Integrating Numerical Weather Predictions in GPS Positioning

A. B. O. Jensen, *National Survey and Cadastre – Denmark*
C. C. Tscherning, *University of Copenhagen*
F. Madsen, *National Survey and Cadastre – Denmark*

BIOGRAPHY

Anna B.O. Jensen holds an M.Sc. in surveying from the University of Aalborg in Denmark, and has since 1995 been employed in the Department of Geodesy at the National Survey and Cadastre – Denmark (KMS). She is presently doing her Ph.D at the University of Copenhagen and KMS with co-supervision from the University of Calgary.

Carl Christian Tscherning is Professor of geodesy at the University of Copenhagen. He has contributed significantly to the theory of geoide determination, and has published numerous papers on physical geodesy and GPS. He is also the general secretary of the International Association of Geodesy.

Frede Madsen is a senior geodetic consultant in the Department of Geodesy at KMS. He is a past head of the department, and was in this position responsible for the introduction of GPS for geodesy in Denmark. He has worked within the area for more than 30 years, the last years with consulting tasks related to high accuracy applications of GPS.

ABSTRACT

When the GPS satellite signals are transmitted through the atmosphere they are affected by the media. In the neutral atmosphere the refraction is a function of the meteorological conditions such as pressure, temperature, and humidity along the signal path, and this effect is in GPS terminology referred to as the tropospheric delay.

In the GPS positioning process this effect is normally handled by utilising global tropospheric delay models. For high accuracy differential positioning based on the carrier phase observables modelling of the error is however not giving sufficiently accurate results, and other techniques must be implemented.

This paper describes a solution where numerical weather predictions (NWP) are introduced in high accuracy GPS data processing. NWPs are predictions of the three dimensional meteorological conditions for a given area and point in time. A NWP can thus be used for predicting the tropospheric delay for a satellite signal by integration along the signal path through the NWP.

This paper describes the theory and the procedure for determining zenith delays from NWP data, and for integrating these in a GPS positioning process.

The zenith delays are evaluated by a comparison with zenith delays determined using GPS data and the Bernese software. The evaluation shows that the NWP zenith delays correspond to the GPS derived zenith delays with an RMS of 1.7 cm.

The NWP zenith delays are finally used in a static positioning process to correct for the tropospheric delay, and for 15 of the 26 baselines processed an improvement in position accuracy is obtained when the NWP approach is used instead of the global Saastamoinen model.

INTRODUCTION

When using GPS for high accuracy differential positioning, at least two GPS receivers must be used. One receiver is located at a reference point with known coordinates and the other receiver (the rover) is located at a point with unknown coordinates. The position of the rover is then determined relative to the reference station, and the positioning process is based on both code and phase observations collected by the two receivers. This can be carried out in real time utilising a data link, or in a post processing mode where the data is processed on a computer after the data collection has been carried out.

The carrier phase based GPS positioning process is based on the phase observation equation as described, for example, in Kleusberg and Teunissen (1996). With observations from two receivers and two satellites at the same time epoch, the observation equation can be generated for each receiver-satellite combination, and the equations can be twice differenced (double differenced). The result is the double differenced phase observation, $\Delta\nabla\Phi$, given below in Equation (1):

$$\Delta\nabla\Phi = \Delta\nabla\rho + \Delta\nabla d_{\rho} - \Delta\nabla d_{ion} + \Delta\nabla d_{trop} + \lambda\Delta\nabla N + \Delta\nabla\varepsilon \qquad (1)$$

$\Delta\nabla$ is the double difference operator, ρ is the geometric distance between receiver and satellite, d_ρ is the error in the satellite position, d_{ion} is the ionospheric delay, d_{trop} is the tropospheric delay, λ is the wave length, $\Delta\nabla N$ is the ambiguity, and ε is the noise term including multipath and receiver noise. All elements are given in range units.

The double differenced ambiguity, $\Delta\nabla N$, is by nature an integer number and to obtain position accuracies of a few cm or mm the correct integer number for the double difference ambiguity must be determined. This is not a trivial task, but many different ambiguity resolution techniques do exist, see for instance Han and Rizos (1997) for a review of some techniques.

The sizes of d_{trop}, d_{ion}, and d_ρ (the spatially correlated errors) are reduced when performing double differencing, and for short baselines these errors are generally negligible after double differencing.

For longer baselines the spatially correlated errors do not cancel out, but they must be treated as a part of the positioning proces. For post processed positioning the orbit error can be mitigated by using precise orbit information for instance from the IGS (Neilan et al., 1997).

The ionosphere is dispersive for radio waves, so ionospheric effect is different for the L1 and L2 signals (Johansson, 1997). When working with dual frequency receivers, which is normally the case for high accuracy positioning over longer baselines, the ionospheric error can therefore be mitigated by using the ionosphere free linear combination of the L1 and L2 observations (Seeber, 1993). Hereby the first order ionospheric effects are removed from the positioning process. Higher order effects of the ionosphere will still be present, but for baselines of 50 – 100 km their influence is relatively small. For longer baselines, the higher order effects should, however, be considered (Brunner and Gu, 1991).

THE TROPOSPHERIC ERROR

The neutral atmosphere is the part of the Earth atmosphere that is located between the Earth surface and the ionosphere, which lower boundary is located at an altitude of about 60 km. As the GPS signals travel through the neutral atmosphere, they are affected by a refraction, causing the receiver–satellite distance determined from the satellite signals to be longer than the geometrical path the signal would have followed if it travelled through a vacuum. In GPS terminology, this effect is often referred to as the "tropospheric delay".

The delay experienced by a satellite signal can be determined by integrating the refractivity, N, along the signal path, ds.

$$delay = 10^{-6} \int N \, ds \qquad (2)$$

The integration is carried out from the GPS antenna to the top of the neutral atmosphere.

The refractivity can be determined using the expression below from Davis (1985).

$$N = k_1 R_d \rho + \left(k'_2 \frac{e}{T} + k_3 \frac{e}{T^2} \right) Z_w^{-1} \qquad (3)$$

Where R_d is the gas constant for dry air, ρ is the density of air, T is temperature, e is the partial pressure for water vapour, and Z_w is the compressibility factor for water vapour accounting for the deviation of the gas from an ideal gas. The constant $k'_2 = k_2 - k_1 M_w / M_d$ where M_w and M_d are the molar mass of water vapor and dry air respectively. Values for the k_1, k_2 and k_3 constants are given by for instance Bevis et al (1994).

For modelling purposes, the refractivity is often split into a hydrostatic and a wet part. In Equation (3), the first part is the hydrostatic delay, and the expression in the last bracket is the wet delay.

The total delay for a satellite signal received at any elevation angle can be determined by first estimating the delay for a signal received in the zenith using Equation (2) and (3). The zenith delay is then multiplied by a scaling factor to map the delay down to lower elevation angles.

The scaling factor is determined by a mapping function, *m(elv)*, and often both a hydrostatic and a wet mapping function are used, so the total delay for a signal received at elevation, *elv*, can be determined as:

$$delay(elv) = zhd * m_h(elv) + zwd * m_w(elv) \qquad (4)$$

Where *zhd* is the zenith hydrostatic delay, *zwd* is the zenith wet delay, and the subscripts *h* and *w* denote hydrostatic and wet mapping functions respectively. A comprehensive review of mapping functions is given by Mendes (1999).

In GPS positioning, the tropospheric delay is mitigated by means of global tropospheric delay models. The best global models do, however, leave unmodelled errors of about 3 cm in the zenith (Mendes, 1999), and for high accuracy positioning the modelling is therefore followed by double differencing, where the residual errors will cancel out for shorter baselines, i.e. up to 20 - 30 km (Cannon, 1997).

For longer baselines, the residual tropospheric error after both modeling and double differencing does have an effect, and the residual zenith error can for instance have a size of about 1.5 cm for a 68 km baseline under normal conditions as shown by Raquet (1998).

If this residual effect is not taken into account, it can interfere with the ambiguity resolution by extending the time to resolve the ambiguities and thereby degrade the positioning performance.

One way of minimizing the residual tropospheric delay is to take advantage of regional weather forecasts, and use the predicted weather data for predicting the tropospheric delay. The global tropospheric delay models can thus be replaced by regionally predicted tropospheric delays, a solution suggested by Schueler et al. (2000) for WADGPS, and by Pany et al. (2001) for static GPS positioning.

Behrend et al. (2001) tested the use of NWPs for static positioning and they obtained an improvement in GPS position accuracy when using the GIPSY software, and when not estimating the residual tropospheric error as a part of the adjustment process. It is therefore anticipated that the most significant improvement by introducing NWPs for GPS positioning will be seen when using software or a processing set up where the residual tropospheric error present after modelling is not estimated. This is the case with most commercial software packages, and in this paper the idea of using Numerical Weather Predictions (NWP) for static GPS positioning is therefore tested using a commercial software package.

NUMERICAL WEATHER PREDICTIONS

NWPs are three dimensional models of the conditions in the atmosphere, extending from the surface of the Earth up to an altitude of about 30 km. The models contain information about temperature, humidity, wind speed and direction, amount of precipitation, etc.

NWPs form the basis for weather forecasts and they are based on numerous meteorological observations collected globally. The observations are mainly collected by ground-based equipment, but a few radio sondes, transatlantic airplanes, and microwave radiometers also provide information about conditions higher up in the atmosphere.

The refractivity, and thereby the tropospheric delay, is a function of the meteorological parameters pressure, temperature and humidity (see Equation (3)), and since these parameters are included in NWPs, the tropospheric delay can be predicted using data from a NWP as described in the following.

DMI-HIRLAM-E

For the following tests a NWP from the DMI-HIRLAM forecasting system was provided by the Danish Meteorological Institute (DMI). The DMI-HIRLAM[1] system is described by Sass et al. (2000).

The NWP called DMI-HIRLAM-E was used. It is a grid based model that covers most of Europe. The grid spacing is 0.15° corresponding to approximately 16 x 9 km in latitude and longitude, respectively, for the region of interest. Vertically, DMI-HIRLAM-E consists of 31 layers that are extending up to an altitude of about 30 km. The distance between the layers is small close to the Earth surface and is increasing with an increasing altitude.

Data assimilation for DMI-HIRLAM-E is carried out twice daily, at midnight and at noon UTC, and for these tests data was available from both the analyses, carried out in connection with the data assimilation, and from a number of predictions run with a 1-hour interval. So at 1:00 UTC a one hour prediction was available, at 2:00 UTC a two hour prediction was available etc. until noon, where the prediction cycle was re-initialised.

Data was available from September 5, 2000, and this day was characterized by low ionospheric activity with a Kp-index of 2-3. See for instance Campbell (1997) for a description of the ionospheric Kp-index. With this low level of ionospheric activity no higher order ionospheric effects are expected to be present.

The weather, and thereby the tropospheric activity, was moderate with a light wind and scattered showers.

Equation (3) can be rewritten to obtain the following expression, which is based on Vedel et al. (2001):

$$\begin{aligned} ztd &= 10^{-6} \int_0^{p_{surface}} \frac{k_1 R_d}{g}\, dp \\ &+ 10^{-6} \int_0^{p_{surface}} \frac{R_d}{\varepsilon} \left(k_2 - \varepsilon k_1 + \frac{k_3}{T} \right) \frac{q}{g}\, dp \end{aligned} \qquad (5)$$

where R_d is the gas constant for dry air, g is gravity in m/s^2, p is pressure in Pascal, ε is the ratio between molar weight of water vapor and dry air, q is specific

[1] HIRLAM (High Resolution Limited Area Model) is a cooperation between several European meteorological institutions. The implementations of the HIRLAM-system vary in the different countries, but they are all built on the same basic models.

humidity in kg/kg, and T is temperature in Kelvin. For the k-constants the values given by Bevis et al. (1994) are used.

The compressibility factor is not included in Equation (5), since it is anticipated that the atmosphere behaves as an ideal gas. This is an approximation, but it is reasonable for determining the delay in the zenith direction. If working with signals received at lower elevation angles, the compressibility factor must be taken into consideration, because of the composition in the lowest parts of the atmosphere.

Values for pressure, temperature and specific humidity are extracted from the NWP by horizontal interpolation within each layer, and by vertical interpolation to determine the values for the height of the GPS antenna. The altitude is determined from the pressure using Equation (6) as suggested by H. Vedel, the Danish Meteorological Institute (personal communication).

$$g \cdot dh = R_d T\left(1-q+\frac{q}{\varepsilon}\right)\cdot d\,ln(p) \qquad (6)$$

Where dh is change in height and $dln(p)$ is the corresponding change in the logarithmic value of the pressure. Gravity, g, is determined using the expression for normal gravity given by Torge (1989). This procedure is described in more detail in Jensen (2002).

The delay experienced by the satellite signals in the part of the neutral atmosphere that is above the NWP is modelled as a function of temperature and pressure given for the upper most layer in the NWP. This procedure is described by Vedel et al. (2001).

When the zenith delay has been determined using Equation (5) a mapping function is applied to determine the delay at lower elevation angles.

In the following this approach has been used for correcting for the tropospheric error in a static GPS positioning process.

ZENITH DELAYS

Before introducing zenith delays determined using Equation (5) and the DMI-HIRLAM-E data into the GPS positioning process it is relevant to evaluate the quality of these zenith delays. This was carried out by comparing the HIRLAM derived delays with zenith delays determined from GPS data.

The GPS data used for these tests originates from 14 permanent GPS reference stations located in Denmark and southern Sweden (see Figure 1). The Swedish stations form a part of the SWEPOS network, which is described on: http://swepos.lmv.lm.se/english/.

The Danish stations were equipped with Ashtech Z-XII dual frequency receivers and Ashtech antennas with choke rings. The Swedish stations are equipped with the same type of receivers and with Dorne Margolin antennas.

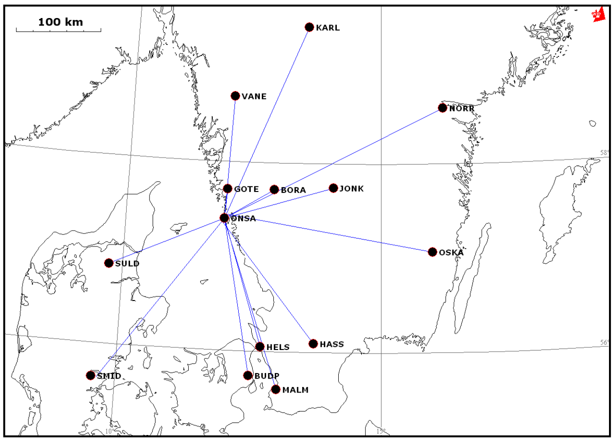

Figure 1. Location of GPS stations, and baselines processed.

Accurate coordinates for the GPS stations are necessary in order to determine zenith delays based on the GPS data, so the Bernese software version 4.2 from the University of Berne (Beutler et al., 2000) was used to determine the station coordinates. In order to obtain accurate coordinates precise orbits were used along with one week of data (September 3 - 9, 2000) from all the stations. The coordinates for the IGS station ONSA were constrained to their IGS values in the ITRF97, whereby the resulting coordinates for the other stations are in concordance with the reference frame used for the IGS orbits.

It is with the Bernese software possible to estimate tropospheric zenith delays with high accuracy for a GPS station, if the influence of all other error sources is minimized. So following the computation of the station coordinates a new processing was carried out using only the data from September 5.

For this processing the Saastamoinen global tropospheric delay model was used to generate a priori values for the tropospheric delay. The residual tropospheric error was then estimated as a part of the adjustment process, and by adding it to the Saastamoinen delays, the total tropospheric delay for the given station and time epoch was obtained. The total tropospheric delays were estimated for every hour.

In order to avoid any residual effects from other error sources in the GPS data processing, it is important that these error sources are handled sufficiently well. The noise and multipath level is expected to be low because of the high quality receivers and antennas used. The residual orbit error is negligible for these baseline lengths when final precise orbits from the IGS are used. The station coordinates are in concordance with the orbits, as previously mentioned, so no geometrical misalignment, caused by reference frame mix up, is introduced. Earth tide is modelled, and the ionosphere is handled by introducing the ionosphere free linear combination of the observables. Any remaining error effects should thus be caused by either differential ocean loading effects, that are expected to be negligible, or by higher order ionospheric effects, that are also expected to be negligible because of the low ionospheric activity.

A 10° elevation mask was used for the processing. Selection of elevation mask is, in this connection, a compromise between eliminating some of the noisy low elevation data, and introducing a bias in the results. The presence of a bias has been illustrated for instance by Emardson et al. (1998), where the influence of the GPS elevation mask on water vapor estimated from zenith wet delays derived from GPS data was shown. When they changed the elevation mask from 15° to 10° an offset of about 2 kg/m^2 in integrated water vapor, corresponding to approximately 1.3 cm in wet zenith delay, was basically removed.

Table 1. shows the mean, standard deviation, and RMS of the differences between zenith delays determined with the Bernese software and with DMI-HIRLAM-E data. The statistics is based on differences for the 14 GPS stations for each time epoch where zenith delays were available from both DMI-HIRLAM-E and from the GPS data processing.

Tabel 1. Mean, standard deviation, and RMS of zenith delay differences. Bernese minus HIRLAM delays.

Time UTC	Mean [meter]	Std. Dev. [meter]	RMS [meter]
0:00	-0.019	0.013	0.022
1:00	-0.014	0.013	0.019
2:00	-0.016	0.012	0.020
3:00	-0.016	0.014	0.021
4:00	-0.013	0.015	0.019
5:00	-0.014	0.016	0.020
6:00	-0.010	0.012	0.015
9:00	-0.010	0.010	0.014
12:00	-0.015	0.009	0.017
13:00	-0.009	0.010	0.013
14:00	-0.007	0.010	0.012
15:00	0.011	0.008	0.014
16:00	0.005	0.008	0.010
17:00	0.006	0.008	0.010
18:00	-0.004	0.009	0.010
21:00	0.010	0.015	0.018

For DMI-HIRLAM-E data assimilation is carried out twice daily, at midnight and noon UTC, and it is interesting to see that the size of both the mean and the standard deviation of the differences varies before and after data assimilation at noon.

The overall RMS for all differences throughout the day is 1.7 cm.

Comparing the Bernese derived zenith delays with delays determined using the Saastamoinen global delay model and standard meteorological parameters, which is most often the case with commercial software packages, the differences are larger. Based on the same time epochs and the location of the 14 GPS stations, the RMS of the differences between the Benese and the Saastamoinen delays is 2.3 cm.

For this day the zenith delays determined based on DMI-HIRLAM-E are thus generally better than the delays determined with the Saastamoinen model.

With these improved estimates of the tropospheric zenith delays it is interesting to see whether an improvement in the position domain is obtained.

STATIC POSITIONING TESTS

The positioning tests were carried out using the commercial software package GPSurvey version 2.35 from Trimble Navigation (Trimble, 1996). Since the source code of the software was not available, the HIRLAM zenith delays were applied to the GPS data before the data was processed by GPSurvey.

The raw GPS data was corrected for the tropospheric delay by initially determining the elevation angle of the received satellite signal. Then the zenith delay for the given epoch in time was determined using a linear temporal interpolation between the zenith delays determined for each full hour. The zenith delay was then mapped to the appropriate elevation angle using the Niell mapping function (Niell, 1996), and finally the slant delay was subtracted from the raw code and phase observations in the data files. Hereby a set of tropospherically corrected data files were generated.

This way of dealing with the tropospheric delay can cause an increased level of detected cycle slips when the data is cycle slip screened by the processing software. At the lowest elevation angles the change in tropospheric delay within 15 seconds (the data rate with this data set) can be larger than the wavelength for L1 or L2. For example, the speed of a GPS satellite is 4 km/s, corresponding to an angular velocity of approximately 0.009° pr. second, or 0.13° for 15 seconds. The mapping factor, determined with the mapping function, changes with about 0.2 when the elevation angle of the received satellite signal is changed from 5° to 5.1°. With a zenith delay of 2.4 meter, the change in total tropospheric delay is then approximately 0.2 x 2.4 meter = 0.5 meter, within the 15 seconds. Hereby a jump of more than two cycles is introduced in the phase observations, and if the cycle slip detection function in the software is not tolerant to this kind of "dynamics" in the phase observations, the jump will be interpreted as a cycle slip.

The purpose of the tests is to investigate whether the HIRLAM zenith delays are better than zenith delays determined using a global tropospheric delay model. So in order to avoid problems with the increased level of cycle slips for low elevation angles the GPSurvey tests are carried out using a 15° elevation mask.

POSITIONING RESULTS

The raw and tropospherically corrected GPS data was processed in static mode. During the processing the ONSA station was used as reference and was kept fixed on its ITRF97 coordinates. The rest of the stations were treated as rovers, and the resulting baselines are shown in Figure 1. The processing was, as mentioned, carried out with a 15° elevation mask, a 15 second data rate, and precise orbits.

When processing the raw data files the Saastamoinen model was used in the software to compensate for the tropospheric error. When processing the tropospherically corrected data files, any correction for the tropospheric error within the software was de-activated.

Two times six hours of data was processed and they will in the following be denoted as the morning data set (0.00-6:00 hours UTC), and the afternoon data set (12:00-18:00 hours UTC).

The positions determined using the raw and tropospherically corrected data files are compared with the known station coordinates determined using the Bernese software. The coordinate differences are then used to determine the 3D error, σ_{3D}, as given by Equation (7).

$$\sigma_{3D} = \sqrt{dX^2 + dY^2 + dZ^2} \tag{7}$$

Where dX, dY, dZ are the coordinate differences between the coordinates determined with GPSurvey, and the coordinates determined using the Bernese software and the seven days of data as previously described.

This was done for both the HIRLAM and the Saastamoinen solutions, and then by subtracting the HIRLAM σ_{3D} from the Saastamoinen σ_{3D} a measure of the improvement is obtained.

The diagrams in Figure 2 and 3 show the improvement in position accuracy by using the new HIRLAM approach instead of using the Saastamoinen model. Negative values imply that the Saastamoinen model gave best results. The baselines are listed according to baseline length and the station names have been abbreviated using only the first two letters of the names given in Figure 1.

With the morning data set the ambiguities for three of the baselines (ON-BU, ON-MA, ON-SM) were not fixed when using the Saastamoinen model, and when using the NWP approach ambiguities for four of the baselines were not fixed (ON-HE, ON-BU, ON-MA, and ON-SM). For the afternoon data set the ambiguities for all baselines were fixed to integer values.

A position determined based on float ambiguities is unreliable since all unmodelled errors in the positioning process will propagate into the floating ambiguities. However, if a satellite is observed without interruptions for longer time intervals, 30 - 60 minutes, and if the noise level in general is low, and if no sudden changes in the receiver-satellite geometry occur, then the floating ambiguities will converge towards the correct integer number, and the accuracy of the resulting position can be as good as a few cm as shown in for instance by Jensen and Cannon (2000).

This explains why the results for the three baselines, where the ambiguities were not fixed to integer values when using both tropospheric approaches (ON-BU, ON-MA, and ON-SM), show both the largest and the smalles improvements in position accuracy.

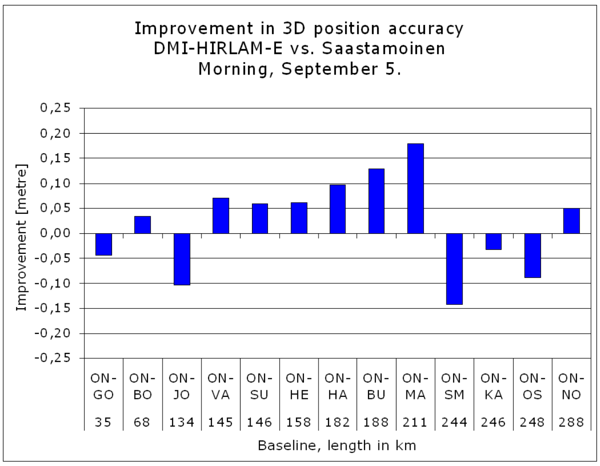

Figure 2. Improvement in 3D position accuracy with the morning data set, using the HIRLAM approach versus the Saastamoinen model.

According to Zhang (1999) the residual tropospheric error is generally larger than multipath and receiver noise, when the GPS baselines are longer than 100 km. An improved tropospheric correction approach is therefore not expected to have any influence for baselines shorter than about 100 km.

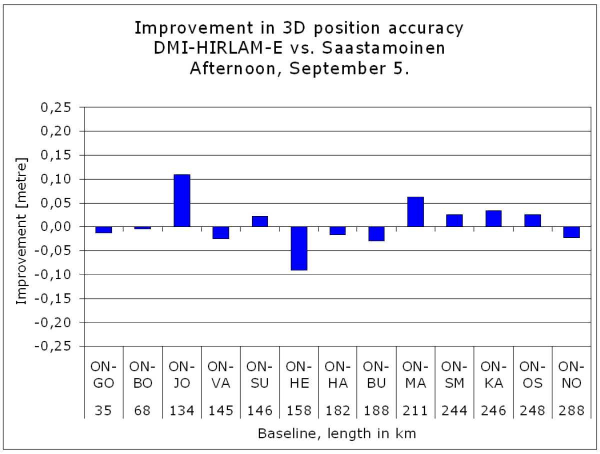

Figure 3. Improvement in 3D position accuracy with the afternoon data set, using the HIRLAM approach versus the Saastamoinen model.

It was expected, however, that the improvement would increase with an increasing baseline length or an increasing height difference between reference and rover. The improvements in 3D position accuracy are listed according to baseline length in the figures, and no indication of a correlation of improvement with baseline length can be seen from the diagrams.

Considering the height difference there is no trend in the results either. The largest height differences are for the baselines ON-BO and ON-JO, and they do not show a larger improvement than the rest of the baselines. A height difference of about 200 meter is, however, not significant, and when the baselines as here are only 68 and 134 km in length, the difference in atmospheric conditions is not large enough to cause a significant improvement by using DMI-HIRLAM-E instead of the Saastamoinen model.

Thus, the results do not show any correlation with either baseline length nor height difference. However for 15 of the 26 baselines processed an improvement in the 3D position accuracy is obtained when using the HIRLAM approach as compared to using the global Saastamoinen model for correcting for the tropospheric error, and this indicates that the method does have a potential.

The plots in Figure 2 and 3 show the improvement in 3D position accuracy, but as described by Brunner and Welsch (1993), the size of the tropospheric delay has a larger influence on the height component than on the horizontal components of a position solution. The improvement in the height component was therefore also analyzed. The results for the height are, however, similar to the 3D position results, and they are therefore not discussed separately.

CONCLUSION

With this paper a procedure for determining zenith delays based on numerical weather predictions (NWP), and for using these zenith delays in a static GPS positioning process, is described.

The zenith delays determined based on NWPs were verified by comparison with zenith delays determined based on post processed GPS data, and the comparison shows that the NWP zenith delays have an accuracy of about 1.7 cm. This is almost a factor of two better than zenith delays determined by the global Saastamoinen model using standard meteorological parameters. The verification thus showed that this new method for determining zenith delays from a NWP is superior to the global delay model, when using this data set.

Results from the static positioning tests showed that when the NWP approach is used instead of the Saastamoinen model for correcting for the tropospheric error, the 3D position accuracy was improved for 15 of the 26 baselines processed. This demonstrates that the use of NWPs does have a potential for GPS positioning. These tests are, however, based on only one day of data, so further tests must be carried out in order to draw any final conclusions on the method.

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