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Computation of the geoid of the Central El Salvador Area using gravity, GPS/levelling and a detailled DTM.

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Abstract:

Least-squares collocation has been used for the computation of a geoid for the Central El Salvador area. Free-air gravity anomalies and height anomalies based on GPS and levelling have been used. A digital terrain model has been used for the smoothing of the data using residual terrain modelling. The GPS/levelling data is of good quality, but during the preparation of the data one possible gross-errors have been detected. This values were not used when computing the geoid.

The geoid has a relative error of 0.10 m if we suppose that the GPS/levelling data has an error of 0.05 m. The geoid solution is delivered in the form of a 0.02 degree grid, which can be used for interpolation purposes using the GRAVSOFT program geoip.

Introduction.

GPS may be used for ellipsoidal height determination with a relative error at the cm level. However for most practical purposes heights above mean sea-level (orthometric or normal heights) are needed. Such heights may be obtained by subtracting the height of the quasi-geoid (or geoid for short) from the ellipsoidal heights.

The geoid may only be determined (in an absolute manner) using a global distribution of gravity data. However, in a local area such as El Salvador a relative geoid may be determined which serves the purpose that it makes the conversion from ellipsoidal heights to orthometric heights possible. This requires that we know the ellipsoidal and the orthometric height in at least one point. In Central El Salvador we are in the lucky position that 29 such points recently have become available. Also gravity anomalies are available. Furthermore a high resolution (3”) digital terrain model is available, which makes it possible to smooth the data and de-smooth the geoid solution.

The method of Least-Squares Collocation makes it possible to combine gravity anomalies and GPS/levelling data, see Moritz, 1981, Torge, 2001. Furthermore the method makes it possible to compute error-estimates which takes both the data-distribution and the varying data quality into account.

The method has been used for the determination of a (relative) geoid for Central El Salvador. It is relative because the zero-level of the height datum (as for all other countries) has a bias with respect to a theoretical world height datum.

In the following we describe the procedure used to compute the geoid.

Least-squares collocation.

LSC determines an approximation to the anomalous potential T as a linear combination of the observations,

$$T(P)=\sum_{i=1}^{n}a_i(P)\cdot x_i=\{\mathrm{cov}(T(P),x_j\}^T\{\mathrm{cov}(x_i,x_j)+d_{ij}\}^{-1}\{x_i\}=C_{Pi}^T\cdot\overline{C_{ij}}\cdot x \quad (1)$$

where x_i are the observations (here gravity anomalies and GPS/levelling height anomalies) and a_i are quantities (functions) which depend on the position of the point P. The quantities cov(T(P), x_i) and cov(x_i , x_j) are the covariances between the quantities. The quantity d_{ij} are the variances and the covariances of the noise. The approximation to T is determined by solving a set of normal equations with as many unknown as the number of observations, plus the number of contingent parameters. When T has been determined, the geoidal heights are determined using Bruns equation,

$$\zeta(P)=T(P)/\gamma$$

where γ is normal gravity. Other quantities like deflections of the vertical or gravity anomalies may also be determined.

The same equations holds when using the remove-restore method (Forsberg & Tscherning, 1981). Here the contribution from a spherical harmonic expansion, EGM96, has been subtracted from both the gravity data and the GPS/levelling data, and subsequently restored. The effect of the residual topography (rtm-effects) has also been removed and restored using the program TC.

Data used.

Gravity anomaly data from which had been subtracted EGM96 to degree 360 and the residual terrain effect, were received from El Salvador, Bureau Gravimetrique International and NIMA. These data contained several errors, such as datum errors and many values were dublicate. A dataset of 393 values was selected with an approximate data density of 0.05 degrees.

GPS/levelling quasi-geoidal or geoidal heights referring to WGS84 and a local height datum, were received from El Salvador authorities and Kapsax A/S. A standard error of 0.05 m was assigned to the data. This value is possibly too low, but aids in forcing the solution to agree with the input data.

A 3” resolution digital terrain model for the land areas was received from Kampsax A/S. For the ocean areas depths from the ETOPO5 model was used.

All data were converted to GRAVSOFT format. All data were reduced by subtracting the contribution of EGM96 and the rtm-effect. These effects were later restored.

Covariance computation and modelling.

The program empcov was used for the estimation of the empirical covariance function. In order to obtain the auto and cross-covariances occurring in Equation (1) a general analytic covariance model (Tscherning & Rapp, 1974) was determined. It has the form

$$\mathrm{cov}(T(P),T(Q))=a\sum_{i=2}^{N}\sigma_{ei}^2\left(\frac{R^2}{rr'}\right)^{i+1}P_i(\cos\psi)+\sum_{i=N+1}^{\infty}\frac{A}{(i-1)(i-2)(i+4)}\left(\frac{R_B^2}{rr'}\right)^{i+1}P_i(\cos\psi) \quad (2)$$

a I and A are scale factors, r, r’ are the radial distances of P, Q from the origin, ψ is the spherical distance between the points, P_i are the Legendre polynomials, σ_{ei}^2 are the error-degree variances of EGM96 and R is the mean radius of the Earth. The estimated values were then used to determine the constants N, a, A and the Bjerhammar-sphere radius R_B using the program covfit, (Knudsen, 1987) see Table 1.

Spherical distance, degrees	Empirical covariance (mgal**2)	Model covariance (mgal**2)
0.0	234.5	211.2
0.021	195.7	198.7
0.054	152.2	154.4
0.088	108.2	110.0
0.121	81.1	75.5
0.154	58.7	48.5
0.188	34.1	27.1
0.221	6.5	11.4

Tabel 1. Empirical and model covariance function.

The model covariance function was estimated to have N=360 a=0.08, depth of Bjerhammar sphere=2155 m and gravity anomaly variance 211.2 mgal^2. This value is much smaller than the empirically estimated value and indicates that noise of between 4 and 5 mgal is present.

Error-estimation.

LSC permits the calculation of the errors of prediction,

$$error(\zeta(P))^2 = Var(\zeta) - C_{i\zeta}^T \cdot \overline{C^{-1}} \cdot C_{j\zeta} \quad (3)$$

Var(ζ) is the variance of the rtm-height anomalies and the vectors C are the covariances between the height anomaly and the data.

Using equation (1) as implemented in the program GEOCOL (Tscherning, 1974) (geocol15f.f) the reduced GPS/levelling heights were predicted and their errors estimated using eq. (3) from the 393 reduced gravity anomalies, see Equation (3). A bias of 0.574 m was detected. The standard deviation of the differences were only 0.19 m, a very good result

The residuals were inspected and one value which was 4 times larger than the prediction error, eq. (3) was removed (point G13).

Computation of the reduced and the restored quasi-geoid.

The set of 28 GPS/levelling heights were then used in a new prediction. The heights had assigned a standard error of 0.05 m.

The data were used to compute a grid of height anomalies and error estimates in grid points covering the area bounded by 13.3N and 14.3N , 270E and 271.5E with a 0.02 degree spacing. The GPS/levelling values were also predicted The residuals (difference observed minus predicted) had a mean value of -0.04 and a standard deviation of 0.13 m. The minimal and maximal difference were -0.27 and 0.40 m, indicating that some gross errors may be left. The formal error in the area which include the GPS-points is around 0.05 m. This value is related to the input value of the same magnitude, which possibly is a little optimistic.

The contribution of EGM96 and of the residual topography was computed in the grid and the values plus bias were added.

The final solution is delivered in the form of a FORTRAN program, geoips.f, which when given the grid (file named elsalva.gri), the coordinates as latitude and West longitude in radians in a file INPUT.USR will interpolate in the grid and deliver the geoid heights in a file OUTPUT.USR (See Appendix 1 and 2). The FORTRAN program is delivered both in the form of the source code and in the form of an executable, geoips.exe, which may run as a DOS program under Windows. All programs and files may be retrieved using anonymous ftp to ftp.gfy.ku.dk. Log in as anonymous, passwd is your e-mail adress. Then change directory

```
cd  pub/People/cct
bin
get <insert file name>
```

Contour maps of the geoid grid and the geoid error grid are found on the web as http://www.gfy.ku.dk/~cct/elsalvg.pdf and http://www.gfy.ku.dk/~cct/elsalvger.pdf .

Conclusion.

The error-estimates in the area of interest are 0.05 m. This is a formal error. The data contain maybe more gross-errors than the one found here in this study. The data should then have assigned realistic error estimates and the computations may then be repeated.

References

Forsberg, R. and C.C.Tscherning: The use of Height Data in Gravity Field Approximation by Collocation. J.Geophys.Res., Vol. 86, No. B9, pp. 7843-7854, 1981.

Knudsen, P.: Estimation and Modelling of the Local Empirical Covariance Function using gravity and satellite altimeter data. Bulletin Geodesique, Vol. 61, pp. 145-160, 1987.

Moritz, H.: Advanced Physical Geodesy. H.Wichmann Verlag, Karlsruhe, 1980.

Torge, W.: Geodesy. 3. ed., de Gruyter, Berlin, 2001.

Tscherning, C.C.: A FORTRAN IV Program for the Determination of the Anomalous Potential Using Stepwise Least Squares Collocation. Reports of the Department of Geodetic Science No. 212, The Ohio State University, Columbus, Ohio, 1974.

Tscherning, C.C. and R.H.Rapp: Closed Covariance Expressions for Gravity Anomalies, Geoid Undulations, and Deflections of the Vertical Implied by Anomaly Degree-Variance Models. Reports of the Department of Geodetic Science No. 208, The Ohio State University, Columbus, Ohio, 1974.

Tscherning, C.C., P.Knudsen and R.Forsberg: Description of the GRAVSOFT package. Geophysical Institute, University of Copenhagen, Technical Report, 1991, 2. Ed. 1992, 3. Ed. 1993, 4. ed, 1994.

Appendix 1. Sample input file INPUT.USR. Format is latitude (radians), west longitude (radians). Same points as in Appendix 4 and 5.

```
  0.236216   1.564588
  0.235686   1.561591
  0.235392   1.559032
  0.234261   1.555419
  0.237691   1.562275
  0.236899   1.560421
  0.237063   1.558043
  0.236974   1.556899
  0.236911   1.555546
  0.240011   1.561942
  0.239486   1.559560
  0.238549   1.558886
```

```
  0.238555   1.556668
  0.238239   1.555879
  0.242324   1.561988
  0.240966   1.559060
  0.240773   1.556441
  0.239911   1.553815
  0.241763   1.559646
  0.242117   1.556625
  0.241314   1.553839
  0.243911   1.559234
  0.243723   1.556645
  0.243711   1.555453
  0.244858   1.560802
  0.245834   1.558574
  0.246105   1.556802
  0.246176   1.555933
  0.238162   1.560395
```

Appendix 2. Sample output file OUTPUT.USR. Output is the geoid height in meters in the points given in the file INPUT.USR . Same points as in Appendix 1, 4 and 5.

```
       -.781
       -.360
       -.064
       -.187
        .702
        .679
       1.022
       1.066
       1.166
       1.422
       1.750
       1.829
       1.808
       1.701
       1.936
       1.979
       2.071
       2.093
       1.959
       2.095
       2.238
       2.104
       2.181
       2.267
       2.226
       2.339
       2.459
       2.574
       1.439
```

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Appendix 3. Heading and first data-line of file elsalva.gri. Heading gives minimum and maximum latitude in degrees, minimum and maximum longitude (East) in degrees, the grid-spacing in latitude and longitude. This is the followed by grid values in meters.

```
   13.300000   14.300000  270.000000  271.500000    0.020000    0.020000
```

1.884	1.878	1.898	1.997	2.031	2.058	2.060	2.054
2.123	2.156	2.200	2.167	2.179	1.932	1.963	2.004
2.033	2.043	2.199	2.261	2.362	2.462	2.563	2.456
2.554	2.665	2.745	2.878	2.964	2.994	3.077	3.131
3.181	3.545	3.502	3.457	3.396	3.342	3.150	3.213
3.240	3.277	3.345	3.895	4.132	4.391	4.556	4.606
4.393	4.430	4.442	4.405	4.356	4.268	4.222	4.173
4.097	4.059	4.160	4.230	4.289	4.322	4.294	4.171
4.151	4.123	4.090	4.081	4.055	4.096	4.135	4.147
4.168	4.474	4.527	4.555				

Appendix 4. Original geoid heights used as input to the computations of the grid. Point 13000 not used in final solution.

Latitude	Longitude (E)		H	Geoid height
	O	o	m	m
1000	13.534161	270.355697	6.617	-0.983
2000	13.503841	270.527398	53.717	-0.380
3000	13.486951	270.674033	15.903	-0.216
4000	13.422151	270.881082	7.008	-0.329
5000	13.618670	270.488240	758.970	0.686
6000	13.573293	270.594452	393.451	0.663
7000	13.582708	270.730740	560.676	1.066
8000	13.577621	270.796263	528.781	1.144
9000	13.573979	270.873794	522.721	1.163
10000	13.751596	270.507298	512.826	1.563
11000	13.721552	270.643814	590.293	1.720
12000	13.667864	270.682438	1116.867	1.725
13000	13.668192	270.809514	710.569	2.244
14000	13.650092	270.854707	742.180	1.726
15000	13.884133	270.504673	783.026	2.090
16000	13.806312	270.672465	506.508	1.965
17000	13.795274	270.822483	443.292	2.050
18000	13.745869	270.972953	764.596	2.176
19000	13.852021	270.638869	424.171	1.981
20000	13.872293	270.811961	339.797	2.105
21000	13.826271	270.971575	512.894	2.349
22000	13.975082	270.662482	311.952	2.127
23000	13.964297	270.810824	295.915	2.122
24000	13.963633	270.879086	313.569	2.240
25000	14.029319	270.572655	301.689	2.162
26000	14.085237	270.700301	287.354	2.196
27000	14.100806	270.801788	263.069	2.325
28000	14.104846	270.851598	281.592	2.580
29000	13.645673	270.595948	1313.468	1.391

Appendix 5. Differences between original geoid heights and values interpolated from grid (see Appendix 2). Note the large difference for point 13000.

	Latitude	Longitude (E)	H	Difference
	O	o	m	m
1000	13.53416	270.35570	6.62	-0.202
2000	13.50384	270.52740	53.72	-0.020
3000	13.48695	270.67403	15.90	-0.152
4000	13.42215	270.88108	7.01	-0.142
5000	13.61867	270.48824	758.97	-0.016
6000	13.57329	270.59445	393.45	-0.016

7000	13.58271	270.73074	560.68	0.044
8000	13.57762	270.79626	528.78	0.078
9000	13.57398	270.87379	522.72	-0.002
10000	13.75160	270.50730	512.83	0.141
11000	13.72155	270.64381	590.29	-0.031
12000	13.66786	270.68244	1116.87	-0.104
13000	13.66819	270.80951	710.57	0.436
14000	13.65009	270.85471	742.18	0.025
15000	13.88413	270.50467	783.03	0.154
16000	13.80631	270.67247	506.51	-0.014
17000	13.79527	270.82248	443.29	-0.021
18000	13.74587	270.97295	764.60	0.083
19000	13.85202	270.63887	424.17	0.022
20000	13.87229	270.81196	339.80	0.010
21000	13.82627	270.97158	512.89	0.111
22000	13.97508	270.66248	311.95	0.023
23000	13.96430	270.81082	295.92	-0.059
24000	13.96363	270.87909	313.57	-0.027
25000	14.02932	270.57266	301.69	-0.064
26000	14.08524	270.70030	287.35	-0.143
27000	14.10081	270.80179	263.07	-0.134
28000	14.10485	270.85160	281.59	0.006
29000	13.64567	270.59595	1313.47	-0.048