OSGM02: A NEW MODEL FOR CONVERTING GPS-DERIVED HEIGHTS TO LOCAL HEIGHT DATUMS IN GREAT BRITAIN AND IRELAND

J. C. Iliffe, M. Ziebart, and P. A. Cross
University College London

R. Forsberg and G. Strykowski
Kort og Matrikelstyrelsen, Denmark

C. C. Tscherning
University of Copenhagen

ABSTRACT

The background to the recent computation of a new vertical datum model for the British Isles (OSGM02) is described. After giving a brief description of the computational techniques and the data sets used for the derivation of the gravimetric geoid, the paper focuses on the fitting of this surface to the GPS and levelling networks in the various regions of the British Isles in such a way that it can be used in conjunction with GPS to form a replacement for the existing system of bench marks. The error sources induced in this procedure are discussed, and the theoretical basis given for the fitting procedure. Results for each major region (Great Britain, Ireland, and Northern Ireland) as well as the various independent island datums are described. The problems to be expected when working between datums are discussed.

INTRODUCTION

In the summer of 2001, the three Ordnance Surveys of Great Britain, Ireland, and Northern Ireland joined together in a consortium to commission a set of models of the separation between the ETRS89 ellipsoid and the different vertical datums that exist in the British Isles. The Ordnance Surveys had recognised the potential for using GPS for heighting since 1994 [**4**] but that this would require a detailed model of the geoid. The development of this was let on a competitive tender on the understanding that the successful bidders would be given access to whatever terrain data was deemed appropriate, from data bases of the three organisations as well as data for regions such as the Isle of Man; in addition, all available gravity data for the land and surrounding seas was to be obtained from the many different bodies that held it, and made available for the project. The successful bidders were a team from Kort og Matrikelstyrelsen (the Danish national mapping and cadastral agency) and the Department of Geomatic Engineering at University College London. The project commenced in September 2001, and the final products were delivered to the consortium of Ordnance Surveys in April 2002.

In April 2001, a paper in this journal [**6**] had given a comprehensive review of the development of geoid models over the British Isles. After reviewing the successive attempts at determining geoid models of differing levels of accuracy, from astro-geodetic solutions, through global geopotential models, to different attempts to combine local gravity and terrain data with long wavelength solutions, the authors concluded that previous geoid models were inadequate for GPS purposes, and instead set forth a set of proposals. In brief, these accorded with the aims of the Ordnance Surveys' commission: to produce a high accuracy gravimetric geoid from all available gravity and terrain data, and a separate series of models tailored to the local height datums.

As envisaged by [**6**], the project essentially split into two main phases. In the first phase a pure gravimetric geoid was computed using long wavelength geoid models, and local terrain and gravity data. No information about the local vertical datums, in the form of levelling or GPS data, was used in this phase. In the section that follows, a brief description of the methods and summary of the data will be given; a condensed account of this phase of the project, with references to the theoretical background, may be found in [**9**]. It is the intention of the authors to focus in this paper on the second phase of the project, that of fitting the gravimetric geoid to the different local vertical datums, and describing the problems that were encountered in this procedure and assessing the accuracy of the final product. Thus, the emphasis of this paper is on what might be termed the professional use of the product: how well does it function as a "correction surface" that can be used to convert GPS data into the equivalent of levelling?

This combination of the computation of an independent gravimetric geoid, followed by fitting to local GPS and height measurements, is a good compromise between intensive computations (for the gravimetric geoid) and more easily managed updates, as new levelling and GPS points become available. The procedure has been used extensively in Scandinavia [**8**] as well as in the United States [**19**].

THE GRAVIMETRIC GEOID

The gravimetric geoid has been computed from gravitational and topographic data using spherical fast Fourier transforms (FFT). A remove-restore technique was used, in which the anomalous gravitational potential T [**11**] is split into three components [**9**], thus:

$$T = T_{EGM96} + T_{RTM} + T_{res} \qquad (1)$$

Here, T_{EGM96} represents the contribution from a global field (EGM96) that provides the long wavelength information; T_{RTM} is the contribution from the topography; and T_{res} is the effect of the residual gravity anomaly field after reduction for the effect of the first two components.

Taking each of these elements in turn, the global field used in the computation was the EGM96 spherical harmonic model, which is complete to degree and order 360 [**14**]. The EGM96 effects were all computed at the surface of the topography, technically as quasi-geoid heights (subsequently converted to classical geoid heights at the end).

For the computation of the terrain effect, the initial requirement was a dense digital elevation model (DEM) covering the land areas of the British Isles: the requirement set was for a 100 m grid spacing. This data was duly supplied by the Ordnance Surveys and by the government of the Isle of Man, mostly in the form of 10 m or 50 m grids that were then thinned by an averaging process to give the required density whilst making data volumes manageable. Some gaps in the data for The Republic of Ireland were present, where some individual tiles of data were not available. There were around five of these that covered areas of a few 10s of square kilometres each. To replace the missing data, a 30" x 30" gridded DEM was obtained from the National Geophysical Data Center in the United States [**10**].

Having created a combined DEM for the whole region, its effect T_{RTM} was computed using prism integration from a residual terrain model (RTM) that gave the terrain height above a smoothed reference surface. The latter had been created by smoothing the original dense DEM to a resolution of around 66 km, slightly larger than the half wavelength of EGM96 (it being a principle of the RTM terrain method that the resolution does not have to match the spherical harmonic resolution exactly: 66 km is a trade-off which gives slightly higher smoothing of the residual anomalies).

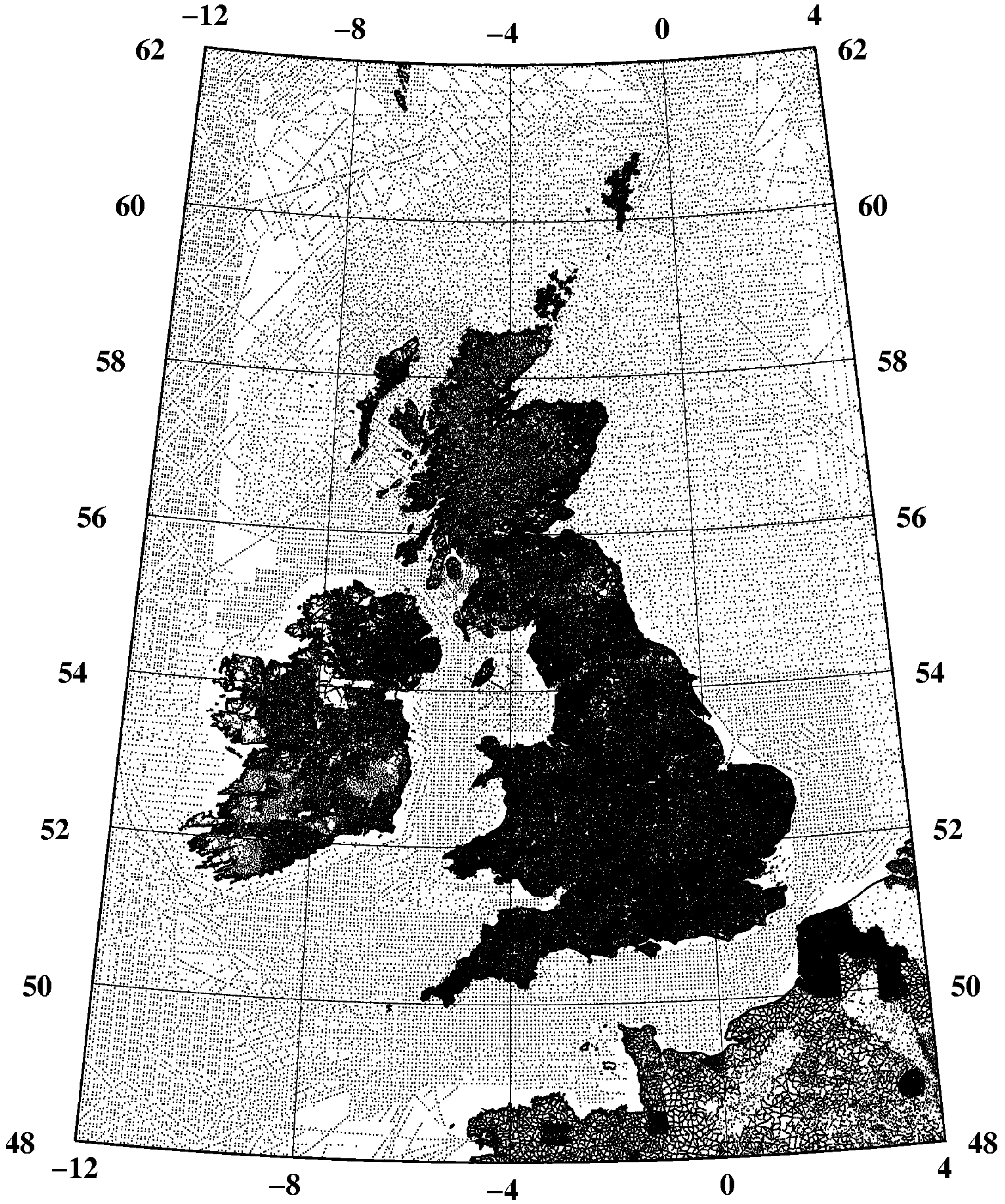

Fig. 1. Station locations for the selected gravity data. Satellite altimetry derived gravity was used to supplement the marine gravity in the westernmost area.

For the gravity anomalies, data was obtained from several different organisations. The original owners of the data included: KMS itself; the Dublin Institute of Advanced Studies; the British Geological Survey; the United Kingdom Hydrographic Office; the Geological Survey of Northern Ireland; and the national survey authorities of France, Belgium, the Netherlands, and the Faeroe Isles. After checking for outliers, a total of 142826 land gravity observations and 15900 marine gravity observations were used after selective thinning and homogenisation of very dense data; these were supplemented by 14719 values derived from the KMS99 model that itself was derived from satellite altimetry observations [**1**]. Gravity anomalies were derived from IGSN71 absolute gravity values, and in the case of BGS anomalies had to be converted from GRS67 into the gravity system based on the GRS80 ellipsoid. Away from the coast, altimetry-derived anomalies were verified using marine gravity values. The tidal system of the gravity data was not described, so it was assumed data were given in the zero-tide (absolute) system.

The distribution of the final gravity anomaly data set is illustrated in Figure 1. As indicated by [**6**], this is one of the densest coverages of gravity data in the world on this scale. In most parts of Great Britain, for example, the point density is above 0.75 points per square kilometre, falling to around 0.3 as a representative value for the Scottish highlands. In Ireland the points are sparser, but still above the required density of 0.25 points per square kilometre.

In a similar way to Equation (1) the quasigeoid ζ is split into components:

$$\zeta = \zeta_{EGM96} + \zeta_{RTM} + \zeta_{res} \quad (2)$$

and similarly for the classical geoid N:

$$N = N_{EGM96} + N_{RTM} + N_{res} \quad (3)$$

The conversion from the quasigeoid to the geoid is given by the following equation:

$$\zeta - N \approx \frac{\Delta g_B}{\gamma_0} H \quad (4)$$

where Δg_B is the Bouguer anomaly, H is the orthometric height and γ_0 is the normal gravity on the ellipsoid [**11**]. It should be noted that (4) is in principle exact if Helmert orthometric heights are used; however, the levelling systems and computations of the up to 100 year old British Isles levelling are not sufficiently well-defined to treat (4) as more than an approximation.

A full description of the construction of the gravimetric geoid is beyond the scope of this paper, but further information can be found in [**9**]. Briefly, ζ_{RTM} is constructed by spatial domain prism integration on a spherical Earth and ζ_{res} by Stokes' integration in the Fourier domain (spherical FFT using the multi-band method, cf. [**7**]). Different modifications of the Stokes' kernel were tried. The best results were obtained by a very weakly modified kernel (assigning full weight and no errors to EGM96 only for spherical harmonic degrees less than 12), which indicates that the residual gravity data contain long wavelength information after subtraction of the global field. In the final step the classical geoid was derived from the quasi-geoid using Equation 4.

The final form of the OSGM02 gravimetric geoid is illustrated in Figure 2.

FITTING PROCESS – PRACTICAL ASPECTS

As stated in the introduction, the primary focus of this paper is on the process of fitting the gravimetric geoid to the local datums. This section will give a general introduction to the important issues in this process; the following section will cover the mathematical basis.

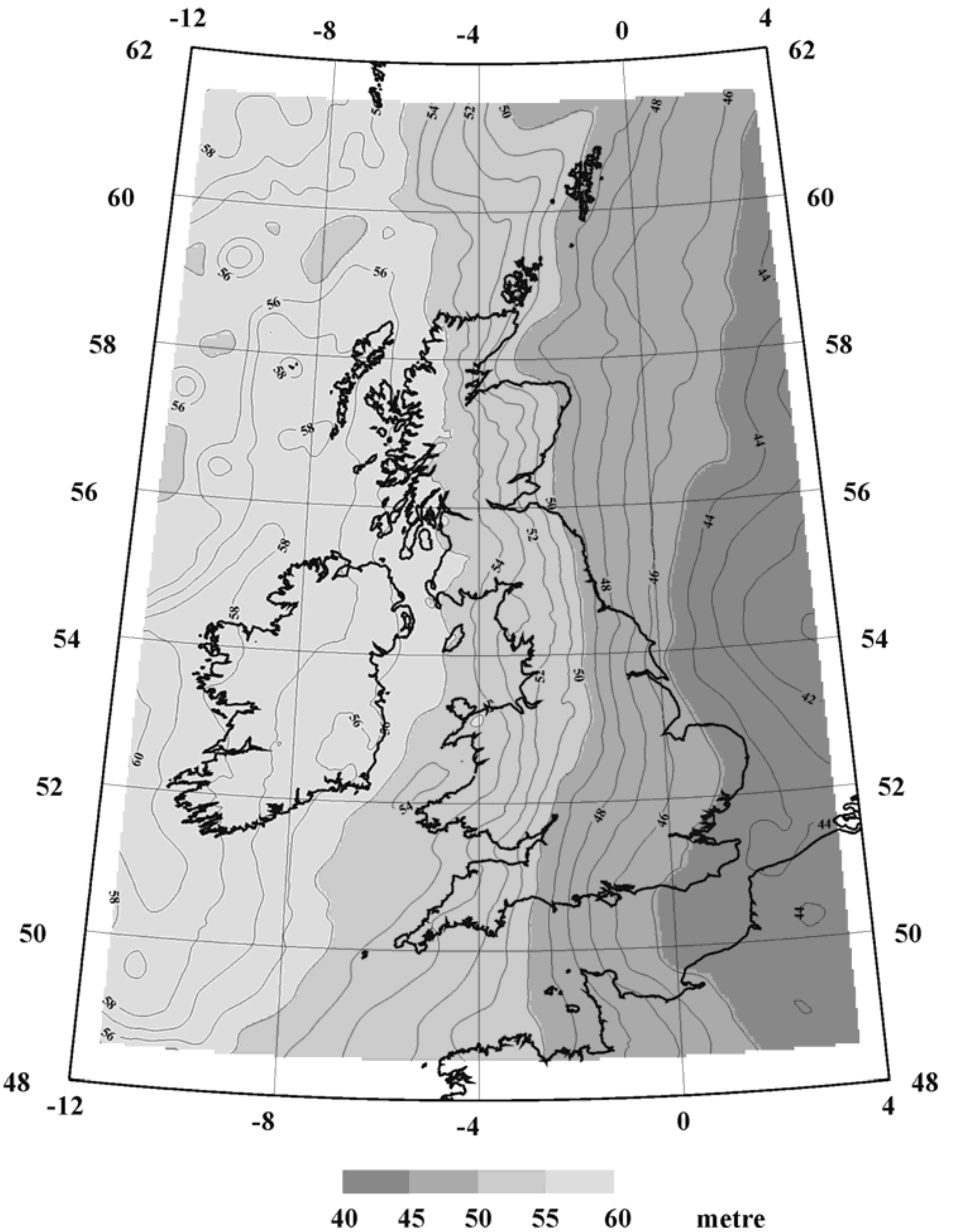

Fig. 2. The final OSGM02 gravimetric geoid, referred to WGS-84.

Definitions and datums

Apart from scientific studies, which will generally use a pure gravimetric geoid, the primary reason for doing work of this kind is in order to be able to use GPS observations to measure heights. As is well known, the conventional method for surveying heights involves levelling to one or more nearby benchmarks, which are themselves realisations of a vertical datum established with respect to a reference tide gauge. Satellite systems such as GPS, on the other hand, give relative or absolute heights with respect to an ellipsoid such as WGS84. The requirement for a fitted geoid is therefore stated in its simplest form as: *to provide values that can be used to convert GPS heights into what would have been achieved through levelling.* The values of the surface required can hence be defined as:

$$N_L = h_{ETRF89} - H_{Local\ Datum} \quad (5)$$

In this expression, h_{ETRF89} is the height obtained through GPS or other satellite techniques in the datum that is appropriate for the region. It is slightly too simplistic to refer to H as the orthometric height above the geoid. Although the intention may have been to provide Helmert orthometric heights [**6**] or normal orthometric heights, the practical realisation of these through published benchmark values is affected not only by a biased evaluation of mean sea level at the original tide gauge, but also by the gradual accumulation of errors in the levelling networks and theoretical limitations to the definition of these height schemes. Since the intention is to provide a seamless transition between existing height data and new work established with GPS, the existing networks have to be adopted with their errors intact, and these therefore form part of the fitting procedure.

It is therefore apparent that the values of N_L so found cannot strictly be referred to as geoid separations. It is inevitable and understandable that they will, and the authors are guilty of contributing to this by such solecisms as using the symbol N. However, it should be placed on record here that the preferable – if less appealing – term is *GPS conversion surface*.

In the United Kingdom and Ireland there are three main vertical datums and several minor ones in use. For Great Britain the datum is Ordnance Datum Newlyn (ODN), which is based upon measurements of mean sea level at Newlyn tide gauge between 1915 and 1921 [**6**,**4**]. The datum in the Republic of Ireland is based upon mean sea level measurements made at Malin Head in County Donegal between 1958 and circa 1970 [**6**,**17**]. In Northern Ireland heights required on an all Ireland basis (typically for small scale mapping) are related to Malin head; otherwise the primary datum has been established by tide gauge readings at Clarendon Dock, Belfast, over the six years 1951 – 1956 [**17**].

Separate datums are in use on several of the other island groups. On the Isle of Man, a datum has been established at Douglas; the Outer Hebrides use a datum at Stornoway; the Shetlands one at Lerwick; and the Scilly Isles one at St. Mary's. The island of St. Kilda also has its own separate height system. The islands that form an exception to this are the Inner Hebrides and the Orkneys, where connections to the mainland have largely brought their height systems into the Newlyn datum. The connections were not of geodetic quality, however, and these islands required careful treatment which will be discussed in the results for Great Britain below [**5**].

This proliferation of separate datums required one early decision to be made. In theory, it would be possible to create a single continuous correction surface across all the different regions of the United Kingdom and Ireland by warping the gravimetric geoid to all GPS/levelling points simultaneously. In practice, however, this is likely to lead to significant problems, particularly in border regions, where a smoothed transition would be created rather than the abrupt one required. Therefore, it was decided at the outset that several *separate* GPS correction surfaces would be created, each to be applicable in separate regions.

GPS and levelling data

To derive a correction surface in each region, the basic requirement is a set of points where both levelled heights in the existing datum and GPS heights in the global system are known, in such a way that individual evaluations of Equation 5 can be implemented. Some comments on the levelling and GPS networks in each region are necessary.

In Great Britain, the heights of the fundamental bench marks are initially related to the second geodetic levelling that was carried out from 1912 to 1921 over most of England and Wales [**13**]. This was extended to northern England and Scotland between 1936 and 1952, and was adjusted by holding fixed the junction points with southern sections [**4**]. The third geodetic levelling was carried out between 1952 and 1956 [**12**], including all existing bench marks from the second levelling, but extending more comprehensively over the whole country. Although analysis of loop misclosures indicated that both networks were within the recommended tolerance of 2 mm √km, a discrepancy of 175 mm became apparent between the two solutions in northern England and Scotland. Combined with an apparent rise in sea level of 250 mm from Newlyn to Dunbar, this led to suspicions of a systematic north-south bias in the work [**6**,**13**]. Rather than publish a sudden change in the bench mark heights, the Ordnance Survey of Great Britain re-adjusted the third geodetic levelling by holding fixed the heights of the fundamental bench marks that had been derived from the second geodetic levelling [**12**], to which orthometric corrections had been applied [**4**]. In summary, the dominant effect upon the vertical datum is the second geodetic levelling of 1912 – 1921 and 1936 – 1952.

The ETRS89 coordinates of the fundamental bench marks in Great Britain were established as part of a GPS campaign carried out between January and March 1999 [**18**]. The formal standard errors for the height component of this work are at the 10 – 20 mm level, although with some higher outliers [**18**].

The geodetic levelling for Northern Ireland was completed between 1951 and 1957, and included nine fundamental bench marks: of these, five still exist [**20**] but only one was subsequently used as a GPS tie point. The geodetic levelling in the Republic of Ireland was never carried out at a single identifiable epoch, but rather as a rolling programme up to the mid 1970s. Links from the Northern Irish levelling to that in the Republic and to the Malin Head datum were made in 1956; after application of orthometric corrections by the Ordnance Surveys of Ireland and Northern Ireland, a least squares adjustment of the primary levelling for the whole island of Ireland was carried out by the OSGB in 1970 [**20**]. However, although some heights in Northern Ireland are hence known in the Malin datum, as pointed out above it is the earlier adjustment with respect to the Belfast datum that forms the official set of heights for the north shown on detailed large scale mapping.

The ETRS89 coordinates of points in both Northern Ireland and the Republic of Ireland were observed as part of the IRENET95 campaign [**2**]. The accuracy of the zero order and densification points has been estimated as 20 mm in height [**2**]. However, given that only one fundamental bench mark and six second order points were used as GPS tie points in Northern Ireland, and all other northern and southern Irish GPS tie points were based on tertiary levelling, it is clear that levelling is likely to present the major part of the error signal.

Figure 3 is a map showing all the points where both GPS and levelling data are available, and hence where evaluations of N_L can be made. For clarity, points on separate island datums, or additional points in the Inner Hebrides and Orkneys that were subsequently included are not shown at this stage. On Great Britain, there are 180 points available, all of which are fundamental bench marks. The average point density is 1280 km^2/point, giving a mean inter-station distance of approximately 35 km. The maximum inter-station distance is 80 km, meaning that any one point can be up to 40 km from a GPS/levelling station, although a more characteristic range is 15 – 20 km. In Northern Ireland there are 38 points; the maximum distance to a GPS/levelling point is 20 km, and the characteristic distance is 10 km. For the Republic of Ireland, there are 149 points; the maximum distance to a GPS/levelling point is 30 km, but 10 – 15 km is a more representative figure.

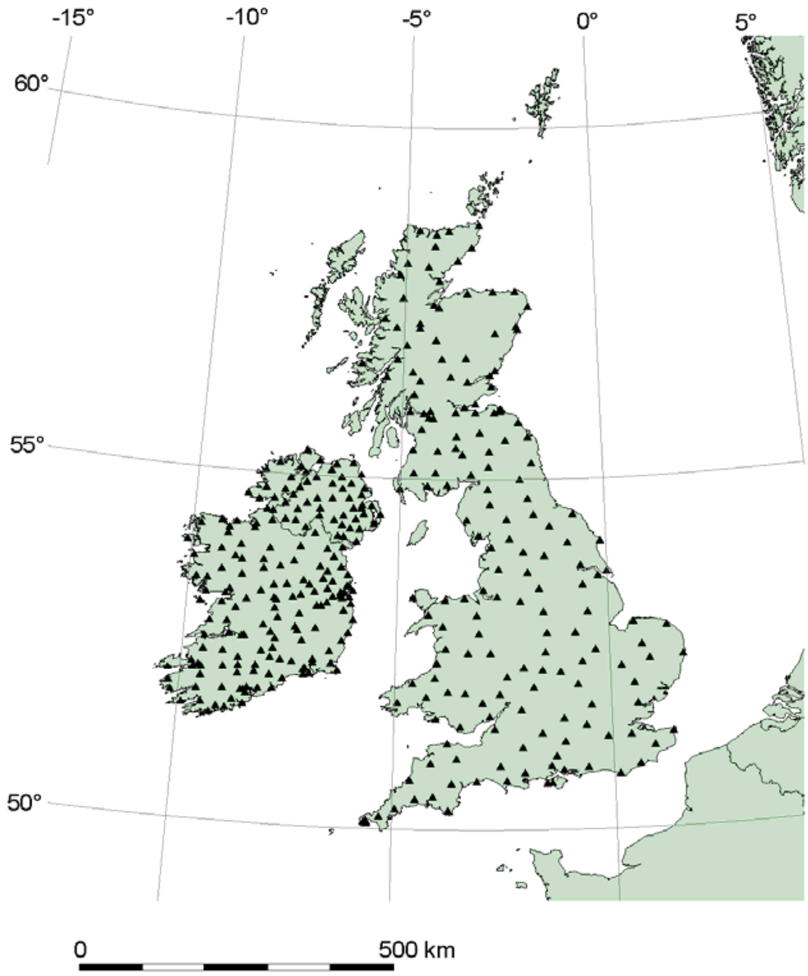

Fig. 3. GPS and levelling points available for fitting.

Error sources

The previous sections have commented on the quality of the GPS and levelling networks. However, it is necessary to have a complete understanding of all the errors that can affect the process of predicting the values of N_L. It is useful to think of these in terms of long and short wavelength, with the critical distance being the (admittedly variable) distance between the points where GPS and levelling information is available.

There are several potential sources of long wavelength error, such as the systematic trends in the levelling networks noted above, or indeed in the GPS networks. It is also possible that regional systematic trends are present in the gravity networks. However, the important point about all of these is that they are effectively absorbed into the fitting process: that is, since the aim of the procedure is to recover the bench mark heights and the latter are themselves used in the computational procedure, any errors present will be worked into the correction surface.

A similar point applies to the effect of regional uplift or subsidence, such as post-glacial rebound [**13**]. These phenomena are present due to the difference in the epochs at which the levelling and the GPS were observed. In effect, a hypothetical measurement of ellipsoidal height made at the same epoch as the levelling would have produced a different result to the late 20th century ones used here. Once again, however, the use of the bench mark required as the final answer ensures that this effect is cancelled out. What may be the case, however, is that changes in the epoch at which the levelling was observed (for example the time difference between the second geodetic levelling of northern England and Scotland compared to the rest of Great Britain) will lead to differences in the value of the correction surface. Comments about the phenomena observed are made in the results sections below, although it should be noted that this effect is difficult to isolate from general systematic trends in the levelling.

In passing, it should be noted that any *future* ground movement *will* introduce errors when using the correction surface to recover bench mark heights, although the effect will be largely mitigated if the GPS observations are essentially relative ones using a consistent base network.

A more important concern for these purposes is ground movement on a localised scale, and the most significant potential source of this is mining subsidence. There are two forms to this problem: the first is that between the levelling and the GPS observations one of the points used as a tie point has been affected in this way, and hence the danger that the correction so derived will be applied over a wider area than is strictly valid. The second is that a bench mark *not* used as a control point has been so affected: the subsidence will not be modelled, and therefore the correction surface applied to future GPS observations will give a different answer to the levelling.

The chances of an error of the first type are lessened to a certain extent by the fact that, in Great Britain at least, most levelling of secondary and higher order has been deliberately planned to avoid areas of potential subsidence [**12**]. Additionally, errors of this type can to an extent be controlled by monitoring outliers when fitting the gravimetric geoid.

The second type of error is therefore both more likely to occur and to be undetected if it does. [**3**] indicates that the most serious subsidence (that over a large area due to collapse following extraction mining) usually occurs within three years of the cessation of mining activity; older pits can cause subsidence at any time, but their area of influence is limited. Maps are available of general coal mining areas, but no efficient method has been found of identifying points where subsidence is a potential problem. It is therefore quite likely that some points will have been affected in this way, and will not have been identified. What is questionable, however, is whether this should be seen as a consequence of the adoption of GPS technology. It has never been the policy of Ordnance Survey Great Britain, at least, to maintain consistent bench mark heights in areas affected by subsidence [**3**]: the problem is usually expressed in terms of the expense of re-levelling. Thus, a user who has consistently tied surveys to a local bench mark that had been affected by subsidence and who finds that corrected GPS observations do not then agree, would in an alternative world have had an identical problem when periodic re-levelling eventually took place.

Another source of short wavelength errors is the random errors in the levelling and GPS networks, of which the most significant is likely to be the levelling. Although the form of the gravimetric geoid in between the tie points has been modelled, there is no way of predicting the errors in the levelling and yet these are in effect a part of the vertical datum that it is required to recover. As these errors are effectively cancelled out wherever the vertical datum and the geoid are “tied together” at a GPS/levelling point, the effect will be at its most extreme in between the tie points. Therefore, the density of the control points used is the critical factor.

From this point of view, it can be seen from Figure 3 and the accompanying discussion that the two Irish networks are better placed than the British one, as they have a significantly greater point density. Set against this, however, is the fact that this has only come about because they are incorporating lower order levelling surveys into the set of tie points. The tie points on Great Britain, on the other hand, are using only fundamental bench marks. The effect is likely to be that the greater point density is cancelled out to a greater or lesser extent by the increased noise at the tie points.

To give this error source an order of magnitude, recall that the worst case in Great Britain is that a point can be up to 40 km from the nearest tie point (this actually occurs in the Shropshire/Staffordshire area). If it is assumed that the levelling that connects this point to the nearest tie point is predominantly of second order, then by the propagation of second order errors over a single levelling run this would reach 0.06 m. Assuming minimal redundancy gives the approximate figure 0.04 m. Slightly smaller figures result for the Irish networks, but given the higher standard errors of the tie points themselves, it seems safer to adopt 0.04 m as a general indication of the discrepancy likely to occur (in the more extreme cases) when comparing existing levelling data against corrected GPS observations, *in the absence of other errors such as disturbance of the bench marks.*

FITTING PROCESS – THEORETICAL BASIS

Following the terminology of Equation (5), values of N_L are required continuously throughout the region; point values have been obtained at discrete locations where GPS and levelling data coincide. The process then proceeds by modelling the difference function

$$\varepsilon = N_L - \mathrm{N}_{\mathrm{gravimetric}} \tag{6}$$

and adding the modelled ε-correction to the gravimetric geoid. A necessary first step in this procedure is to remove a trend surface. The simplest of these is a constant bias, and this has been used when dealing with small island datums with only a few points. More generally, a four parameter Helmert model has been used:

$$\varepsilon = a_1 \cos\phi \cos\lambda + a_2 \cos\phi \sin\lambda + a_3 \sin\phi + a_4 R + \varepsilon' \tag{7}$$

where ε' represents a residual to be modelled by least squares collocation, ϕ is latitude, λ is longitude, and R the Earth radius. In order to do this a covariance model is defined in order to determine the covariance as a function of distance, *s*. In this case, a relatively simple second order Gauss-Markov model was used:

$$C(s) = C_0 (1 + \alpha s) e^{-\alpha s} \tag{8}$$

C_0 is the variance at zero distance, and α is a constant to be specified. The effect of specifying α is to set the correlation length (the distance where the covariance falls to half the zero variance). The importance of this value is that it determines the "stiffness" of the gravimetric geoid when fitting to the GPS and levelling data, provided that the N_L values are assigned realistic a priori standard deviations. A short correlation length implies that the gravimetric geoid does not have the "strength" to over-ride short wavelength errors in the GPS and levelling networks. It was decided in this case to define a relatively "stiff" surface with a correlation length of 50 km as standard throughout the project. The effect of this is that in the final surface produced the shorter wavelengths will be controlled by the gravimetric geoid, and it is only the long wavelength trends that are adjusted to follow the levelling.

Having defined the covariance function, the solution proceeds by a standard least squares collocation procedure [**15**].

RESULTS – GREAT BRITAIN

The first phase of fitting the gravimetric geoid in Great Britain involved a straight comparison of the values of N_L with N_G at the 180 fundamental bench marks on the mainland. At this stage, one point was identified as an outlier: it was subsequently discovered that the marker had been damaged and never re-connected to the network with geodetic accuracy. The differences between the two determinations of N at the remaining 179 points are shown in Figure 4.

In this figure the main feature that is apparent is the presence of a distinct systematic trend, here having a range of 0.51 m. This recalls the comments previously made about the presence of a north-south systematic bias, but it is clear from the figure that a closer description would be that it lies along the longitudinal axis of the country. Although the theoretical causes of this effect include errors in the gravimetric geoid and the GPS network, we believe that the only realistic cause is errors in the levelling network.

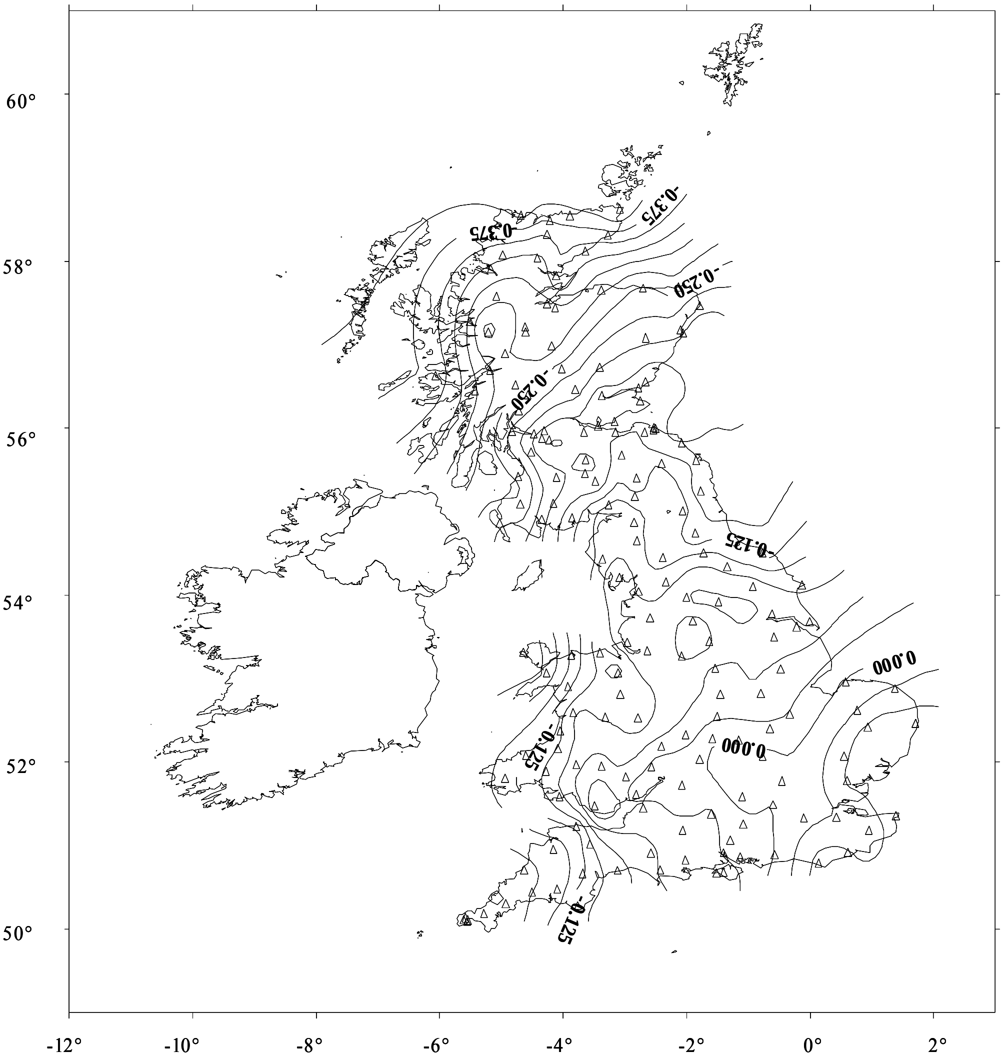

Fig. 4. Residual geoid values on Great Britain.

Interestingly, the increase in the slope of the surface northwards of the north of England is consistent with the change in epoch between the two phases of the second geodetic levelling if a gradual tilt of the north of the country is postulated. However, we have no independent evidence of this effect, and the magnitude would need to be investigated further before coming to definite conclusions.

In the next phase of the process, the gravimetric geoid was fitted to the fundamental bench marks using the methods previously described, in order to compensate for the long wavelength errors. Before the fit (after subtracting a 4-parameter trend surface as in equation 7) the standard deviation of the residuals was only 0.032 m (without the trend surface the residuals had had an r.m.s of 0.170 m). After the fit, the extremely high quality of the gravimetric geoid became finally apparent, as the overall standard deviation of the post-fit residuals was 0.004 m, with the greatest being 0.018 m. Of course, it can be argued that if the tie points are sufficiently far apart then the residuals will naturally tend to zero as the long wavelength errors are absorbed. One of the best methods of assessing the accuracy is therefore to examine pairs of tie points that are close together and to see how effectively the gravimetric geoid models the differences in N_L. Table 1 shows all pairs of fundamental bench marks that are closer to each other than 20 km, a distance that is much less than the correlation length of 50 km.

Table 1. *Geoid differences between closely spaced tie points.*

From	To	Distance (km)	ΔN_L (m)	$\Delta(N_L - N_G)$ (m)
Loch Naver	Betty Hill	18.3	- 0.08	0.001
Aberdeen North	Aberdeen South	4.9	- 0.04	0.007
Scorguie	Daviot	9.5	- 0.21	- 0.007
Glen Moriston	Loch Tarff	7.3	0.12	- 0.008
Partick	Necropolis	7.3	- 0.08	0.027
Bowling	Milngavie	11.4	- 0.14	- 0.004
Wester Broomhouse	Dunbar West	2.8	- 0.08	0.020
Dunbar West	Dunbar East	2.2	- 0.10	- 0.014
Immingham	Patrington	15.2	- 0.52	0.000
Sancreed	Tolcarne	4.6	- 0.08	- 0.001
Paul	Madron	5.2	0.05	- 0.011
Freshwater	Calbourne	8.4	- 0.19	- 0.004
Southampton	Winchester	18.4	0.08	- 0.001

N_G is the value of the fitted gravimetric geoid. N_L is defined in the text.

It is noted that even quite sharp variations in the geoid, such as the 0.52 m change over 15 km between Immingham and Patrington, are well modelled by the gravimetric geoid.

Although the network of fundamental bench marks is the most accurate set of points available, it is noted that it does not extend to the Inner Hebrides (where the Newlyn datum is used) or indeed to the west coast of the highlands of Scotland. Thus, although interpolation between tie points is likely to be well controlled over most of the mainland, there is a risk of serious errors arising in the Inner Hebrides through uncontrolled extrapolation. A strategy was therefore developed whereby an additional

18 tie points (from lower order levelling) were introduced on the western fringes of the mainland and on the Inner Hebrides (particularly on Mull and Skye). Since these new points were of a lower order, and because problems were anticipated with the connections to the mainland (trigonometric heighting has been used extensively in this area) they were down-weighted to a standard error of 0.05 m, compared to 0.02 m for the fundamental bench marks.

The resulting statistics of the fit (for the whole of Great Britain, including the original points) show a standard error of 0.013 m and a maximum residual of 0.08 m. As a sudden deterioration in the quality of the gravimetric geoid is unlikely, it is therefore apparent that a much noisier set of levelling data is being used here. The down-weighting of the points is a reflection of the fact that there are dangers in attempting to follow too closely the errors in the levelling network and thus perpetuating them in the GPS correction surface: the surface derived effectively represents a compromise between attempting to follow the existing bench mark heights and not deviating too far from an orthometric height system. Nevertheless, a greater deviation between the correction surface and the gravimetric geoid can be expected in this region, and it may be appropriate for some precise engineering applications to explore the possibility of using the gravimetric geoid alone.

A similar procedure was initially attempted for the Orkney Islands, until it became apparent that there were unacceptable inaccuracies in the links to the mainland. For this reason, the Orkneys were given the same status as other independent island datums, and a separate fitting of the gravimetric geoid was carried out.

Summary statistics for the fittings to the different island datums are given in Table 2. Note that the Isle of Man is included here due to the similarity of treatment to other islands.

Table 2. *Statistics of fit for island datums*

Area/ Datum	No. points	Pre-fit st. dev. (m)	Post-fit st. dev. (m)	Maximum residual (m)
Shetland/ Lerwick	13	0.038	0.030	0.053
Orkneys/ Modified Newlyn	15	0.098	0.084	0.199
Outer Hebrides/ Stornoway	8	0.209	0.088	0.143
Isle of Man/ Douglas	3	0.109	0.027	0.028
St. Kilda/ St. Kilda	3	0.055	0.055	0.067
Scilly Isles/ St. Mary’s	1	-	-	-

For all the datums shown in Table 2, the pre-fit standard errors are computed from the raw comparison between the gravimetric geoid and the GPS/levelling values. The post-fit residuals are computed after fitting by the standard procedure, using a 50 km correlation length; all points that had been spirit levelled were given a weight of 0.05

m, and those that had been trigonometrically heighted were given a weight of 0.10 m. The modulus of the maximum residual is given in the last column.

It is evident from Table 2 that some of these island height networks contain significant systematic effects and that they are of significantly poorer quality than was encountered on the mainland. Once again a compromise has been reached between following the existing height system and maintaining an orthometric system. An examination of the way in which the standard error on small islands (e.g. St. Kilda) has hardly been affected by the fitting procedure shows that the 50 km correlation length ensures that over short distances it is the gravimetric geoid that predominates.

Comments on the relationships between the different datums are given in a later section.

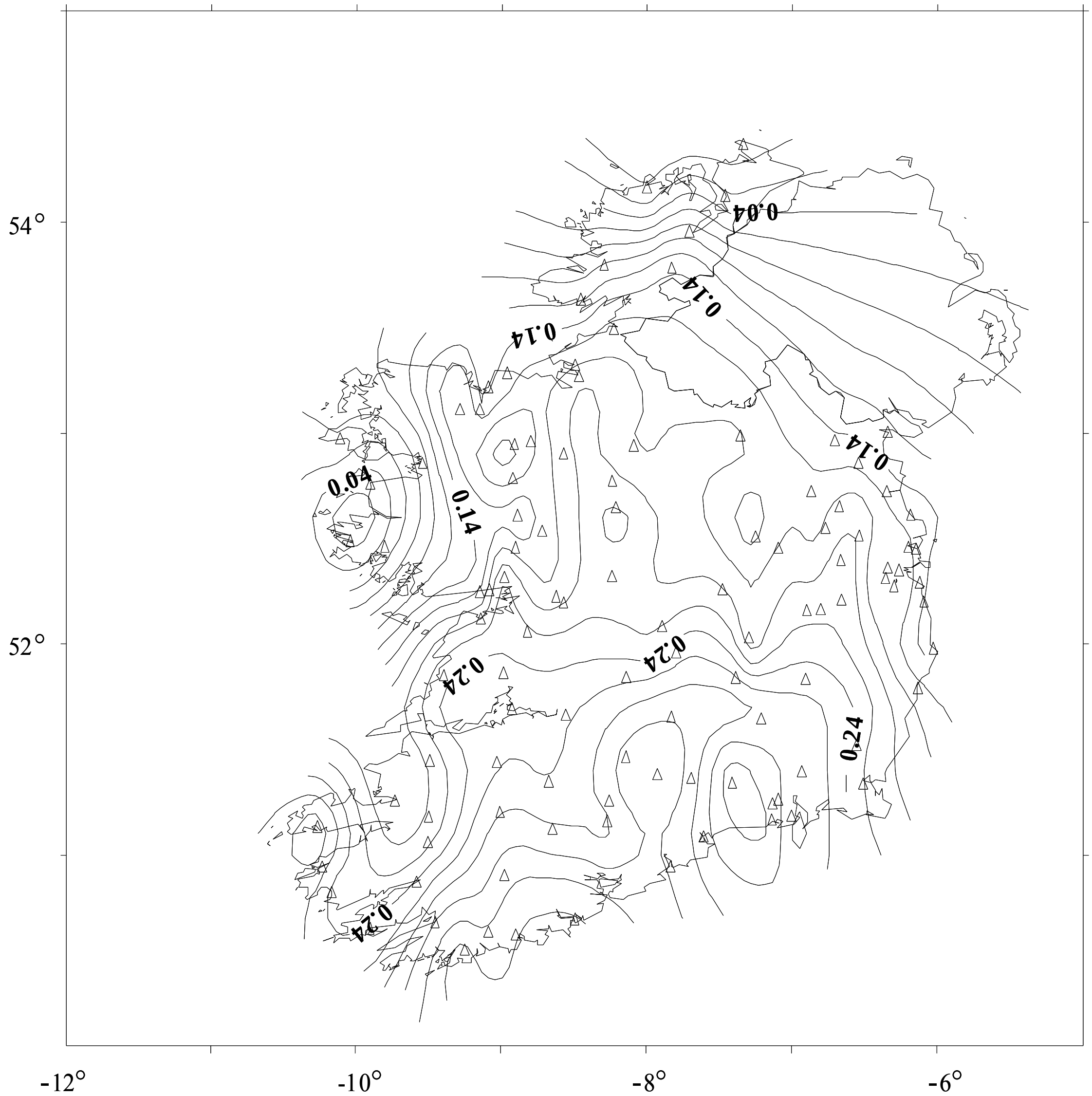

Fig. 5. Residual geoid value in the Republic of Ireland (filtered data set).

RESULTS – REPUBLIC OF IRELAND

The procedure adopted in the Republic of Ireland was essentially similar to that used for Great Britain, although modifications had to be made as problems were encountered with outliers. With only minor differences in the data sets used to compute the gravimetric geoid, we believe that these outliers are due to the lower overall quality of the levelling used (secondary and tertiary points as opposed to uniformly primary) and more serious problems at individual stations.

Initially, 133 points were available where GPS and levelling data were known. An initial comparison with the gravimetric geoid showed residuals with a standard error of 0.095 m and a range of 0.502 m. By using a 2.5σ rejection criterion (applied iteratively) during the fitting procedure, 23 points were filtered from the data set. The direct comparison between the gravimetric geoid and the correction surface using only the filtered data set is shown in Figure 5. As with the data on Great Britain, some systematic effects are once again visible: a general trend in an approximately north-south direction is accompanied by a separate trend in the north west (the Mayo/Galway peninsula).

The gravimetric geoid was then fitted to the tie points using the same techniques as had been used in Great Britain. The same 50 km correlation length was used, but this time a weighting strategy was adopted whereby secondary points were given a standard error of 0.030 m; tertiary points with spirit levelling 0.050 m; and tertiary points with trigonometric heighting 0.100 m. The final standard error of the residuals of the fitted surface was 0.024 m; the range of residuals was 0.114 m. It is clear that potential problems still exist with this correction surface: sharp discontinuities in the residuals of several centimetres exist over quite short distances, and it would be surprising if these were not reflected in subsequent discrepancies between archive height data and GPS data reduced using the correction surface.

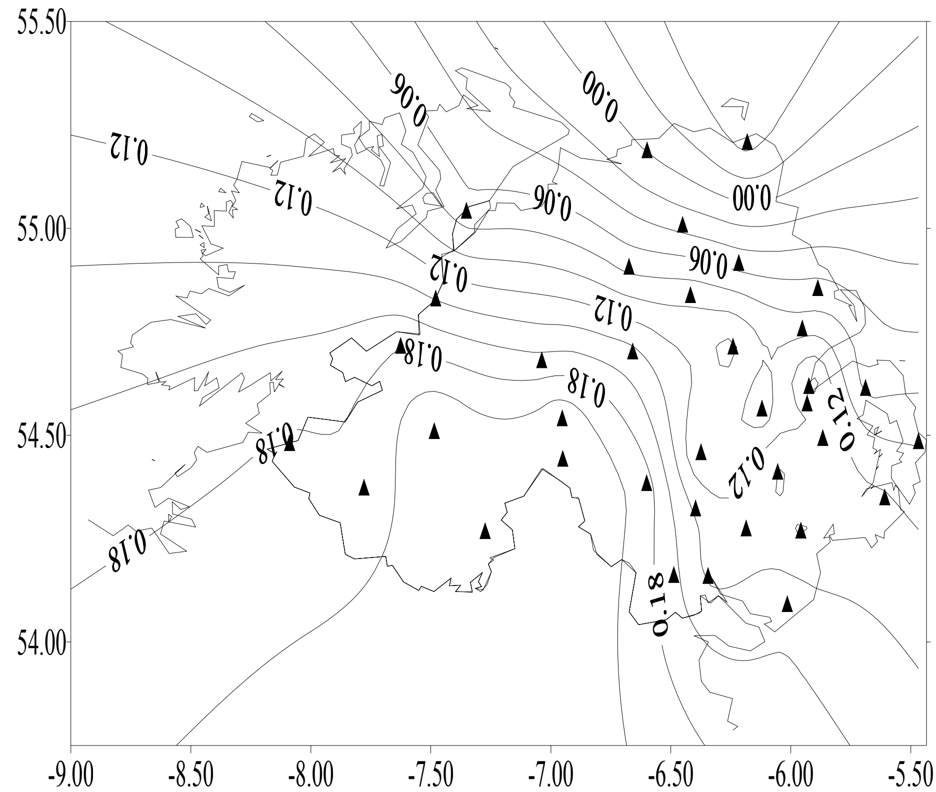

Fig. 6. Residual geoid values in Northern Ireland.

RESULTS – NORTHERN IRELAND

In Northern Ireland 38 tie points were initially available for fitting the geoid. After an initial comparison between the gravimetric geoid and the GPS/levelling data, two points were identified as outliers and removed from the data set. The comparison for the remaining 36 points is shown in Figure 6.

Once again a pronounced systematic effect is visible. The approximate orientation of this is 25°: it is noticeable that it is far from being parallel to the "north-south trend" in Great Britain, and that it is more pronounced. These facts are likely to eliminate some potential explanations of these phenomena.

The gravimetric geoid was fitted to the tie points by the previously used method, with the same weighting strategy as had been adopted for the Republic of Ireland. The residuals showed a standard error of 0.019 and a range of 0.076. As expected, with lower orders of levelling being used, the agreement is not as good as was the case on the mainland of Great Britain; however, fewer outlier problems have been identified than for the Republic of Ireland. It should also be recalled that the inter-station distances (between tie points) are generally less than in Great Britain, and the development of levelling errors between points is likely to be more constrained.

DATUM COMPARISONS

As previously stated, the vertical datum is established at a tide gauge and realised by a levelling network. Due to the observational errors in the levelling, the offset between *the* geoid (in the sense of an internationally accepted vertical datum) and the datum in use in any particular region is therefore a variable quantity. Diagrams such as Figure 4 are essentially maps of the differences between the global and a local datum (although incorporating some GPS and geoid errors). A single average value for the offset of each local datum to the global vertical datum of the gravimetric geoid has been calculated and shown in Table 3. This was computed from the mean difference between the geoid separation implied by the GPS/levelling points and that given by the gravimetric geoid.

Table 3. *Estimated vertical datum offsets.*

Datum	Offset to global datum (m)
Newlyn (GB FBMs)	- 0.160
Malin (Eire)	0.201
Belfast	0.124
Orkneys	-0.461
Stornoway	-0.219
Lerwick	- 0.209
Douglas	0.064
St. Kilda	-1.761
St. Mary's	- 0.71

Although Table 3 shows the offsets between the datums as an overall average, it follows from the previous argument that local distortions are likely to have just as great an effect when making comparisons across borders or short stretches of sea. For example, the data above imply that the Stornoway datum is offset by 0.059 m from Newlyn. However, comparing the values of the correction surface across the shortest stretch of water between the two regions (from Skye to Benbecula) shows a change of 0.352 m, which should be mostly attributed to the datum change rather than a change in the gravimetric geoid. Similarly, the mean difference between the Newlyn and Malin datums is 0.361 m; an extrapolation of the mainland British correction surface to five points in the Dublin region shows a mean separation between the datums of 0.330 m. The mean difference between the Malin and Belfast datums is 0.077 m, whereas at the border where it is possible to level between bench marks in the different systems the best estimate of the offset is 0.038 m.

It must be emphasised that any attempt to derive such values will be influenced by distortions in the local networks (particularly on the island datums) and different values will result depending on the geographic selection. For scientific or engineering applications, the use of the gravimetric geoid will give the best estimate of the orthometric height difference between points.

CONCLUSIONS

A new gravimetric geoid model of Great Britain and Ireland has been computed, using for the first time all the existing gravity data and high resolution DEMs. The high quality of the agreement between this geoid and the fundamental bench mark network on the mainland of Great Britain indicates the high accuracy of this computation.

This surface has been individually fitted to the different datums used across the region, resulting in a GPS correction surface that is termed OSGM02. The degree of accuracy depends upon the quality of the original levelling network: in the best of cases (the fundamental bench mark network of Great Britain, excluding some of the Scottish Isles and parts of the west coast of Scotland) this is with a standard error of 0.004 m. However, in all cases the agreement that users will notice between corrected GPS observations and existing height data will vary geographically as a function of the distance to the nearest tie point and the quality of the original levelling connections. In practice, on both islands (Great Britain and Ireland) the worst areas are likely to show standard errors of 0.04 m for the disagreement. However, this is *not* the same as an error in the derived orthometric height: in reality the corrected GPS observations are more likely to give a better value for changes in orthometric height than differences in archive bench mark values. It is likely that some transitional problems may occur, but in the long run the most efficient results will come from a consistent adoption of the applicable correction surfaces.

OSGM02 is now freely available from the Ordnance Surveys in the form of the set of different correction surfaces, as a web based tool [**16**]. The gravimetric geoid is available for specific scientific uses on direct application to the relevant Ordnance Survey.

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