and 0.130 m, respectively, are very similar. This indicates that the data-set received probably has been screened for outliers.

Table 8. Prediction results from Northern Area. Altimeter data close to coast downweighted. Gravity at sea predicted from altimetry and land gravity. 153 Parameters solved for. Units mgal.

	Observations	Predictions	Difference	Error
Mean	3.17	-1.83	-1.33	10.81
Standard Deviation	12.67	12.53	6.24	12.01
Maximum	47.00	51.32	19.66	99.35
Minimum	-30.48	-26.38	-24.62	7.76

However, we also have available the formal error-estimates of the predicted gravity from LSC, see Figure 3-6.

When comparing the maps in Figure 3 and 5 with the maps in Figure 4 and 6 we see a striking difference. Fig 3 and Fig, 5 show the prediction error estimate from altimetry only, while the two other figures show the error estimate based on altimetry and land gravity. It is obvious, that we in the coastal area, between islands, and in fjords see a large improvement.

6. Comparison with gravity obtained using a Fourier based method.

A fast method for the estimation of gravity anomalies from altimetry used the Fourier method is implemented using the Fast Fourier Transform, (Andersen et al., 2004). The method uses, like above, data from which a global model (EGM96) has been subtracted. An area bounded by parallels and meridians is selected, and data are gridded. This results in zero residual values on land due to the use of local LSC, which will predict small – or zero values if no data are close to the grid point. ("Close" here means at a distance twice the correlation distance). Consequently the results at sea may be erroneous if for example a large anomaly is located at the coast.

We have compared gravity predicted from LSC and from the Fourier based method with observed gravity at sea, see Table 8 and 9.

Table 8. Comparison between KMS02 gravity and sea-gravity in Northern area.
Points predicted: 211.

	Mean	Standard Deviation	Minimum	Maximum
Original data	13.74	10.27	-2.0	33.90
Predictions	16.29	8.12	5.26	33.60
Differences	-2.55	6.62	-14.33	12.67

Table 9. Comparison between KMS02 gravity and sea-gravity in Northern area.
Points predicted: 292.

	Mean	Standard deviation	Minimum	Maximum
Original data	-6.81	15.83	-44.50	77.10
Predictions	-5.15	14.97	-42.04	33.99
Difference	-1.66	8.04	-24.83	60.68

If on compares the differences with the values of Table 5 and 6, there is a small, but not significant difference. For the southern area the combination with land data using LSC seems to give an improvement.

7. Conclusion.

The use of altimetry combined with land gravity in costal areas have the potential of improving the quality of the predicted gravity anomalies. Obviously the results are only examples from a limited area. In other areas similar improvements are possible. The improvement in terms of the square-root of the error-estimate will be proportional to the gravity signal standard deviation in the area. This value may be decreased - and the prediction results better, if the data are pre-smoothed using the bathymetry which generally is well known in many coastal areas. The effect of the bathymetry on the altimetry is subtracted from the altimetry. Subsequently the effect of the bathymetry is added to the predicted gravity.

An alternative area of investigation would be Japan, where there are new very fine land and sea gravity. Here is a problem because of the sea-surface topography created by the Kuroshio-current. Here the method of down-weighting data should be used not only taking into account the distance to the coast-line, but also the distance to the main flow of the current. Such distances may be computed using the variability of repeat-track altimetry to identify the general location of the current.

Finally, it might be worthwhile to consider also to mix altimetry and sea-gravity, where the sea gravimetry is sparse or of lower quality. However, in areas where there is much sea-gravity, we obviously do not need altimetry to calculate gravity.

7. Acknowledgement.
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Figures. Are not included in this electronic version of the document. All units in Fig. 3-6 are mgal.
Figure 1. Altimetry in the test area, see http://cct.gfy.ku.dk/oev6.7.pdf
Figure 2. Gravity in the test area, see http://cct.gfy.ku.dk/oev_6.6.pdf
Figure 3. Error of prediction, Altimetry only http://cct.gfy.ku.dk/oev6.8.pdf
Figure 4. Error of prediction from altimetry and land gravity, see http://cct.gfy.ku.dk/oev6.9.pdf
Figure 5. Error of prediction from altimetry,see http://cct.gfy.ku.dk/oev6.10.pdf
Figure 6. Error of prediction from altimetry and land gravity, see http://cct.gfy.ku.dk/oev6.11.pdf

Appendix. Files and conventions. All the files are available at http://cct.gfy.ku.dk/nima/<file-name>. A survey of the files are found in http://cct.gfy.ku.dk/nima/nimals.txt .

```
Files
nordso.dat      altimetry
nordso1.dat     altimetry from smaller area (55.0 56.25  6.5 10.0)
nordso3.dat     downweighted altimetry
tgr.dat         free-air gravity, selected with cells 0.05 deg. x 0.1 deg.
tgregm.dat      gravity minus EGM96 to deg. 360 contrubution
tgregml         land gravity
tgregms         sea-gravity

Input files naming conventions:
<data1><data2>2<data3>.inp
with data: dgl for land gravity (or just “l”)
           dgs for sea gravity (or just “s”)
           a for altimetry.

Output files have suffix *.out.

Empirical covariance function: empcov.inp
fadk96.ct
tdgegm.ct

Covariance fitting:
covfit.job
```

Appendix 2. Track biases and error estimates.

Track-biases and associated error estimates were determined for all tracks in the two selected areas. Below are shown for 10 tracks the track number, bias and error estimates (column 2 and 3) obtained using altimetry only and altimetry+ land gravity (column 4 and 5). All units m. Note that the error estimate of the bias decreases when gravity observations are added. The corresponding estimates for all tracks are found in http://cct.gfy.ku.dk/nima/trackb.dat .

Northern area.

3690	.014	.107	.009	.085
3759	-.004	.107	.146	.152
4178	-.011	.107	.006	.084
4247	.036	.111	-.042	.089
4264	-.045	.108	.025	.084
4333	-.029	.107	-.040	.097
4666	.025	.114	.023	.086
4752	-.030	.107	.033	.085
4821	-.002	.107	.027	.084
4907	-.015	.107	.000	.085

Southern area:

3587	.122	.111	.049	.096
3673	.304	.123	.033	.096
3920	.119	.110	.025	.096
4006	.081	.114	.072	.101
4161	.138	.110	-.011	.097
4408	.069	.119	.009	.096
4494	.135	.111	.062	.105
4649	.146	.111	.009	.096
4982	.140	.110	.035	.097
5068	.110	.111	.022	.096