OSGM02: A new geoid model of the British Isles

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Abstract. This paper briefly describes the construction of a new geoid model of the British Isles. The new model, the Ordnance Survey Geoid Model 2002 (OSGM02), covers the area 45.5°N-61.5°N and 11.5°E-3.5°W (approximately 1445 km × 980 km) with the grid spacing 0.01333° × 0.02° (approximately 1.5 km × 1.5 km). A dense set of gravity and height data was used. The spatial data resolution was 100 m × 100 m for heights, one gravity station per 1.5 km × 1.5 km on land and one station per 5 km × 5 km offshore. These require-ments were met by the data except in few areas. A quasi-geoid was constructed and, subsequently, converted to geoidal heights. The OSGM02 gravimetric geoid was then fitted to GPS/levelling. To accommodate different vertical datums, the GPS fitting was done in patches corresponding the various datums in use (i.e. Newlyn, Belfast, Malin Head and various island datums). The final postfit error r.m.s. of the fitted geoid is 2 cm for the Great Britain and 3-4 cm for other areas.

Keywords. Geoid, quasi-geoid, fitting to GPS/levelling, vertical datums

1 Introduction

The years since the 1950's have seen several projects aimed at determining geoid models for the British Isles, whether as part of regional or local initiatives or more specifically focussed on parts of these islands. Featherstone and Olliver (2001) give an overview of the history of geoid models over the British Isles.

In 2001, a consortium consisting of the Ordnance Surveys of Great Britain (GB), Northern Ireland (NI) and Ireland invited tenders for the computation of a new geoid model for the British Isles, the Ordnance Survey Geoid Model 2002 (OSGM02). This new project would for the first time utilise all available gravity, terrain and GPS/levelling data. The work was subsequently carried out by a team of contractors consisting of the National Survey and Cadastre (KMS), Denmark, Department of Geomatics Engineering, University College London, UK, and Department of Geophysics, University of Copenhagen, Denmark.

The objective was to construct a cm-geoid for the whole area. It was estimated that these requirements would be met with a data distribution of a 100 m × 100 m Digital Elevation Model (DEM), a gravity station coverage of one point per 1.5 km × 1.5 km on land, and one station per 5 km × 5 km offshore. In section 2 it is seen that, except for few areas, these requirements were fulfilled by the available data. The methodology used is described in section 3. Section 4 describes the procedure leading to the construction of OSGM02 gravimetric geoid. In section 5 the fitting to the GPS/levelling data is briefly described.

2 DEM and gravity data

Members of the Ordnance Surveys Consortium provided DEMs for their respective territories in the national grids. A DEM for Isle of Man was provided directly by the Isle of Man Government. For Northern Ireland and Eire, heights were given in the Irish Grid (OSI & OSNI, 2000), a Transverse Mercator projection on a modified Airy ellipsoid. For Great Britain, the DEM data were given in the British National Grid on the OSGB36 datum; a Transverse Mercator projection on the Airy ellipsoid.

The DEM for Great Britain, originally provided on a 50 m × 50 m grid (OSGB, 2001a), was a uniform commercial product. The Isle of Man DEM, which is defined in the same national (horizontal) grid, but with different vertical datum, could be directly patched on top of the above grid. Eire and NI use the same national grid, the Irish Grid (OSI & OSNI, 2000), which made it quite straightforward to connect DEMs of both countries. The vertical datum associated with the Irish Grid is Malin Head (OSI & OSNI, 2000, Appendix C). Eire provided height data on a 10 m × 10 m grid, and Northern Ireland on a 50 m × 50 m. Some patches of DEM data were missing in Eire, and these voids were closed by interpolating from GTOPO30, a coarser 30''×30'' DEM downloaded through the National Geophysical Data Center, USA (see http://edcdaac.usgs.gov/gtopo30/README.html).

The gravity data were obtained from three main data sources: the British Geological Survey (BGS), KMS (Denmark) and Bureau Gravimetrique International (BGI), France. Proprietary data from several sources were included: The Institute of Advanced Studies (IAS, Dublin, Eire), the Geological Survey of Northern Ireland (GSNI) and the UK Hydrographic Office (HO).

The gravity data from all sources were subjected to consistency tests, checking absolute gravity value, station height, location, free-air gravity anomaly and the Bouguer anomaly, and data converted from GRS67 to GRS80 as required. Generally, the gravity data supplied were of high quality. One additional test was to compare the station heights of the gravity points with the interpolated height from the detailed DEMs. This comparison showed a mean bias below 2 m and a r.m.s. fit of 4-7 m, illustrating the high quality and internal consistency of the data set.

Unadjusted marine gravity data from HO were supplied via BGS; other marine gravity data were supplied via BGI. These data were screened for gross errors using KMS99, a 1999 version of the global free-air gravity anomaly field derived from satellite altimetry (Andersen and Knudsen, 1998). A few problematic marine data lines were removed. Marine areas void of marine gravity data in the area west of 10.5°W and in the area south of 50°N and west of 8°W were filled in by KMS99 data.

Another source of information about off-shore areas around GB were free-air gravity anomaly grids, produced by BGS. Correction from GRS67 to GRS80 was also necessary for these data. The BGS marine grids were used instead of the original point data simply because of economic reasons, as the BGS data were acquired by the OSGB at cost.

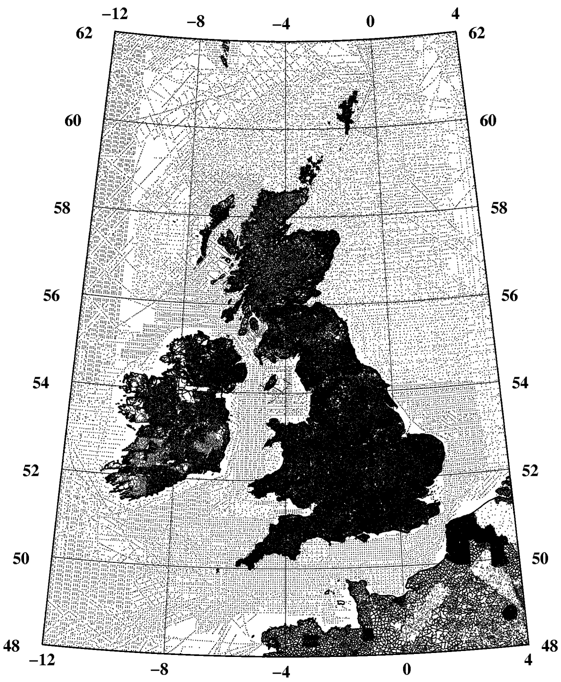

Fig. 1. Location of gravity data used in the project.

The BGS marine grids were checked against point marine data available from the same area from the KMS database, yielding acceptable agreement between the two (mean of the difference between measured and interpolated value was –0.21mgal and std. dev. was 1.39 mgal). Subsequently, the knowledge of the point locations of the original BGS gravity data was used to 'back-interpolate' the GRS80 free-air gravity values from the BGS marine grid for the remaining marine areas where no point data existed. The back interpolation is not as good as the original data, but results showed the lack of resolution made little difference in the final geoid product for areas with both sources of data.

Figure 1 shows the coverage of the gravity data used in OSGM02. The original dense set of land gravity data (especially near the urban areas in the UK) were thinned to the one gravity station per 1.5

km × 1.5 km on land, and to one gravity station per 5 km × 5 km in marine areas. Only a few areas in Eire have significant gravity data voids, resulting in risk of geoid errors, see Figure 1.

3 Methodology

The methodology for geoid construction is based on remove-restore techniques. The anomalous gravity potential T is split into three parts.

$$T = T_{EGM96} + T_{RTM} + T_{res} \quad (1)$$

where T_{EGM96} is the anomalous gravity potential of the EGM96 global field (Lemoine et al., 1998), T_{RTM} is the anomalous gravity potential generated by the residual topography and T_{res} is the residual anomalous gravity potential residual, i.e. the the unmodelled part of the residual gravity field.

In this project, the height anomaly ζ, i.e. the quasi-geoid, was modelled via Bruns's equation

$$\zeta = \frac{T(\phi,\lambda,H)}{\gamma(\phi,H)} \quad (2)$$

where γ is normal gravity, φ and λ are the geographical latitude and longitude, and H is the Helmert orthometric height.

From Eq.(1), the height anomaly ζ, i.e. the quasi-geoid, can also be split into three parts

$$\zeta = \zeta_{EGM96} + \zeta_{RTM} + \zeta_{res} \quad (3)$$

Nevertheless, the overall goal is to model the classical geoid heights N, i.e. the geoid. The relation between N and ζ is given approximately by

$$\zeta - N \approx \frac{\Delta g_B}{\gamma_0} H \quad (4)$$

where Δg_B is the Bouguer anomaly.

Computation of ζ_{EGM96} is straightforward using

$$\zeta_{EGM96} = \frac{GM}{a\gamma} \sum_{n=2}^{N} \left(\frac{a}{r}\right)^n \times \sum_{m=0}^{n} \left((C_{nm} - C'_{nm}) \cos m\lambda + S_{nm} \sin m\lambda\right) P_{nm}(\sin\phi) \quad (5$$

where G is the gravitational constant; M and a is the mass and the radius of the Earth; γ is the normal gravity; C_{nm}, S_{nm} are harmonic coefficients; and C'_{nm} are zonal coefficients of the normal potential.

Computation of ζ_{RTM} is done relative to the mean elevation surface, see Figure 2. The effect of the high-frequency topography above this surface, both the gravity disturbances and the gravity potential, is modelled. The mean elevation surface was obtained from the DEM by filtering, i.e. by moving 9×9 cell average across a 4'×6' averaged DEM, cf. Figure 3.

The geoid effect of the topography is

$$\zeta_{RTM} = \frac{G\rho}{\gamma} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \times \int_{z=h_{ref}(x,y)}^{z=h(x,y)} \frac{dx_Q dy_Q dz_Q}{\sqrt{(x_Q - y_Q)^2 + (x_Q - y_Q)^2 (x_Q - y_Q)^2}} \quad (6)$$

where ρ is the topographic mass density.

In practice, the computations of ζ_{RTM} were done in the frequency domain by Fourier methods. Equation (6) can be expanded into infinite series of convolutions in powers of h and h_{ref}. In this project, a 3rd order term was included

$$\zeta_{RTM} = \frac{G\rho}{\gamma}\left[(h - h_{ref}) * \frac{1}{s} - 3h(h - h_{ref}) * \frac{1}{6s^3} + + 3h(h^2 - h_{ref}{}^2) * \frac{1}{6s^3} - (h^3 - h_{ref}{}^3) * \frac{1}{6s^3}\right] \quad (7)$$

where s is the horizontal distance between the gravity station in P and the DEM grid point in Q, and where * is the (2D-)convolution. The 3rd order expansion was an improvement to the earlier used linear approximation (Forsberg, 1985), with difference up to few cm in the mountains. The corresponding RTM effects on gravity data were computed directly by space-domain prism integration using the dense height data.

The computation of ζ_{res} was performed using Stoke's integration (Heiskanen and Moritz, 1967), using the residual free-air anomaly Δg_{res}, i.e. what is left in the gravity data after the contributions of the RTM-effect, Δg_{RTM}, and the global field, Δg_{EGM}, were subtracted. In order to optimise modelling of ζ_{res} different modifications of Stokes's kernel S(ψ), where ψ is the spherical distance, were tried. A modified version of the Wong and Gore (1969) method, supplemented with a linear taper to avoid abrupt spectral discontinuities was used, i.e.

$$S_{mod}(\psi) = S(\psi) - \sum_{n=2}^{N_2} \alpha(n) \frac{2n+1}{n-1} P_n \cos(\psi) \quad (8)$$

with a linear tapering coefficient

$$\alpha(n)=\begin{cases} 1 & for \quad 2 \le n \le N_1 \\ \dfrac{N_2-n}{N_2-N_1} & for \quad N_1 \le n \le N_2\,,\, n=2,\ldots,N \\ 0 & for \quad N_2 \le n \end{cases} \qquad (9)$$

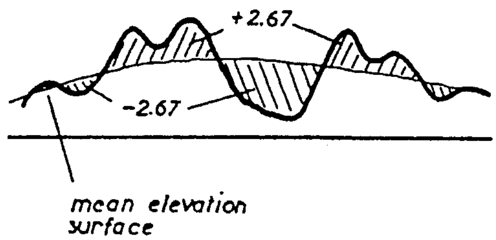

Fig. 2. RTM effect principle, from Forsberg (1985). Gravitational effect of the mass surplus and mass deficit of the detailed topography with respect to a mean elevation surface is removed computationally.

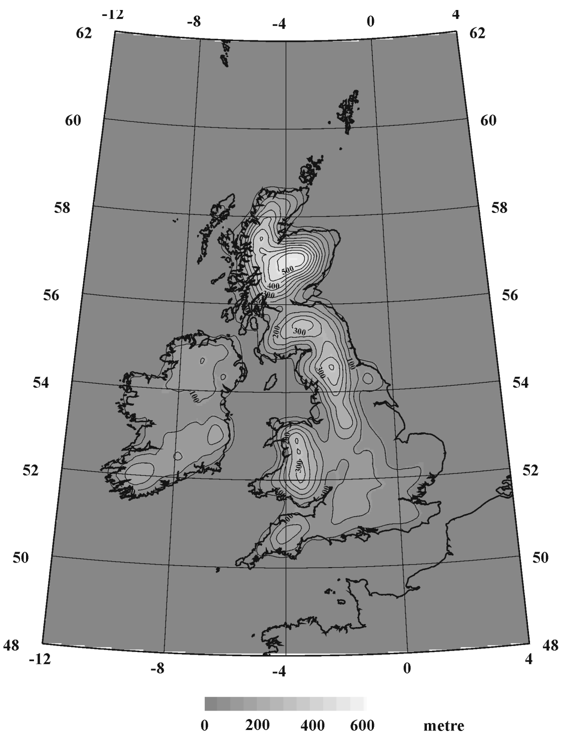

Fig. 3. Reference DEM, i.e. a mean elevation surface, obtained from detailed DEMs by filtering, see above, to approximately 66 km spatial resolution.

Geoid heights derived from GPS-levelling were used to find "optimised" values of N_1 and N_2, i.e. where the fit between the gravimetric geoid N_{grav} and the geoidal heights from GPS-levelling

$$N_{GPS} = h_{ellipsoidal} - H_{orthometric} \qquad (10)$$

is best in the least squares sense. The results indicated that the residual gravity field relative to EGM96 had some long wavelength components, and values of $N_1 = 12$ and $N_2 = 15$ were used for the final gravimetric geoid computation. Given the size of the area (NS×WE: 1445 km × 980 km) the relatively small values of N_1 and N_2 indicate that there is long-wavelength information left in the residual gravity data after the subtraction of EGM96.

4 OSGM02: gravimetric geoid

The OSGM02 gravimetric geoid was constructed based on methodology described in Section 3. After reductions, the gravity data was gridded by least squares collocation with empirical correlation length 25 km and rms noise as defined by the data. A final screening for errors in the gravity data was carried out on Δg_{res}, cf. Figure 4.

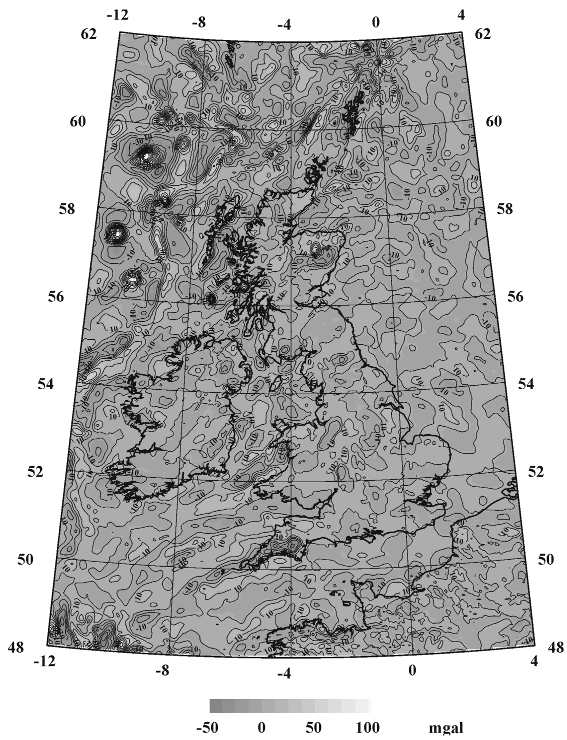

Fig. 4. Residual free-air gravity anomalies Δg_{res}.

Possible inconsistencies in the gravity data, which sometimes can pass undetected in the measured free-air gravity anomalies, often appear as spikes in the smoother Δg_{res} field. On the other hand, if the value of Δg_{res} is consistent with the values at neighbouring stations, and especially if these values correlate with known geology (e.g. continental shelf structures or igneous intrusions), these data are not in error. Only relatively few detected individual spikes were removed from the data set in this final screening. For the marine data 11 suspicious locations with anomalous Δg_{res} were investigated in details. It resulted in removal of 57 erroneous marine data points (often a part of a marine survey line). Similarly, for the land data 7 suspicious locations with anomalous Δg_{res} were investigated. 5 outliers were detected and removed.

Height anomalies ζ_{res} were computed using the modified Stokes' integration, see section 3. A multi-band spherical FFT method (Forsberg and Sideris, 1993) was used with 100% zero-padding to avoid the FFT periodicity effects. The atmospheric correction on gravity was applied at this stage taking into account the dependence of height.

Table 1. OSGM02 gravimetric geoid. Basic statistics of the different components of the model construction.

mgal or metre	mean	std.dev.	min.	max.
Δg (mgal)	9.5	20.7	-75.8	167.9
Δg_{res}	1.0	8.9	-67.2	129.7
ζ_{res} (m)	0.11	0.26	-0.67	1.63
ζ_{RTM}	0.00	0.05	-0.24	0.50
$\zeta_{Gravim.}$	51.68	5.07	40.92	60.63
ζ - N	0.00	0.00	-0.06	0.04
N	51.68	5.07	40.92	60.63

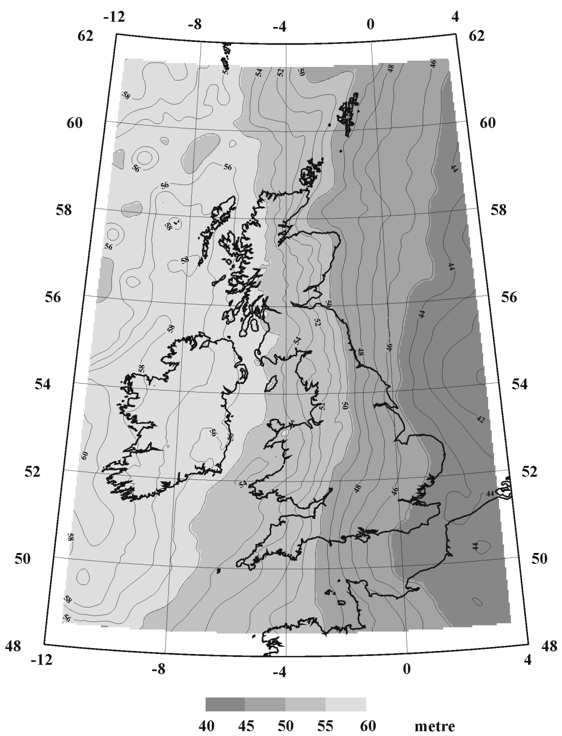

Fig. 5. OSGM02 gravimetric geoid. Contour interval: 1m.

ζ_{RTM} was computed by the third-order expansion spherical FFT methods, cf. Eq. (7), on the 1.5 km basic resolution grid. Finally, ζ_{EGM96}, computed at the surface of topography was added to obtain the final quasigeoid.

The quasigeoid was subsequently converted into the classical geoid N, referring to sea level, using the approximative formula Eq. (4), with Bouguer anomalies derived from the cleaned and gridded free-air anomalies.

Table 1 shows the basic statistics of the different stages of the model construction. Figure 5 shows the final OSGM02 gravimetric geoid.

5 Geoid fitting to GPS/levelling

The Ordnance Surveys Consortium requested the new gravimetric geoid to be fitted to their GPS/levelling data and to transfer the vertical datum across the British Isles.

The quantities to compare are the gravimetric geoidal heights N_{grav} and the geoidal height N_{GPS} from GPS/levelling, $N_{GPS} = h - H$. The misfit ε

$$\varepsilon = N_{GPS} - N_{grav} \tag{12}$$

includes datum differences, systematic errors and subsidence/uplift in the levelling, as well as errors in the gravimetric geoid.

In practise, ε was modelled using a four-parameter Helmert model for the long wavelength trend and a second-order Gauss-Markov collocation fit for the residuals. For the UK an apriori error of 20 mm and a correlation length of 50 km was used. Table 2 shows the misfits of the fundamental (FBN) GPS levelling points for the mainland UK. The misfit ε has a mean of 13 cm and r.m.s. of 11 cm. The bias is unavoidable since N_{GPS} and N_{grav} refer to different datums. The r.m.s value is likely due to the well-known Ordnance Datum Newlyn (ODN) levelling problem in the mainland UK; a NS-slope of some 0.053 m/degree of latitude (Thompson, 1980; Featherstone and Olliver, 1994). Figure 6 supports these results.

The problem with different datums was handled by constructing individual geoid patches for each datum. Also, even for the same datum there can be problems. For example the Orkneys are in ODN, but the crossing from the mainland was done by trigonometric heighting. The geoid did not show a good fit across, indicating that the height transfer must be inaccurate. A separate Orkney geoid patch was done.

Table 2. GPS levelling geoid fits of UK FBN network (179 points).

unit: meter	Mean	std. dev.	min.	max.
misfit N_{grav}	-0.129	0.110	-0.431	0.081
misfit after 4-par. fit	0.000	0.032	-0.089	0.092

Table 3. Main datums. Post-fit GPS/levelling statistics. (Newlyn, GB: 179 points; Belfast, NI: 36 points; Malin Head, Eire: 110 points)

Unit: meter	mean	std. dev.	min.	max.
Newlyn	0.000	0.004	-0.011	0.018
Belfast	0.002	0.019	-0.035	0.041
M. Head	0.003	0.024	-0.064	0.050

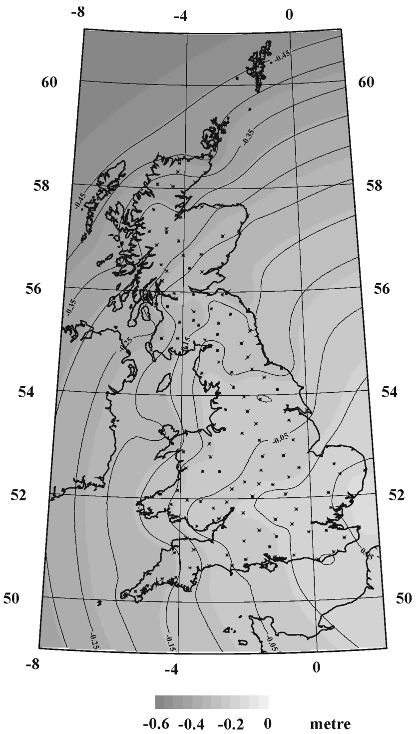

Fig. 6. Location of 179 ODN GPS/levelling points used in post-fit GPS/levelling statistics and the modeled geoid fit surface ε. Contour interval: 0.05 m.

In Eire, several problems with GPS-levelling were discovered, in part due to levelling and in part due to GPS problems yielding a larger standard deviation. Table 3 shows the post-fit statistics for the main datums. Figure 6 shows the distribution of the GPS/levelling points.

The off-sets between the main datums were investigated by two methods: rigorous least-squares collocation with parameters (using a thinned-out data set covering all of the British Isles in one batch solution), and by direct comparison through height transfer to the OSGM02 gravimetric geoid. Collocation yielded 0.42 m between Newlyn datum and Malin Head, while OSGM02 yielded 0.31m between the Newlyn datum and Belfast datum and 0.38m between the Newlyn datum and Malin Head datum. A height value in Newlyn datum would be numerically larger then the same height in the Irish datums, meaning the Irish geoid is above the Newlyn geoid. Due to the apparent tilt in the Newlyn datum in England, relative to the OSGM geoid model, an more accurate definition of the height difference seems not possible, and will depend on latitude.

6 Conclusion

The OSGM geoid surface has been derived from dense height and gravity data, and fitted to available GPS datums. The quality of the gravimetric solution seems excellent, with a 3.2 cm r.m.s. misfit across the UK after subtracting a trend, taking into account an apparent slope of ODN.

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