Simulation results from combination of GOCE gridded SST and SGG data.

C.C.Tscherning, *University of Copenhagen, Department of Geophysics*,
Juliane Maries Vej 30, DK-2100 Copenhagen Ø., Denmark.
Phone ++4535320582, Fax: ++4535365357.

Abstract: EGM96 spherical harmonic coefficients from degree 25 to 60 have been used to produce grids of values of the anomalous potential,. T, the first order derivatives, T_r, T_n, and various second order derivatives in an East, North, radial frame, (T_{rr}, T_{re}, T_{ee}, T_{nn})Various combinations of the data has been used to recover the potential coefficients using the method of Fast Spherical Collocation (FSC). A covariance function with the terms up to degree 25 equal to the EGM96 error degree-variances was used. Terms from degree 25 to infinity were chosen so that the covariance function could be evaluated using a closed expression.

The simulation shows that using T_{re} ,T_{ee}, T_{nn}, T_{rr}, and combining T_{re} ,T_r, T_n nearly give the same result. That T_{nn}, T_{ee} , gives the same results, but combining T_{ee} with T_{nn} improves the results. T_r, T_{rr} ,and a joint use of T_{rr}, T_r also gives nearly the same results, while the results using the "horizontal" derivatives are up to a factor 2 worse per degree.

Since the covariance function contain terms from degree 2 to infinity, spherical harmonic coefficients in the range 2 – 24 will be predicted with results which are non-zero, despite the input data do not contain corresponding harmonic coefficients different from zero. Consequently the predicted values show the effect of how the different data types "aliases" back into degrees 2 – 24. Here T and T_r give the smallest aliasing.

1. INTRODUCTION.

The GOCE satellite (Johannesen et al., 2003) will fly a gravity gradiometer, which will observe the gravity gradients (Satellite Gravity Gradiometer, SGG, data), and a GPS receiver which provide (Satellite-to-Satellite, SST data) positions, velocities and accelerations of the satellite. Using the so-called common mode output of the accelerometers used to measure the gradients, the non-inertial frictional force may be measured.

The gradiometer data will be given in the instrument frame; they will include colored noise, but the data may be filtered and transformed to an earth-fixed East, North, radial up frame, see e.g. (Tscherning, 2003).

In each data point, we will have 6 gradiometer measurements, 3 accelerations and the kinetic energy from the velocity vector. The diagonal components of the gravity gradient matrix are supposed to have a much lower noise than the off-diagonal components. And in between the diagonal elements the radial second order derivative is the one with the smallest error.

In theory one may combine all the data in order to determine a model of the gravity field, e.g. expressed as a series in spherical harmonic coefficients. If a method such as least squares is used to estimate the coefficients, the number of equations to be solved is equal to the number of coefficients. But if a method as least-squares collocation (LSC) (Moritz, 1980) is used a system of equations equal to the number of observations has to be solved. Considering the large amount of data expected to be collected by GOCE this is obviously not be possible to use LSC in this manner. Instead, the data may be gridded at satellite altitude and the method called Fast Spherical Collocation (FSC), (Sansò & Tscherning, 2003), can be used. Using this method a number of equations, with unknowns equal to the number of parallels times the number of data-types, must be solved. One limitation of the method is, however, that the variance of the data-error must be identical for all data (of the same type) on one parallel.

FSC has been used to study how much each (gridded) GOCE observation, or combination of observations, will contribute to a solution.

2. COVARIANCE MODEL AND ASSOCIATED SIGNAL STANDARD DEVIATIONS.

The anomalous potential T is the difference between the gravity potential W and a reference potential U, see (Torge, 2001).

In the first implementation of FSC only 0, 1 and 2. order radial derivatives of the anomalous potential, T, could be used (program sphgrid.f). Meanwhile the software has been expanded to handle T_{ee}, T_{nn}, T_{rn} and T_n (program sphgric.f, see http://cct.gfy.ku.dk/sphgric.pdf , 2004-02-22).

The program has been tested using EGM96 (Lemoine et al., 1998) from degree 25 to degree 60, i.e. data was generated using these coefficients. The results presented here are from numerical experiments where data were generated at altitude 255000 m, in a 2 degree grid. In latitude the grid covered latitudes from -88 to 88 degrees. The data are noise-free, but in the FSC experiments a noise standard deviation of 5 mE was used for the 2.-order derivatives T and T_{re} ,$T_{ee,}$ $T_{nn,}$ T_{rr}, 0.01 mgal for the first derivatives and for T 0.001 m^2/s^2. This means that the corresponding noise variances were added to the diagonal elements of the matrices in the equations to be solved.

The following covariance function was used

$$COV(\psi,r,r') = \sum_{i=2}^{24} \sigma_e^2 \left(\frac{R_B^2}{rr'}\right)^{i+1} P_i(\cos\psi) + \sum_{i=25}^{\infty} \frac{A}{(i-1)(i-2)(i+4)} \left(\frac{R_B^2}{rr'}\right) P_i(\cos\psi)$$

$R_B = 6369939m, a = 1.0,\ A = 115.92\ mgal^2 \times R_E^2,$

ψ the spherical distance,

σ_E^2 error – degree – variances of EGM96.

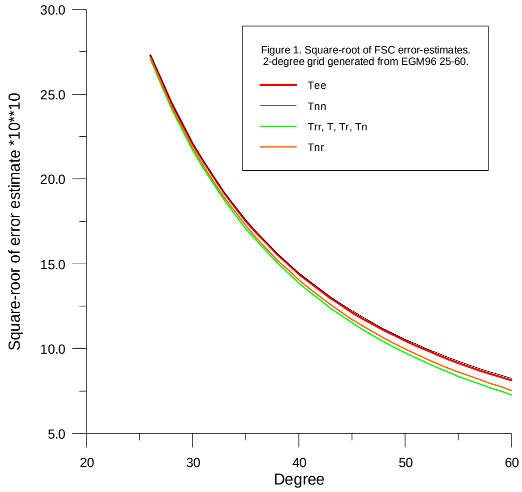
Hence the covariance does not reflect that we only use coefficients with degrees between 25 and 60 when generating the simulated data. In fact, the simulations could have been carried out without the simulated data, if we were just interested in the error-estimates of the coefficients to be determined. On the other hand, the covariance function reflects the setting in which the GOCE observables are going to be used, i.e. a low degree and order reference field will be subtracted from the observed data in order to obtain a data set which is statistically more homogeneous than the original measurements, and which may aid in reducing errors with a long correlation distance, see (Migliaccio et al., 2004).

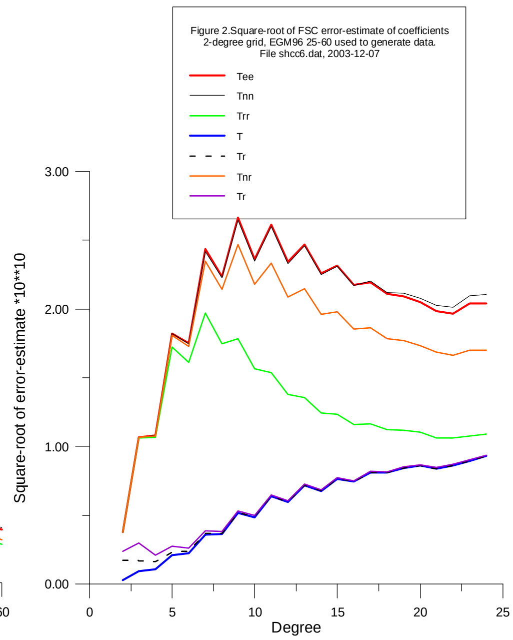
The associated gravity anomaly variance at the mean earth surface is in this model 579 $mgal^2$. The standard deviation of the signal corresponding to the model for the different quantities are at satellite altitude T_{rr} 143 mE, T_{ee} and T_{nn} 86 mE, T_{nr} 100 mE, the gravity disturbance T_r 2.5 mgal, T_n 1.74 mgal and for the anomalous potential T, it was 4.5 m^2/s^2.

3. Results.

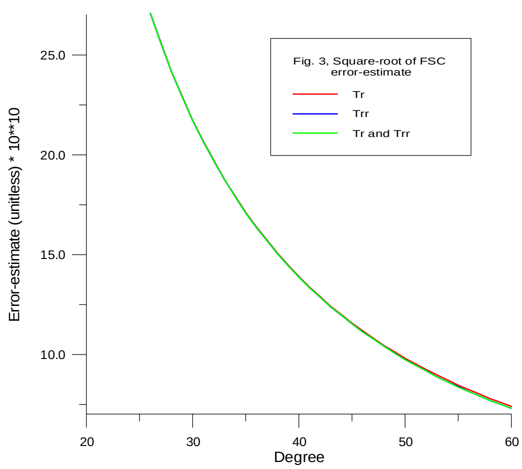
The results are illustrated by two kinds of figures: The first kind show the formal standard deviation of the prediction error obtained from FSC. The second kind shows the square-root of the differences between coefficients obtained using simulated observed data and the “true” coefficients per degree.

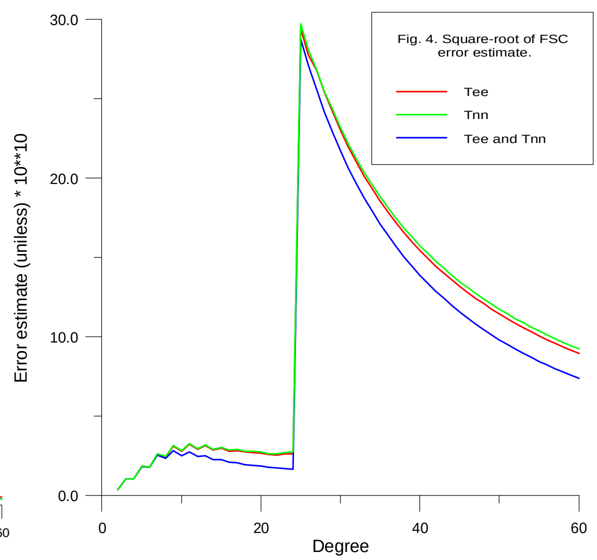
The first four figures show the square-root of the formal FSC error-estimate. These estimates correspond to a realistic situation, where all degrees and orders are present, but where EGM96 has been subtracted up to degree 24.



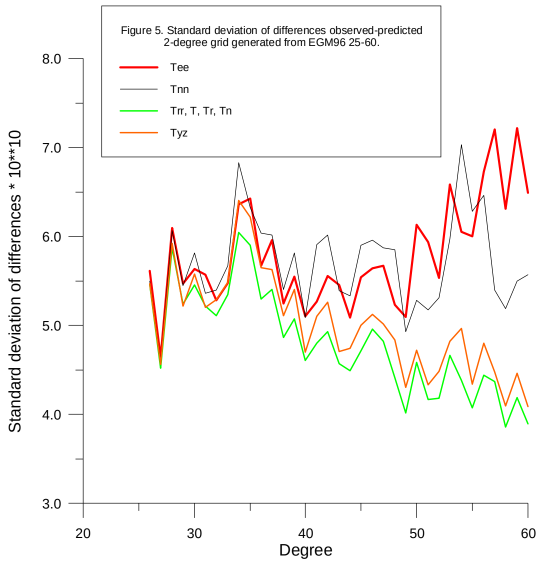


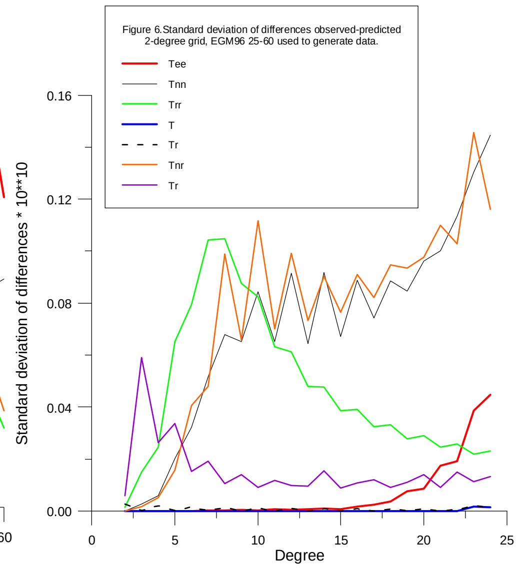
We see (in Fig. 1) that the results for the 7 kinds of data are nearly identical for degree 25-60. But Fig. 2 show the differences in the range 3-24, and here are the results quite different. The smallest aliasing is obtained for T, while the largest are for the second order derivatives, as should be expected, see (Tscherning, 2003a).

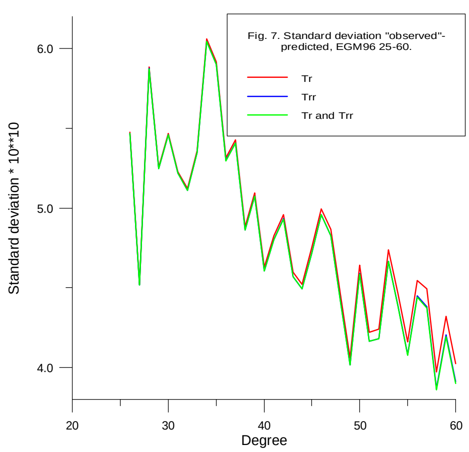


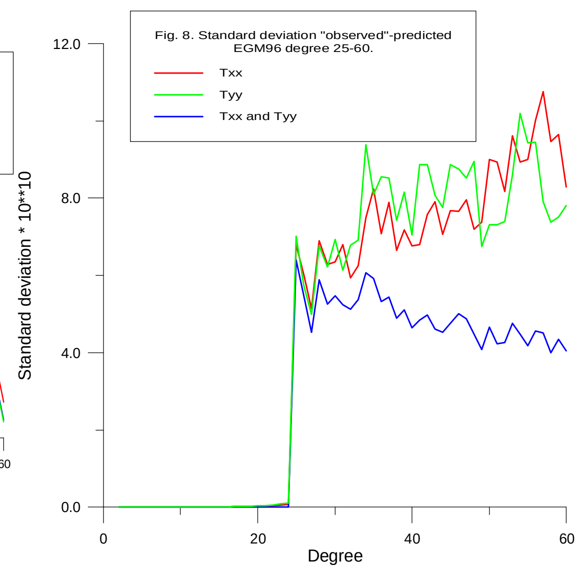


A similar picture is seen in Fig. 3 and 4. Most interesting is Fig. 4, which show how a combination of two diagonal components improve the results both in the range 25-60 and in the "aliasing" interval. However, their associated error is larger than the one obtained for the radial components.









The second set of figures illustrate the standard deviation of the difference between the "predicted" coefficients and the original EGM96 coefficients. The results in Fig. 5 corresponds to the values in Fig. 1, and we see that T and the radial derivatives give the best results in the range 25-60. In Fig. 6, we again see that the zero and first order derivatives are less influenced by aliasing. Fig. 7 and 8 illustrates the influence of combining two quantities. Combining $T_{rr,}$ T_r seems not to improve the result, while the combination of the two other "diagonal" components give an improvement.

3. CONCLUSION.

When using the GOCE data we have a choice between different data-combinations. Or one may chose to use all the data. But some of the data are, as illustrated above, so physically correlated, that they give similar nearly identical results. There are differences between solutions using the radial derivatives or combinations of these and the "horizontal derivatives $T_{nn,}$ $T_{ee\ ,.}$ The latter do not give as good results as the first. On the other hand, the data my have different error-characteristics, which may aid in the filtering performed implicitly when using FSC.

The non-zero predicted coefficients for degree 2 – 24 illustrate the fact that we are going to encounter aliasing when determining spherical harmonic coefficients. This is an unfortunate fact, which no method can avoid. However, some of the data-types, like values of the anomalous potential, T, are less prone to aliasing than other.

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