Testing frame-transformation, gridding and filtering of GOCE gradiometer data by Least-Squares Collocation using simulated data.

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Abstract: Least-squares collocation (LSC) has been used to test frame transformation, filtering and gridding of GOCE gradiometer data. Initially the function of the GRAVSOFT program GEOCOL was checked by using noise-free data with 5 s sampling provided by IAG along a 1 month realistic orbit. Data in a 2^o x 2^o area was used. For these data it was verified that 0.1^o frame transformation and gridding in an interior 1^o x 1^o area of one quantity from the same quantity could be done with an error below 5 % of the signal standard deviation. If several components of the gravity gradient were used, the error decreased slightly.

Noise with standard deviations of 0.050 and 0.150 Eötvös Units ($E=10^{-9}/s^2$), respectively, and a 1 s correlation distance was generated and added to the data. For the 0.050 E noise the result of the gridding was as for the data without noise, when 1 s sampling was used. For the 0.150 E data, the use of totally 2 months of simulated data and all 3 diagonal components of the gradient matrix were necessary in order to obtain a result below 10 % of the signal standard deviation. Further improvements are possible if data from a larger area or from the planned second measurement phase of GOCE are used.

1. **Introduction**.

It will most likely not be possible to use least-squares collocation (LSC) in its most general form for the modeling of the gravity field of the Earth using GOCE (ESA, 1999) satellite gradiometer data. In its general form LSC requires the solution of systems of equations with as many unknowns as observations.

Regional solutions may however be constructed, and such solutions may form the basis for the creation of datasets of gridded values at satellite altitude. Certain types of gridded data (e.g. the second order radial derivative, $d^2T/dr^2=T_{rr}$, where T is the anomalous potential) may be used as the basis for the application of the method of Fast Spherical Collocation, see Sansò and Tscherning, (2002).

In order to test the use of LSC, the GRAVSOFT program GEOCOL (Tscherning, 1975, Tscherning et al., 1994) has been used with test data available from the IAG Special Commission 7. These test data have been generated for a period of one month with a 5 s sampling, along a realistic GOCE orbit. It includes all the 6 second order derivatives of the gravity potential, V, given in a local reference frame aligned with the velocity vector of the satellite and the radius vector. No roll of the satellite was included. The data were generated using EGM96 (Lemoine et al., 1998) to degree 300. The data set did not include any noise. However, noise has been simulated and test was carried out using the noisy data. The data set has furthermore been densified time-wise (to 1 s) and space-wise corresponding to 2 months of observations in order to test the influence of the sampling.

In the following we will describe the results of the testing. Not only did the testing reveal some embarrassing programming errors, but it did also give better insight into which errors could be expected when doing frame transformation and gridding. In section 2 the test using data without noise, and in section 3 the results of the gridding using data with added noise are described.

2. **Testing the LSC prediction procedure with data without noise**.

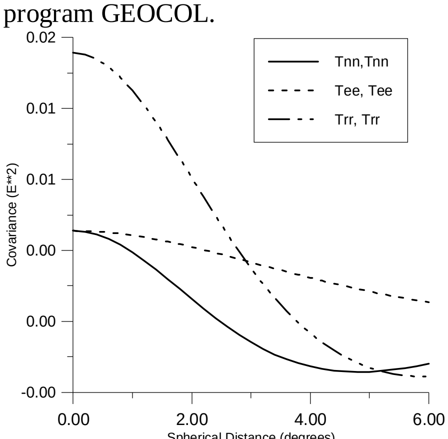
As a reference potential EGM96 to degree 24 was used. This has the effect that the fields become more statistically homogeneous. This is not very important for the gridding, but very important for the interpretation of the error estimates. Furthermore the data decorrelates to a certain extend.

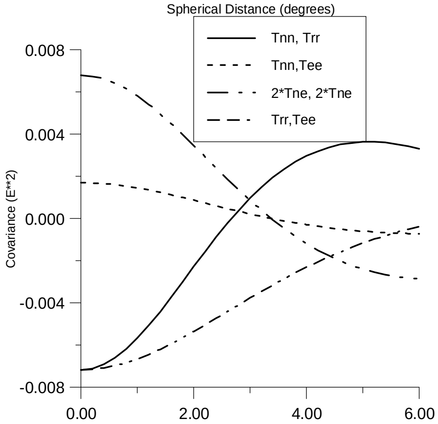
As a covariance function we used a function defined using the EGM96 error-degree variances e_i^2 from degree 2 to 24 scaled with a factor of a=0.5, the depth to the Bjerhammar-sphere of 1061 m and the variance of the gravity anomalies at the mean Earth surface of

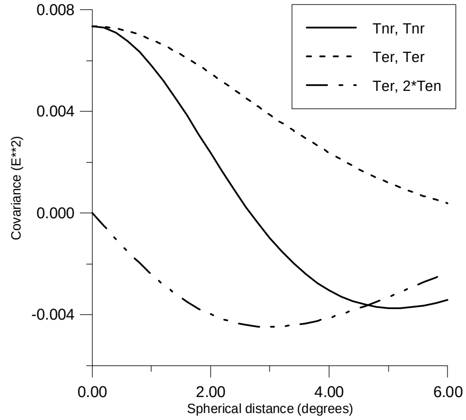
408.13 $mgal^2$. A scale factor on the degree-variances from 25 to infinity was used equal to A= 98.79 $mgal^2/R^2$, where R is the mean Earth radius. Hence the analytic covariance expression for the anomalous potential is with R_B the Bjerhammar sphere radius

$$\mathrm{cov}(P,Q) = \mathrm{cov}(r,r',\psi) =$$
$$\sum_{i=2}^{24} a * e_i^2 \left(\frac{R_B^2}{rr'}\right)^{i+1} P_i(\cos\psi) +$$
$$\sum_{i=25}^{\infty} \frac{A}{(i-1)(i-2)(i+4)} \left(\frac{R_B^2}{rr'}\right)^{i+1} P_i(\cos\psi)$$

P_i are the Legendre polynomials, r and r' are the radial distances from the origin of P, Q, respectively, and ψ is the spherical distance between the points. Covariances of the gravity gradient components (Fig. 1) may be evaluated as described in Tscherning (1976, 1993). Subroutines COVAX, COVBX and COVCX used for these computations are included in the program GEOCOL.







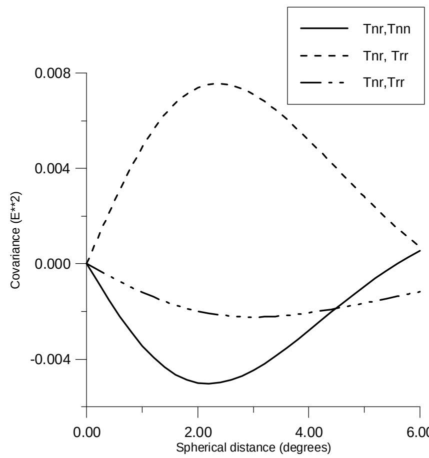


Figure 1. Covariance functions of gravity gradients at 255 km altitude in an East (e), North (n) and radial (r) frame as a function of spherical distance. An azimuth equal to zero was used. This makes covariances of all combinations of gravity gradients not shown identical zero.

Two test areas were selected. One bounded at latitudes -1^o and 1^o and longitude -1^o and 1^o. This area included 29 points. The other area was bounded by latitude 81 and 83 degrees and longitude 45 to 50 degrees and included 294 points. However this area was only used initially because results similar to the first mentioned area were obtained.

The individual gravity gradients $T_{kl}(Q_n)$, with error correlation σ_{mn}, were used to predict the other gradients, $T_{ij}(P)$

$$\tilde{T}_{ij}(P) = \left\{\mathrm{cov}(T_{kl}(Q_m), T_{ij}(P))\right\}^T \cdot$$
$$\left\{\mathrm{cov}(T_{kl}(Q_m), T_{kl}(Q_n)) + \sigma_{mn}\right\}^{-1} \cdot$$
$$\left\{T_{kl}(Q_n)\right\}$$

as well as the components of the gravity vector given in an inertial, Cartesian frame (also computed from EGM96 to degree 300). The evaluation of the covariances require that the functionals associated with the observations (second order derivatives in local cartesian frames) are applied on the fundamental covariance expression given above. We will use two frames: the instrument frame (along-track: x, direction of radius vector: z, y: orthogonal on x,z) and the e: East, n: North and r: radially up frame in a geocentric CTRS. The **LSC** error estimates were also computed, see (Moritz, 1980, eq. 14-42).

Below in Table 1, the results from the first test area are presented. The points in which the values are predicted are the same points, which are associated with the observations. Hence, the test is a kind of consistency test, and it was used to detect errors in the subroutine COVCX. It furthermore shows how well/bad a pointwise frame-transformation may be done.

The results are given as the differences and standard deviations of "observed" minus calculated and the mean value of the error of prediction calculated also by **LSC**. Units are mE. In the calculations an uncorrelated data noise of 5 mE was used, i.e. 25 mE^2 was added to the diagonal of the normal equations.

The table shows that there is a very good consistency between the different quantities in the sense that the analytically computed error of prediction (in bold) is large when the error is large and small when it is small. Components of the gravity gradient, which are spatially close (T_{zz} and T_{rr}) are predicted with a small error (3 mE), while components which are spatially different (T_{xx} and T_{ee}) have a large error (64 mE) compared to the signal variance (71 mE).

It also illustrates the well known fact, that we using only local data obtain biased results when we predict one functional from another. See for example the third row, where the prediction of T_{rr} has a standard deviation of only 5 mE with a bias of 39 mE when predicted from T_{xx}.

Similar results were obtained in the area close to the Pole, where the velocity vector direction varies much. It was furthermore throughout the computations verified that the Laplace-equation was fulfilled both in the East/North/Up (ENU) frame and in the instrument frame. This shows that the program works correctly at least for this type of quantities. However, before this conclusion could be reached a number of errors were detected and corrected.

Pre-dicted	T_{ee}	$2T_{en}$	T_{er}	T_{nn}	T_{nr}	T_{rr}
LSC St.de.	***071***	***082***	***089***	***074***	***089***	***127***
Data						
T_{xx}	-34 005	006 007	-18 022	001 002	046 006	039 005
LSC	**064**	**014**	**060**	**003**	**027**	**063**
$2T_{xy}$	-10 002	000 001	-15 006	-08 005	077 012	019 007
LSC	**044**	**003**	**037**	**044**	**033**	**088**
T_{yz}	001 007	-10 013	000 001	017 009	008 006	-18 011
LSC	**025**	**028**	**003**	**035**	**016**	**046**
T_{yy}	000 001	003 006	-18 003	005 031	132 010	-05 031
LSC	**003**	**016**	**031**	**063**	**061**	**062**
T_{xz}	-10 013	-20 024	-01 007	022 006	000 001	-12 008
LSC	**034**	**029**	**015**	**022**	**003**	**044**
T_{zz}	-10 010	-45 026	-18 008	010 010	044 008	000 001
LSC	**030**	**057**	**028**	**029**	**026**	**003**
T_{xx}, T_{yy}, T_{zz}	000 001	-03 002	000 041	000 000	-06 009	000 001
LSC	**002**	**008**	**021**	**002**	**024**	**002**

Table 1. Results of predicting (e,n,r) gravity gradient components from instrument (xyz) gravity gradients. EGM96 to degree 24 subtracted. The first column gives the kind of data used for prediction. The x, y, z refers to derivatives of T along-track, cross-track and up. The column heading of each of the following columns shows the type of quantity predicted. The second row give calculated variances calculated from the covariance model. Mean and standard deviation of the predicted quantity is found in the upper cell and below in bold the mean value of the prediction error computed using GEOCOL. Units now and later are mE.

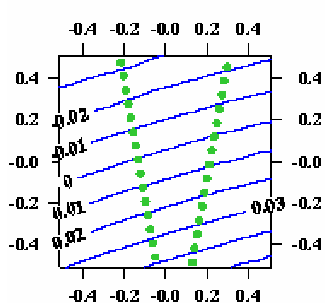


Figure 2. Area used for test of gridding. Contour lines of T_{rr} in E based on EGM96 degree 25-300 coefficients. The dots are simulated GOCE measurement points.

Subsequently data in a 0.1° grid bounded by -0.5° and 0.5° in latitude and longitude (Fig. 2) were generated in an East, North, radially up frame. The excellent results, which are obtained when a quantity is spatially close to the one used for the prediction and when several components are used, are given in Table 2.

Predi-cted Data	T_{ee}	$2T_{en}$	T_{er}	T_{nn}	T_{nr}	T_{rr}
xx	032 004	007 003	-017 010	001 000	042 002	040 002
LSC	**060**	**011**	**057**	**003**	**027**	**059**
2*xy	-011 001	000 000	-014 003	009 001	069 005	020 002
LSC	**043**	**003**	**030**	**043**	**030**	**017**
xz	010 007	016 009	000 003	021 003	000 001	-011 004
LSC	**030**	**028**	**013**	**021**	**002**	**042**
yy	000 000	-002 001	003 001	007 002	-003 006	-007 002
LSC	**003**	**016**	**028**	**059**	**058**	**059**
yz	000 004	007 004	000 000	019 005	005 002	-020 006
LSC	**024**	**026**	**003**	**032**	**017**	**042**
zz	-009 005	-041 011	-018 003	009 005	040 003	000 000
LSC	**027**	**053**	**025**	**027**	**025**	**002**
xx,yy,zz	000 000	002 001	-009 001	001 000	038 001	000 000
LSC	**002**	**007**	**021**	**002**	**020**	**002**

Table 2. The table gives results of predictions of data in a grid with spacing 0.1° totally 121 points. The first row gives the mean and standard deviation of the differences "observed" minus predicted. The second row gives the LSC computed error estimate. Units: mE. Note that the instrument x-axis in this area points close to North or South in an Earth-Fixed frame. This is the reason that results of predicting T_{nn} from T_{xx} or T_{ee} from T_{yy} are excellent. See Fig. 2.

Again the results are striking. The mean values are sometimes large, indicating that we are using a too limited area of "observations". Again the noise-free observations in the calculations were supposed to have an associated noise standard deviation of 5 mE, i.e. 25 mE^2 were added in the diagonal of the normal equations. This mainly affects the LSC error-estimate. Note that the computed error-estimate is large when the bias is large, as it should be.

3. Gridding with noisy data.

An accepted model for the (total) gradiometer noise is given in the Granada report (ESA, 1999, Fig.8.2). This model has been used by (Regguzoni, 2002) for the generation of noise, which had a very large correlation distance. The elimination of this kind of noise requires that data are distributed over a much larger area than studied here, see (Tscherning and Arabelos, 2003) or (Bouman et al. 2003).

Using a procedure similar to the one used by (Regguzoni, 2002) pseudo-random noise was generated with a short correlation distance, so that the noise was uncorrelated after a 2 s time difference.

The generated noise was added to the "observations" used in section 2. These data have a 5 s time-difference; hence they will be regarded as uncorrelated. The variance of 0.003027 E^2 was added to the diagonal of the normal equations. The results are presented in Table 3.

Predi-cted Data	T_{ee}	$2T_{en}$	T_{er}	T_{nn}	T_{nr}	T_{rr}
xx	-012 002	029 005	-011 005	003 002	073 006	009 004
LSC	**068**	**061**	**071**	**016**	**046**	**071**
2*xy	-041 007	013 011	-044 008	071 027	190 015	-030 033
LSC	**059**	**018**	**062**	**063**	**062**	**111**
xz	004 007	015 010	001 004	028 004	004 006	-032 011
LSC	**066**	**052**	**064**	**035**	**017**	**081**
yy	-002 001	-060 005	000 007	023 004	002 014	-021 005
LSC	**015**	**065**	**048**	**069**	**071**	**071**
yz	-015 010	-012 013	001 005	065 005	008 014	-050 013
LSC	**037**	**050**	**017**	**065**	**068**	**082**
zz	-004 004	-029 004	-019 005	-005 013	123 002	009 015
	038	069	043	038	040	018

Table 3. Differences between "observed" and predicted data in a grid. Noisy data with a noise standard deviation of 55 mE was used. First row gives the type of predicted quantities and the first column the kind of data used. Units mE. By a mistake the noise added to the 2*xy component was not multiplied by 2.

In the table we note that not only have the predictions become worse, but also the estimated prediction error from LSC - as it should be. There are still some large biases due to the small area used for the "observations".

The observation and the prediction grid points are the same as used in Section 2.

It is encouraging to see that rather small errors of prediction can be expected using T_{zz} and predicting the same kind of data in a grid. The estimated error is 18 mE. Hence for 5 s means the error will be around 9 mE. Then we are not far from the 5 mE required to reach the goals of the GOCE mission.

A dataset with 1 s sampling was generated and noise added. This improved the results considerably, so when 3 diagonal components were used a "perfect" filtering was obtained, see Table 4.

Predic-ted Data	T_{ee}	$2T_{en}$	T_{er}	T_{nn}	T_{nr}	T_{rr}
xx, yy, zz	007 004	-002 003	-029 006	-001 002	054 003	-006 005
LSC	**006**	**031**	**029**	**006**	**029**	**006**
	xx	xy*2	xz	yy	yz	zz
xx, yy, zz	-003 005	001 010	001 061	004 007	-012 030	-001 010
LSC	**009**	**033**	**033**	**009**	**033**	**010**

Table 4. Prediction (row 2 and 3) and filtering (row 6 and 7) using 1 s T_{xx} , T_{yy} and T_{zz} data with noise standard deviation of 55 mE. Note the very good filtering at the data. The noise is reduced to 20 %.

Here a noise standard deviation slightly smaller equal to 0.116 E was used. The results using the 1 s - 1-month dataset were unsatisfactory, and a dataset corresponding to a 2-month period was generated. The results are given in Table 5.

Pre-dicted	T_{ee}	$2T_{en}$	T_{er}	T_{nn}	T_{nr}	T_{uu}
Data						
xx,yy zz,	-004 002	-028 004	-029 004	-007 001	058 005	011 002
LSC	**008**	**041**	**031**	**008**	**032**	**009**
.	T_{xx}	T_{xy}	T_{xz}	T_{yy}	T_{yz}	T_{zz}
xx,yy zz,	-009 006	026 014	-001 068	-003 005	-009 032	011 004
LSC	**012**	**043**	**036**	**012**	**035**	**013**

Table 5. Gridding and filtering of data with 116 mE standard deviation noise added. T_{xx} , T_{yy} and T_{zz} used. Totally 720 observations, corresponding to 2 months of data, 1 s sampling. Rows 2 and 3 show the results of the gridding and rows 6 and 7 the filtering. The noise is reduced to (12 mE and 13 mE), i.e. 10 %

We see that having the double number of data enables us to obtain very satisfactory results when the data have a twice larger noise standard deviation as shown in Table 5. However a second measurement-phase of 2 months (as planned for GOCE) should give results of both the gridding and the filtering which are very satisfactory, i.e. like working with data with a 5 mE noise standard deviation.

4. Conclusion.

We should keep in mind that the error of prediction varies from area to area due to the variation in signal variance. We have here used a small area with a rather smooth gravity field. However, globally predicted T_{rr} give similar results, see (Tscherning and Arabelos, 2003) where equiangular 20^o x 20^o areas were used. This is probably due to the spatial closeness of the T_{zz} and the T_{rr} vector components.

LSC may (as demonstrated above) be used to combine different components, which will reduce the error, or equivalently improve the filtering of the noise. This naturally requires that the noise is uncorrelated between the gravity gradient components.

The tests carried out are both a test of the GEOCOL program as well as of the possible results to be expected when gridding noisy GOCE gradiometer data in order to use Fast Spherical Collocation.

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