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Computation of a geopotential model from GOCE data using fast spherical collocation - A simulation study

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Abstract.
Using a realistic orbit for GOCE, IAG SC7 has created a one month gravity gradient dataset from EGM96 to degree 300, with gradients referring to an instrument frame aligned with the velocity vector and the z-axis in the plane formed by this vector and the position vector. From the second order derivative of the potential V_{zz} we subtracted the contribution of EGM96 to degree 24. The resulted (noise free) data set was used to predict second order radial gravity gradient values in a 0.5° grid, covering the area of the Earth from $-83°$to $+ 83°$ latitude using local Least-Squares Collocation (LSC). The standard deviation of differences between predicted gridded values and values computed from EGM96 (degree 24 - 300) was between 1.0 and 0.5mEU (Eötvös unit = EU, 1 EU=$10^{-9}s^{-2}$). Correlated noise with a 3 mEU standard deviation and a 35° correlation distance was added to the simulated data and the gridding was repeated. The formal LSC error-estimates were 2 mEU. This was confirmed by comparing second order radial derivatives from EGM96 with the values predicted from the data with noise. The simulated data sets were used to generate spherical harmonic coefficients of the gravity potential to degree 300 using Fast Spherical Collocation (FSC), with a global covariance function. Both the grid of noise-free data and the grid obtained from the data with noise was used. Both results agreed with EGM96 within the error-bounds of the FSC estimate.

Key words: GOCE mission, Geopotential model, Fast spherical collocation

1 Introduction

Let us denote by V the gravity potential of the Earth (without centrifugal part) and by T the anomalous potential obtained by subtracting a reference potential from V. Both functions may be approximated arbitrarily well as the sum of a series in solid spherical harmonics with coefficients C_{ij}

$$V(r,\varphi,\lambda) = \frac{GM}{a}\sum_{i=0}^{n}\left(\frac{a}{r}\right)^{i+1}\sum_{j=-i}^{i}C_{ij}Y_{ij}(\varphi,\lambda),$$

where r is the distance from the origin, φ, λ the latitude and the longitude respectively, GM the product of the gravitational constant and the mass of the Earth and Y_{ij} are the surface spherical harmonics. n is the maximal degree, in this paper generally equal to 300.

The ESA satellite GOCE (Gravity Field and Steady-State Ocean Circulation Explorer Mission, ESA, 1999) is expected to be launched in 2006. It will carry a gradiometer which will measure the 6 gravity gradients, V_{xx}, V_{xy}, V_{yy}, V_{xz}, V_{yz}, V_{zz}, given in the instrument frame with the x-axis aligned with the velocity vector, the z-axis in the plane formed by the velocity and the position vector. The y-axis completes the orthogonal triad. The goal of the mission is to determine 1° mean geoid and gravity anomalies with an error below 0.01m and 1.0mGal, respectively.

Many global and regional simulations (see e.g. Arabelos and Tscherning, 1995) have been carried out, initially using preliminary information available from ESA. Meanwhile the orbit parameters have been fixed, and more realistic simulations can be made. In order to enable the comparison between different methods IAG Special Commission 7 has issued a 1 month dataset with a 5s sampling, totally 718401 points with 6 gravity gradient values given in the above mentioned instrument frame. EGM96 to degree 300 (Lemoine et al., 1998) was used to generate the gradients.

Least-squares collocation may, as demonstrated in Tscherning (2001), be used to predict spherical harmonic coefficients. The use of the method for this purpose requires that a globally distributed set of data is used. Furthermore the optimal use of LSC require that systems of equations with as many unknowns as observations are solved, a procedure which obviously not is feasible today.

Fortunately the method of Fast Spherical Collocation, FSC, (Sansó and Tscherning, 2003) enables us to bypass this problem. But data gridded equidistantly in longitude and at a distance from the origin which is constant for each parallel must be used. The method has also limitations with respect to which data-types may be used. For gravity gradients the second order radial derivative, T_{rr} may be used, for example. In the following we will describe how such values were obtained from the "clean" SC7 data and from data to which correlated noise has been added. Finally we will present the results of the application of FSC to the two interpolated (gridded) data sets.

2 Prediction of gridded T_{rr} data

LSC with regionally distributed data may be used to interpolate/predict values of the gravity gradients with a limited effort. Other studies (Tscherning, 2003) show that excellent results may be obtained using T_{zz} to predict T_{rr} just using data in a neighbourhood which size will depend on the quality of the reference potential used to define T. This is due to the closeness of the radius vector to the instrument $z-$ axis. We will here use EGM96 to degree 24 as our reference potential. Using such a reference potential we will have a correlation distance of 1.5° at mean satellite altitude (255km), see Figure 1.

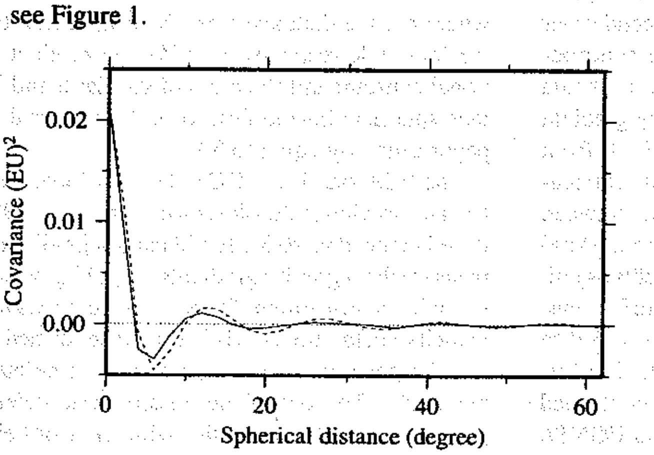

Fig. 1. Covariance function of second order radial derivative of T at 255 km altitude: solid line=from degree-variances to degree 720, dashed line=from analytic expression

Consequently, a region with at least a 1.5° border should be used. We have used 2.5° in latitude and longitude, i.e. the border is smaller at numerically high latitudes, but here will the data density be very high due to the 96° inclination of GOCE. In order to limit the administration of the regional solutions, we decided to divide the Earth in 20° × 20° equal angular blocks, i.e. totally 9 × 18 = 162 regions.

LSC permits us to use covariance models fitted to local statistical characteristics. We decided only partly to use this possibility. In fact we simply scaled a global covariance function (Fig. 1) with respect to the regional variance. The global model was determined in two steps. First a covarianc function of T_{rr} was determined for 255km altitude. It wa determined as a Legendre-series with EGM96 error degree variances from degree 2 to 24, EGM96 degree-variances fror degree 25 - 360 and GPM98A (Wenzel, 1998) degree-var iances from degree 361 - 720. This function was then fitte with an analytic expression using the GRAVSOFT (Tschern ing et al., 1992) program COVFIT (Knudsen, 1987). Thi program fits a closed expression to the values derived fron the finite Legendre series.

Using the GRAVSOFT program GEOCOL, 162 jobs wer run, where values in a 0.5° grid at 255km altitude as wel as error-estimates were computed. The numerically maxima latitude was 83.25°.

Initially the noise-free SC7 T_{zz} data set was used to predic the values which were compared to values computed fron EGM96 (degree 25-300). An agreement was found with a mean value of zero and a standard deviation of typically 1 mEU. In the computations the data were supposed to hav correlated noise with a 3mEU standard deviation and a 35 correlation distance. We have here implicitly pressuposed that the GOCE data have been pre-processed so that contin gent global biases are removed (see Bouman et all., 2003).

Subsequently the computations were repeated with nois with these characteristics added (see Figure 2). The standar deviation between values calculated from EGM96 and pre dicted values now were in the range $1.5-2.0$mEU, whicl corresponds well to the LSC estimated prediction error o 2.0mEU.

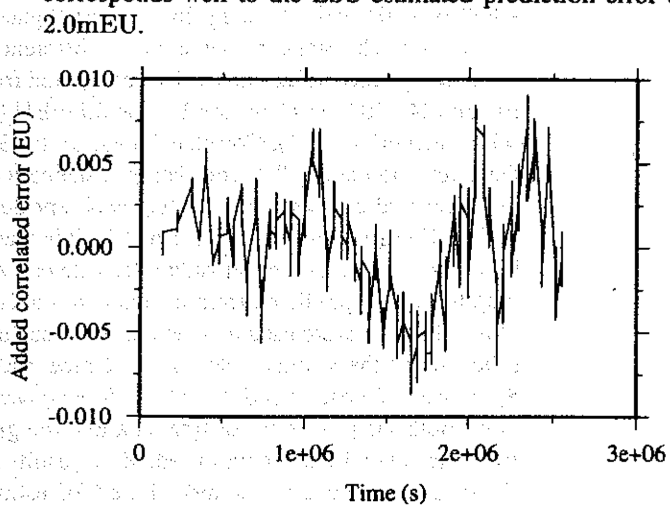

Fig. 2. Error added to EGM96 data (time interval)

In Figure 3 the mean and the standard deviation of the differences between "observed" and "predicted" T_{rr} as a function of latitude is shown. From this figure it is evident tha the error is large at the block-boundaries but not so much a the boundaries and as if expected, increases very much in the area right outside the data area.

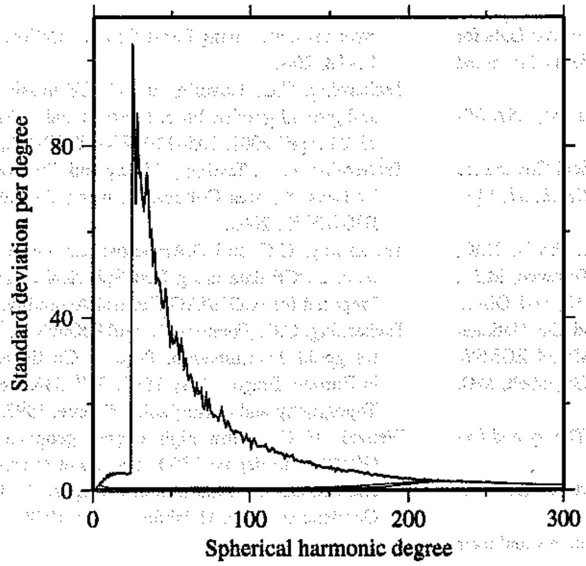

Fig. 4. Results of FSC prediction of spherical harmonics. Black = Square roots of EGM96 degree-variances, Red = Standard deviation per degree of the differences observed - predicted, Blue = Mean collocation error-estimates of the predicted coefficients per degree

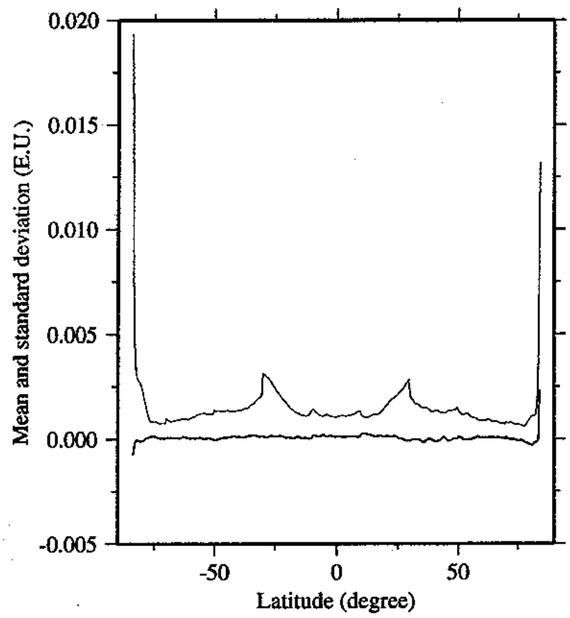

Fig. 3. Mean and standard deviation of the differences between observed and predicted T_{rr}, as a function of latitude. Black=Mean, Red=Standard deviation

3 Estimation of spherical harmonic coefficients using FSC

The gridded data could directly be used by the program "SPH-GRID", which used a covariance function represented by the sum of a Legendre series to degree 720. The error-estimates were for all parallels fixed to 2mEU.

The standard deviation (unitless) per degree of the differences between the predicted coefficients and EGM96 coefficients (red curve), as well as the mean collocation error-estimates per degree of the predicted coefficients (blue curve) are shown in Figure 4. In the same figure the square roots of potential degree-variances of EGM96 are shown(error degree-variances from degree 2 to 24, degree-variances from degree 25 to 300).

We see that the crossing point, where the signal and the noise standard deviation becomes equal are approximately at degree 200 as also found in other investigations.

Figure 5 shows the error-estimates as a function of the order for the coefficients of degree 150. The influence of the polar gap is clearly seen (cf, Tscherning, 2001a).

4 Conclusion

Regional LSC has been used to predict values of gravity gradients (here T_{rr}) in a grid. The grid values were used as input data to Fast Spherical Collocation to determine estimates of spherical harmonic coefficients as well as error-estimates of these quantities.

It is without problems to predict gridded values of other gravity gradients such as $T_{\varphi\varphi}$ and $T_{\lambda\lambda}$, the second order derivatives with respect to latitude and longitude. Further investigations will be executed in order to see whether it is possible to improve the estimates of the coefficients using these additional data types.

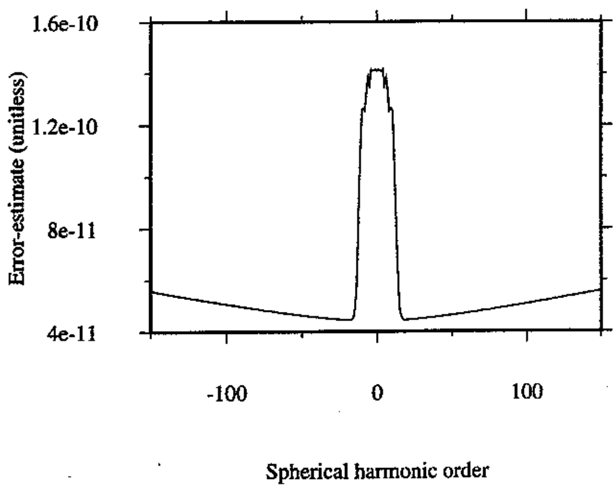

Fig. 5. Error estimates for degree 150 as a function of the order. Note the influence of the polar gaps

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