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Determination of a (Quasi) Geoid for the Nordic Countries from Heterogeneous Data Using Collocation

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by

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Summary

The method of collocation has been applied for the determination of approximations to the anomalous gravity potential for an area covering Denmark, Finland, Norway and Sweden. The approximations have been used for the computation of quasigeoidal heights.

The different approximations have been computed based on various combinations of available data: potential coefficients, approximate geoid undulations, deflections of the vertical, and gravity anomalies. A comparison of computed quantities with quantities observed, but not used in the determination of the approximation, shows that quasi-geoid differences may be determined for the Nordic countries with a standard deviation better than ±0.5 m.

1. Introduction

The Nordic Geodetic Commission, at its meeting in Oslo, 1978, adopted a resolution requesting that Geodætisk Institut carry out the computation of a "standard geoid" for the Nordic Countries (Denmark (+Faroe Islands and Greenland), Finland, Norway and Sweden).

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This task was after consultations with representative of the other countries in the spring of 1982 made more specific: The goal should be the determination of a quasi-geoid covering at least the land areas. The differences between the height anomalies should be determined with a standard deviation below ±0.5 m.

That this kind of goal was fixed made it possible to disregard

(1) (small) errors in reference system parameters
(2) errors due to the use of spherical approximation
(3) the time variation of the gravity field other than these which usually are taken into account (tides, etc.)
(4) errors due to lack of gravity field information in USSR, Poland and DDR.

In this paper, we will describe the status of the computations, which are nearly completed, (August, 1, 1982). A description of the first phase of the project, where only deflections of the vertical were used, is given in (Tscherning, 1982). The determination of a quasi-geoid requires the solution of the geodetic boundary value problem, i.e. the determination of the anomalous (gravity) potential, T. In order to do this, we have selected the method of collocation as developed by (Krarup, 1969) and (Moritz, 1972). A very short outline of the method is given in section 2. The reasons for selecting the method are outlined in (Tscherning, 1982).

In section 3 the data collection and selection process is described, and in section 4 the computational process is sketched. The result is formally given as a set of linear combinations of sets of harmonic functions, each valid in a specific area. However, we have also constructed a set of maps showing the height anomalies and free air gravity anomalies computed at height zero. These maps permit the computation of the height anomaly at terrain level, which per definition is equal to the quasi-geoidal height. This is explained in detail in section 5. The possibilities for further improvement of the results is discussed in section 6. Such improvements will be possible if information about the topography and geology are used and if data becomes available in areas, where the data coverage today is sparse.

2. Collocation

Let W be the gravitional potential of the Earth and U a normal potential of the Somigliano-Pizzetti type. Then T = W-U is the anomalous potential, which we here will suppose is harmonic outside the solid Earth's surface and regular at infinity. An approximation $\tilde{T}$ to T may then be determined using collocation. This approximation is the element of a reproducing kernel Hilbert space, which has the least norm, and which agrees with observed values,

$$x_i = L_i(T) = L_i(\tilde{T}), \quad i = 1, \ldots\ldots\ldots, n \qquad (1)$$

Here n is the number of observations and L_i are the linear (or linearized) func- functionals, which relates T to the observed values, x_i.

Let the reproducing kernel be denoted K(P, Q), where P and Q are points in space. Then

$$\tilde{T}(P) = \sum_{i=1}^{n} a_i \, K(L_i, P). \qquad (2)$$

Here we will use the notation introduced in Krarup (1978), where

$$K(L_i, L_j) = L_i(L_j(K(P,Q))) \text{ and } K(L_i,P) = L_i(K(P,Q)) .$$

The coefficients $\{a_i\}$ are determined from the n equations which arise, when we combine eq. (1) and (2),

$$\{K(L_i,L_j)\} \ \{a_i\} = \{x_j\} \qquad (3)$$

If the observations mainly are related to points in a limited area, we get a <u>local</u> approximation to T.

If T is an element of the Hilbert space, (a condition, which need not to be fulfilled in order to construct $\tilde{T}$), then estimates of the maximal error may be computed. Let a computed (or "predicted") quantity be expressed as the result of a linear functional L, applied on $\tilde{T}$, $L(\tilde{T})$. Then the true value is L(T), and we have

$$|L(T)-L(\tilde{T})| \leq \|T\| (K(L,L) - \{K(L,L_i)\}^T \{K(L_i, L_j)\}^{-1} \{K(L_j,L)\})^{\frac{1}{2}} \qquad (4)$$

see e.g. (Tscherning, 1978). Unfortunately, this equation is of little practical value as $\|T\|$ is unknown and T is mostly not an element of the Hilbert space.

An Approximation $\tilde{T}$, optimal in a least-squares sense, may be determined, if we use as reproducing kernel the so-called empirical covariance function, see Moritz (1980). Then eq. (4) applied with $\|T\| = 1$, gives error estimates of the root mean square error. These estimates are quite realistic, if a good estimate of the empirical covariance function has been obtained.

The quality of the predicted quantities also depends on the covariance function. The dependence is limited, if the same type of quantity is observed and predicted, i.e. in situations where collocation works as an interpolation technique. However, the dependence becomes critical when for example height anomalies are computed from gravity anomalies as reported by (Wenzel, 1980) and (Arabelos, 1980). In order to facilitate the practical determination of $\tilde{T}$, it is convenient to work in spherical approximation, and with a Hilbert space of functions which are harmonic down to a sphere totally enclosed in the Earth, (as proposed originally by A. Bjerhammar). This means, that we are able to evaluate the value of linear functionals related to points inside the masses as long as they are outside the "Bjerhammar sphere". We have used this fact in order to represent the result in a convenient way, see section 5.

In the computations we have throughout used the method of representing the covariance functions developed in (Tscherning, 1972).

$$K(P,Q) = \sum_{n=o}^{\infty} \sigma_n \left(\frac{R_E^2}{r_P r_Q}\right) P_n(\cos\psi). \qquad (5)$$

where ψ is the spherical distance between P and Q, r_P, r_Q the radial distances of these two points, P_n Legendre polynomials and σ_n degree-variances.

These have been modelled by modifying a global model for the degree-variances, using

$$\sigma_n = \frac{A \cdot R_E^2}{(n-1)(n-2)(n+24)} \left(\frac{R_B}{R_E}\right)^{2n+2} \qquad (6)$$

where R_B is the radius of the Bjerhammar sphere, R_E = 6371 km is the mean radius of the Earth and A is a positive constant.

When writing down eq.(1), we did not consider errors in the observations. These may be taken into account, by requiring that the sum

$$\| \tilde{T} \|^2 + \lambda\{(L_i(T)-x_i)\}^T \{w_{ij}\}^{-1} \{(L_j(\tilde{T})-x_j)\} \qquad (7)$$

be a minimum. Here λ is a positive constant and $\{w_{ij}\}$ is the weight matrix associated with the observation errors. λ is mostly put equal to 1, when the empirical covariance function is used as the reproducing kernel, but it is uncertain whether this always is the best choice. The minimalization of (7) is obtained by adding $\{w_{ij}\}$ to $\{K(L_i,L_j)\}$ (the normal equation matrix) in eq. (3).

Also parameters, such as datum shift parameters, may be determined. We will not discuss this here, as this has not been used in this phase of the project.

Note, that the procedure described above may be applied to any harmonic function. This makes it possible to take into account the topographic variations and known geological structures. The simplest method is subtract from T the potential of the masses, T_M, and later on add this potential back again, see (Forsberg and Tscherning, 1981). But also methods may be used, which involves the simultaneous estimation of unknown density variations, see (Sanso' and Tscherning, 1982). These possibilities have not been applied in this phase of the project, but will have to be used in order to attain the goal mentioned in section 1 in areas with a strong gravity variation, see section 6.

3. Data collection

The data collection started in 1978, where a large set of gravity anomalies were received from U.S. Defense Mapping Agency, Aerospace Center. Deflections of the vertical had earlier been collected for Denmark, northern Germany, Norway and Sweden as a part of earlier projects.

A revision and completion of this dataset was initiated in the fall of 1980, and is unfortunately not yet finished. Detailled instructions were distributed concerning the needed data elements and specific physical formats were recommended.

The most important requirement was that the information be complete, and this was also the most difficult for the cooperating agencies to fulfil. The data elements which were requested are described in Table 1.

Data kind	Coordinates (inclusive height)	Observed value(s)	Standard deviation	Source	Unique station number	Reference system
Gravity	x	x	x	x		x
Deflection of the vertical	x	x	x		x	x
Doppler determined (x, y, z)	x	x	x	x	x	x
Satellite altimetry	x	x	x			x
Potentiel coefficients		x	x			x
Topographic mean or point heights	x	x				

Table 1. Data elements collected.

Unfortunately, some of these data elements were sometimes not available, see Table 2. The countries are shown using their well known identifying letters: D, DK, N, S, SF.

Data kind	Coordinates φ, λ	Coordinates H	Observed value	Standard deviation	Source	Unique number	Reference system
Gravity	S*, SF	SF	SF				
Deflections of the vertical		S		S, SF***			
Doppler (x, y, z)				S			
Topographic heights			S** SF** DK**, D**				

Table 2. Lacking data elements.

* Coordinates only given as integer minutes,

** Only mean heights of 5' x 10' blocks available.

*** Only available for groups of stations and for some individual stations.

A few other problems must be mentioned. For sea-gravity, the depth of the ocean was sometimes lacking. The Doppler determined (x,y,z) coordinates were sometimes given as the result of a specific campaign, NORDOC, SCANDOC), for which a well defined transformation to an "absolute" system was not determined (see however (Ekmann, 1980, Table 8)).

It is difficult to determine the exact number of gravity values which are available, because a number of duplicate points exist. We have shown the total number of values in Fig. 1.

Some areas have a dense distribution and some have few observations. In order to handle this large dataset, a basic set was created, with the observations spaced as uniformly as possible. This was done, by selecting from "cells" of a given size the one measurement closest to the center. The cell size was determined in order to achieve a balance between the number available and the minimum number needed in order to obtain the result of ± 0.5 m for the quasi geoid.

This last number is difficult to estimate. But if deflections could be estimated with a standard error of $\pm 1''$, then we should be on the safe side. This however, would be a too strong requirement in mountains, where deflections of the vertical and gravity anomalies vary substantically over small distances. In other words, most of the variation is due to the influence of the higher order harmonics of the spherical harmonic expansion, which do not influence the height anomalies in the same way as it influences the two other types of quantities.

This fact, was then accounted for in the estimation of the number of observations needed by using a fixed value of 7' for the correlation distance ψ_1 of the covariance function. This distance is together with the mean square gravity variation, C_0, the factors which determine the number of required observations, see (Forsberg and Tscherning, 1981a, eq. (13)). The use of this formula, gave the result, that in most cases less than 400 observations were needed in a $1^{\circ} \times 2^{\circ}$ block. Only in South-West Norway was the number larger, the maximal value being 1100. (This number exceeded the actually available number of observations).

A cell size of $3' \times 6'$ (or 9' north of 66°) was then chosen everywhere, except in South-West Norway, where cells of size $2' \times 4'$ were used.

Simultaneous with the creation of this dataset, the mean value and root mean square variation were computed for each cell. This was very useful, as this reveiled a number of errors mainly due to the fact, that some older gravity values still were contained in the original dataset.

Totally 718 meridian components (ξ) and 685 prime vertical (η) deflection of the vertical components are available in D, DK, N, S and SF north of $\varphi = 54^{\circ}$.

We have the following distribution of the Doppler stations DK:4, S:6, N:19, SF:4. (This excludes 2 old norwegian stations, which seem to have large errors).

SEASAT-A data has been obtained from Prof. Rapp, The Ohio State University covering the area $52^{\circ} < \varphi < 72^{\circ}$, $0^{\circ} < \lambda < 32^{\circ}$.

The data collection still continues. Gravity data from Finland are, for example, expected to be received soon using Bureau Gravimetrique International as an intermediary agent and New Doppler observations are also being collected.

4. The computational process

The computational process may be divided in 5 stages:

(1) validation of the data

(2) determination of the covariance functions

(3) selection of observations needed in order to obtain the required precision

(4) determination of a set of local solutions T_k

(5) evaluation of the result.

Phase (1), (2) and (3) was carried out simultaneously. The best way to detect errors is to construct a local approximation to T, from a uniformly distributed part of the observations, and then predict the rest. This primary set may then also be used for the covariance determination.

As only DK, N and S had a reasonable distribution of gravity data, the validation process was limited to these areas. Here we selected gravity data points spaced as close as possible to a grid with side length 6' x 12' (latitude, longitude) for the area below $\varphi = 66^{\circ}$ and 6' x 18' for the area above $\varphi = 66^{\circ}$. Local solutions were then constructed based on a set of potential coefficients complete to degree and order 180, cf. (Rapp, 1978), and the gravity data. This was done in two steps, where the first approximation to T simply was the one determined by the potential coefficients. In the second step, the contribution from the potential coefficients was subtracted, and the residual gravity data were used as observations. In this step the covariance function described in (Tscherning, 1982) was used everywhere. The predicted quantities then consisted of two parts, one computed from the potential coefficients and one computed using the gravity data. The solutions had an overlap of $\frac{1}{2}^{o}$ in latitude, and 1^{o}, or 2^{o} in longitude depending on the latitude. The size of the blocks were then generally $3^{o} \times 6^{o}$ or $3^{o} \times 9^{o}$, with slight variations in order to fit the blocks in reasonable way to the land area.

Block name ≈ (φmax, λmin)	φ min	λ max	Number of filled 6'blocks*	ξ	η	Covariance function par.** C_o mgal²	ψ_1	ψ_o	A mgal	d m
71.5 17°	69°	26°	346	3	3	1464	3'	45'	345.6	175
70 13	67	25	851	22	21	1190	5'	35'	374.0	505
68 11	65	25	818	27	25	1060	5	30	329.3	505
66 07	63	16	665	18	17	711	4	28	162.0	205
66 13	63	22	548	30	29	501	5	28	115.0	245
64 05	61	11	844	15	15	2500	3	28	630.0	200
64 09	61	15	755	21	21	1140	3	37	253.2	150
64 13	61	21	465	32	32	216	8	26	58.0	620
62 04	59	11	884	36	36	2576	3	24	650.0	200
62 09	59	15	807	48	48	780	5	42	216.3	400
62 13	59	19	612	41	41	180	16	44	83.0	3000
60 04	57	11	788	49	45	640	5	28	166.3	350
60 09	57	15	765	59	55	199	9	31	55.0	700
60 13	57	19	642	35	35	142	14	33	50.5	1700
58 07	54	18				170	10	29	49.7	900

Table 3. Block extend, data distribution and covariance function characteristics.

* Longitude extend is for φ max > 64° equal to 18' and below 64° equal 12'

** ψ_1 = correlation distance, ψo = location of first zero point, ψ_o = variance of free-air gravity anomalies, d = $R_E - R_B$, where and A is a scala factor for the degree-variances, see eq (6).

The local solutions where then used to predict the remaining gravity observations, the deflections of the vertical, doppler derived height anomalies and geoid undulations obtained from satellite altimetry. The results for the predicted gravity values and deflections are given in Table 4. Note, that the data distribution generally was so that the observed value was the one in the middle of a grid consisting of maximally nine points spaced with a distance corresponding to 3' in latitude.

Block	Δg mgal mean	σ +/-	ξ mean	σ +/-	η mean	σ +/-
72 17	-6	30	3".5	6".5	-0".3	7".0
70 13	-5	25	-0.3	3.4	-0.2	4.2
68 11	-2	19	0.0	1.6	0.6	2.6
66 07	-2	16	0.5	2.8	0.1	2.2
66 13	0	10	-0.6	1.7	1.0	1.6
64 05	-5	27	-1.3	5.4	-2.5	2.8
64 09	-5	26	0.1	1.5	0.3	2.3
64 13	1	8	-0.1	1.3	1.2	1.9
62 04	-9	38	0.0	4.7	0.0	4.2
62 09	-1	15	-0.7	1.9	0.3	3.2
62 13	-1	6	-0.1	1.1	0.7	1.7
60 04	-1	16	0.1	2.5	-1.3	3.3
60 09	0	5	-0.6	1.5	-0.3	3.0
60 13	0	4	-0.4	1.1	-0.1	1.4
58 07			-0.3	1.0	0.3	1.1

Table 4. Results (observed - predicted) of predictions obtained using only gravity data and the same covariance function everywhere.

A numbers of errors were detected in the areas, where the gravity variation is small (block 5807). However, it is generally very difficult to see whether a large difference between the predicted and the observed value is due to an error in the prediction (due to lack of data or a locally strong gravity field variation) or due to an error in the observations.

In one block (5807) we then subsequently used new observations, which we suspected to be erroneous, and predicted other values. This generally gave a clear distinction between the correct values and those, which were wrong. For observations executed in Denmark it was then possible to go back to the source; and we could then verify for some of the suspected errors, that they were real-namely due to eccentric stations, and with wrong sign.

Height anomalies were also computed, and compared to observed values. This showed that too large height anomalies were predicted in the mountains caused by the too long correlation distance implied by the covariance function.

Height anomalies computed in identical points from different local solutions were also compared. This showed, that the height anomalies at the boundaries generally agreed within ± 10 - ± 20 cm, but with biases amounting to maximally 70 cm. (Note, that only potential coefficients and gravity observations were used when constructing the local solutions).

The empirically determined covariance values, were then used in order to determine analytic representations of the discrete values, cf. eq. (6). The constants used can be found in Table 3.

Low order degree-variances, derived from the error estimates of the potential coefficients, were used if they were smaller than those determined using these constants, (see (Tscherning, 1982, section 4.2)).

New local solutions were then constructed for each block. Here new observations were added, namely all the deflections of the vertical and all gravity anomalies, for which the difference between the predicted value and the observed value were larger than 15 mgal in the first, preliminary, solution. We also used height anomalies computed at the boundary to the block located southeast of the actual block as new observations. This was used to assure the continuity of the quasi-geoid. The only observed height anomalies used were these derived from (x, y, z) determined in the EDOC II campaign and the related mean sea level height.

We will describe the evaluation of the result in section 6.

5. Representation of the result

Formally the individual local solutions $\tilde{T}_i$, are given as the sum of a spherical harmonic series (with coefficients of degree and order less than or equal to 180) and a linear combination of harmonic functions as given by eq. (2). The solutions all refer to an approximate geocentric coordinate system and a normal potential, U, determined using the parameters of GRS1980, however with a semi -major axis equal to 6378135.5 m as discussed in (Tscherning, 1982).

The use of these solutions for the computation of the values of linear functionals applied on the solutions requires the application of various computer programs available in Algol or FORTRAN, see (Tscherning, 1974, 1975, 1978), (Tscherning and Poder, 1982).

The necessary constants $\{a_i\}$ and specifications of the associated linear functionals (coordinates of observation point and type of observation) are available from the author on request. This information then permits the evaluation of any linear functional applied on T_i, if this functional is an element of the Hilbert space dual to the one containing T_i. This space will contain the functionals associated with any derivatives of $\tilde{T}_i$, if the point of evaluation is exterior to the Bjerhammar sphere. A practical restriction is that the computer programs presently available only permit the evaluation of up to the second order derivatives.

Formal error estimates of the predicted quantities may also be computed, but this requires that the reduced normal equations also are transferred to the user.

Using the existing computer programs, it is then possible to evaluate e.g. height anomalies, (free air) gravity anomalies and deflections of the vertical along with their "standard deviation" in any point outside the Bjerhammar sphere, for example in the points of a regular grid. These quantities may then subsequently be used by a contouring program for the construction of maps with isolines showing the variation of these quantities in an area. Note, that we also easily may compute absolute quantities such as the geopotential and the gravity to be used for example in levelling. In fact, the solutions have a large range of applications beside these related to the use of the quasi-geoidal heights.

This height is defined as the value of the height anomaly ζ evaluated at terrain level. The construction of a map of the quasi-geoid (using a computer) then requires, that a digital map of the terrain be available. Such maps have not been available to the author, cf. Table 2. As a compromise, two maps have been constructed, which permit the easy computation of the quasi-geoidal height, if the height of the terrain is given.

One map contains the height anomaly evaluated on the reference ellipsoid. The other map shows the derivative of the height anomaly with respect to height, which approximatly is equal to the free-air gravity anomaly, Δg, divided by normal gravity, γ. Knowing the height of the terrain, h, we then have

$$\zeta(P) \approx \zeta(P_o) - \frac{\Delta g}{\gamma} \cdot h ,$$

where P_o is the projection of P on the ellipsoid. That this is correct is easily seen using Bruns formula, $\zeta = T/\gamma$. Then

$$\frac{\partial\zeta}{\partial h} \approx \frac{\partial\zeta}{\partial r} = \frac{1}{\gamma}\left(\frac{\partial T}{\partial r} - \frac{\partial\gamma}{\partial r}\frac{1}{\gamma} \cdot T\right)$$

$$\approx \frac{1}{\gamma}\left(\frac{\partial T}{\partial r} + \frac{2}{r} T\right) \approx -\frac{1}{\gamma} \Delta g.$$

The value of Δg seldom exceeds 200 mgal (10^{-5} m/s^2) in the Nordic area. This means, that the derivative maximally is 0.0002. For a mountain of height 3000 m, the effect is then 0.6 m. Hence, in order that an error in this quantity does not reduce the quality of the quasi-geoidal height, then we must be sure, that the free-air gravity anomaly is known with a standard deviation below ±25mgal.

This goal is easily attained in the areas having moderate topographic variations (0 – 1000 m), i.e. in areas where we do not need to know the free-air gravity anomaly in order to compute the quasi-geoidal height with a standard deviation of ±0.5 m. In South-West Norway, where some of the highest mountains are located, the standard deviation of the predicted anomalies is up to ±20 mgal, if data in a grid with side length 6' is used, see Table 5.

Figures 2 – 13 show a sample maps of ζ, the standard deviation of ζ, and of Δg evaluated at the ellipsoid. Maps of other areas may be obtained on request from the author. A map of height anomalies evaluated on the ellipsoid covering the Nordic area, however computed only using potential coefficients and deflections of the vertical, can be found in Tscherning (1982).

6. Evaluation of the result and possibilities for further improvements.

The quality of the result may be described in two ways. We may use eq. (4) for the computation of estimates of the root mean square error of prediction; and we may compare predicted values with known values, which have not been used as observations.

The use of the formal error estimates is reasonable, if we use an empirical covariance function, which describes well the local gravity field variation. Such covariance functions have been used, however we only know that they describe well the variation of the gravity anomalies. We have not been able to construct empirical cross covariance functions for gravity and height anomalies. This will in principle be possible e.g. in the Southern part of the Baltic Sea, in the North-Sea, and in the Norwegian Sea, but there has not yet been time available to perform the computations. For some of the blocks we know that the goal of ± 0.5 m has been attained, because we already in the first approximation were able to predict the deflections of the vertical with a standard deviation around $\pm 1''$ (block 6213, 6013, 5807). For the other blocks, we may hope that the addition of new information as described in section 4 (gravity anomalies, deflections of the vertical and doppler derived height anomalies) was sufficient. This was checked by comparing observed gravity and height anomaly values with predicted. The gravity observations in the $3' \times 6'$, $2' \times 4'$ and $3' \times 9'$ grids, which were not used for the construction of the local solutions, were used as control values. Doppler derived height anomalies, from campaigns other than EDOC-II, and "geoid undulations" obtained by using SEASAT-A sea-surface height observations were used to control the height anomalies. The result is summarized in Table 5. We see that the goal of ± 0.5 m has been attained except in the mountains.

This raises the question about how we may improve the solution. One possibility is obviously to take into account the topography as mentioned in section 2. This will enable us to attain the goal of ± 0.5 m. This conclusion can be drawn using the results given in Forsberg and Tscherning (1981), and the fact that we have a relativelv dense data distribution in the mountains, see Figure 1. Some improvements may also be possible, if additional data are included, especially new doppler derived height anomalies. More refined modelling of the covariance functions may also help improve the solution.

Let me finally mention, why satellite altimetry have not been used as observational data. In the Norwegian Sea we do not know whether systematic differences between the geoid and the sea-surface topography exist due, for example to the Gulf Stream. In the Bay of Bothnia and the Baltic Sea they are not needed due to the very smooth variation of the gravity field. However, for these areas, we intend to use the data in future solutions anyway, because we lack gravity data at sea.

	Δg(mgal)			ζ (m)					
				Doppler derived			SEASAT-A		
Block	Number of values	mean	σ +/-	Number of values	mean	σ +/-	Number of values	mean	σ +/-
7217	318	-3.2	10.4	3	0.64	0.81	776	-0.26	0.40
7013	1127	-2.4	9.6	4	1.49	1.03	156	0.20	0.32
6811	1269	-3.6	10.0	2	0.51	3.05	100	-0.02	0.56
6607	748	-1.2	6.4	3	0.77	0.26	360	-0.25	0.36
6613	488	-0.6	7.0	2	1.53	0.24	161	1.03	0.33
6405	947	-4.3	13.1	4	-0.06	1.16	301	-1.49	0.59
6409	525	-4.8	9.6	2	0.01	0.56	0	-	-
6413	261	0.2	5.0	0	-	-	277	1.14	0.39
6204	1105	-3.7	12.5	3	-1.66	1.24	65	-1.27	0.80
6209	791	-2.6	7.6	1	0.79	-	0	-	-
6213	669	-0.5	5.3	2	0.66	0.09	105	1.38	0.36
6004	1072	-1.8	7.6	4	0.51	0.34	222	0.38	0.21
6009	933	-0.3	4.0	1	0.21	-	57	0.74	0.21
6013	357	-0.2	3.7	0	-	-	184	0.81	0.32
5807	486	-0.2	3.6	2	0.21	0.14	384	0.93	0.47

Table 5. Results from the final solution. Mean and standard deviations of (observed-predicted) quantities. Note the mean value of the differences obtained for SEASAT-A, which indicates an East-West bending of the computed geoids.

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References

Arabelos, D.: Untersuchungen zur gravimetrischen Geoidbestimmung, dargestelt am Testgebiet Griechenland. Wiss. Arb. d. Fachricht. Vermessungswesen der Universität Hannover, Nr. 98, 1980.

Ekman, G.: The Scandinavian Doppler Observation Campaign 1979 (SCANDOC-79) Results and Comparisons. Geographical Survey of Norway, May 1980.

Forsberg, R. and C.C. Tscherning: The use of Height Data in Gravity Field Approximation by Collocation. J. Geophys. Res., Vol. 86, No. B 9, pp. 7843-7854, 1981.

Forsberg, R. and C.C. Tscherning: Deflection and Gravity Anomaly prediction using Collocation. Proceedings Sec. Int. Symposium on Inertial Technology for Surveying and Geodesy, Banff, June 1-5, 1981, pp. 89-95, Canadian Institute of Surveying, 1981a.

Krarup, T.: A Contribution to the Mathematical Foundation of Physical Geodesy. Meddelelser Nr. 44, Geodætisk Institut, København 1969.

Krarup, T.: Some remarks about collocation. In: H. Moritz and H. Sünkel (Ed.): Approximation methods in Geodesy, pp. 193-209, H. Wichmann Verlag, Karlsruhe, 1978.

Moritz, H.: Advanced Least-Squares Methods. Reports of the Department of Geodetic Science No. 175, The Ohio State University, Columbus 1972.

Moritz, H.: Advanced Physical Geodesy. H. Wichmann Verlag, Karlsruhe, 1980.

Rapp, R.H.: A Global 1 deg. x 1 deg. Anomaly Field Combining Satellite, Geos-3 Altimeter and Terrestrial Data. Dep. of Geodetic Science Report No. 278, The Ohio State University, Columbus, Ohio, 1978.

Sansó, F. and C.C. Tscherning: Mixed Collocation: A proposal. Quaterniones Geodesia, Vol. 3, No. 1, pp. 1-15, 1982.

Tscherning, C.C.: Representation of Covariance Functions Related to the Anomalous Potential of the Earth using Reproducing Kernels. The Danish Geodetic Institute Internal Report No. 3, 1972.

Tscherning, C.C.: A FORTRAN IV Program for the Determination of the Anomalous Potential Using Stepwise Least Squares Collocation. Reports of the Department of Geodetic Science No. 212, The Ohio State University, Columbus, Ohio, 1974.

Tscherning, C.C.: Covariance Expressions for Second and Lower Order Derivatives of the Anomalous Potential. Reports of the Department of Geodetic Science No. 225 The Ohio State University, Columbus, Ohio, 1975.

Tscherning, C.C.: A Users Guide to Geopotential Approximation by Stepwise Collocation on the RC4000-Computer. Geodætisk Institut Meddelelse No. 53, 1978.

Tscherning, C.C.: Geoid Determination for the Nordic Countries using Collocation. J. Geodetic Soc. of Japan, (in print), 1982.

Tscherning, C.C. and K. Poder: Some Geodetic applications of Clenshaw Summation. Bollettino di Geodesia e Scienze Affini, (in print), 1982.

Wenzel, H.-G.: Recent Results of Geoid Determination by Combination Techniques in the North Sea Test Area. Veröff. d. D.G.K., Reihe B, Heft Nr. 252, pp. 95-121, München, 1980.

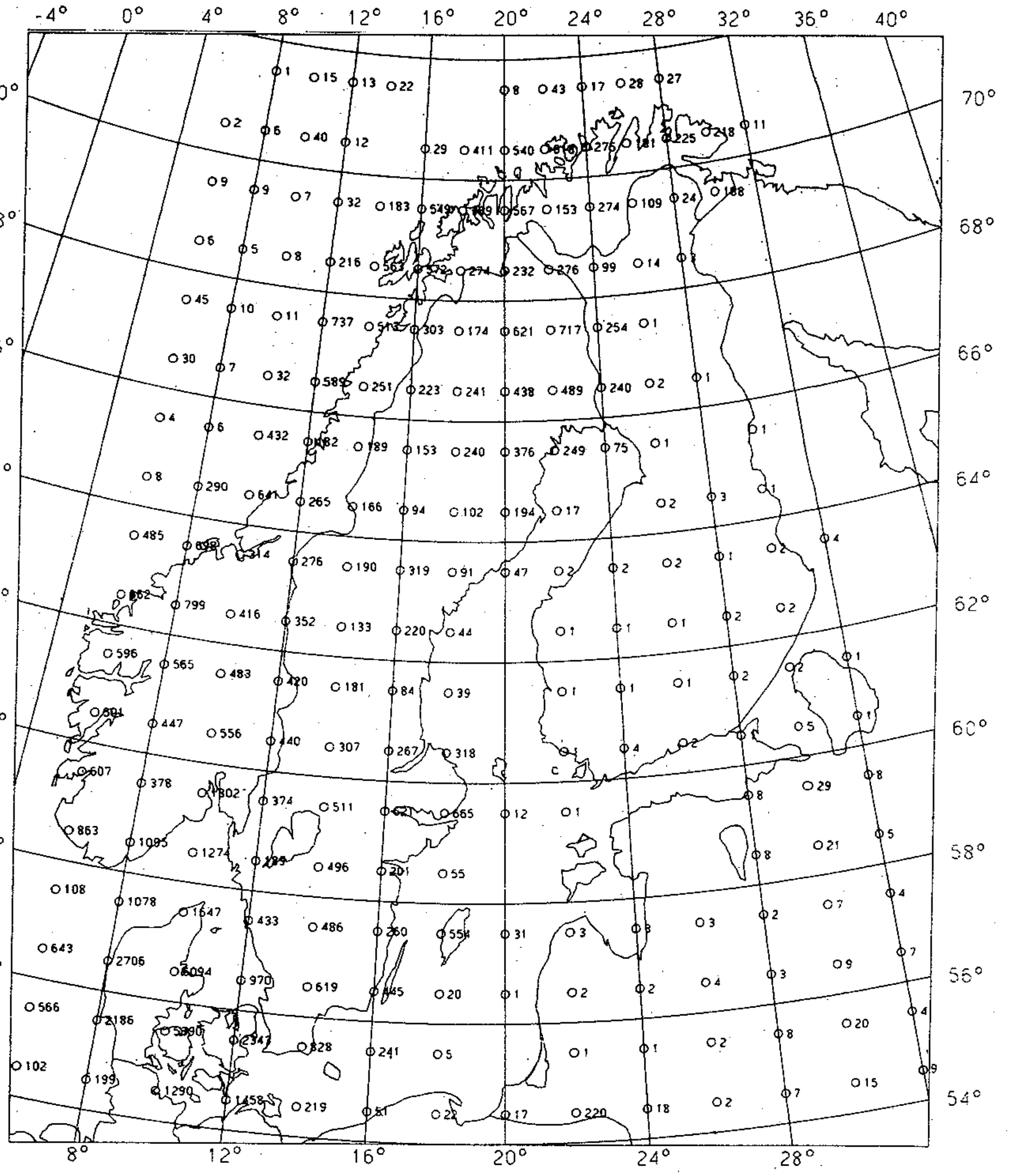

Fig. 1. Number of gravity anomalies available per 1° x 2° square, summer 1982

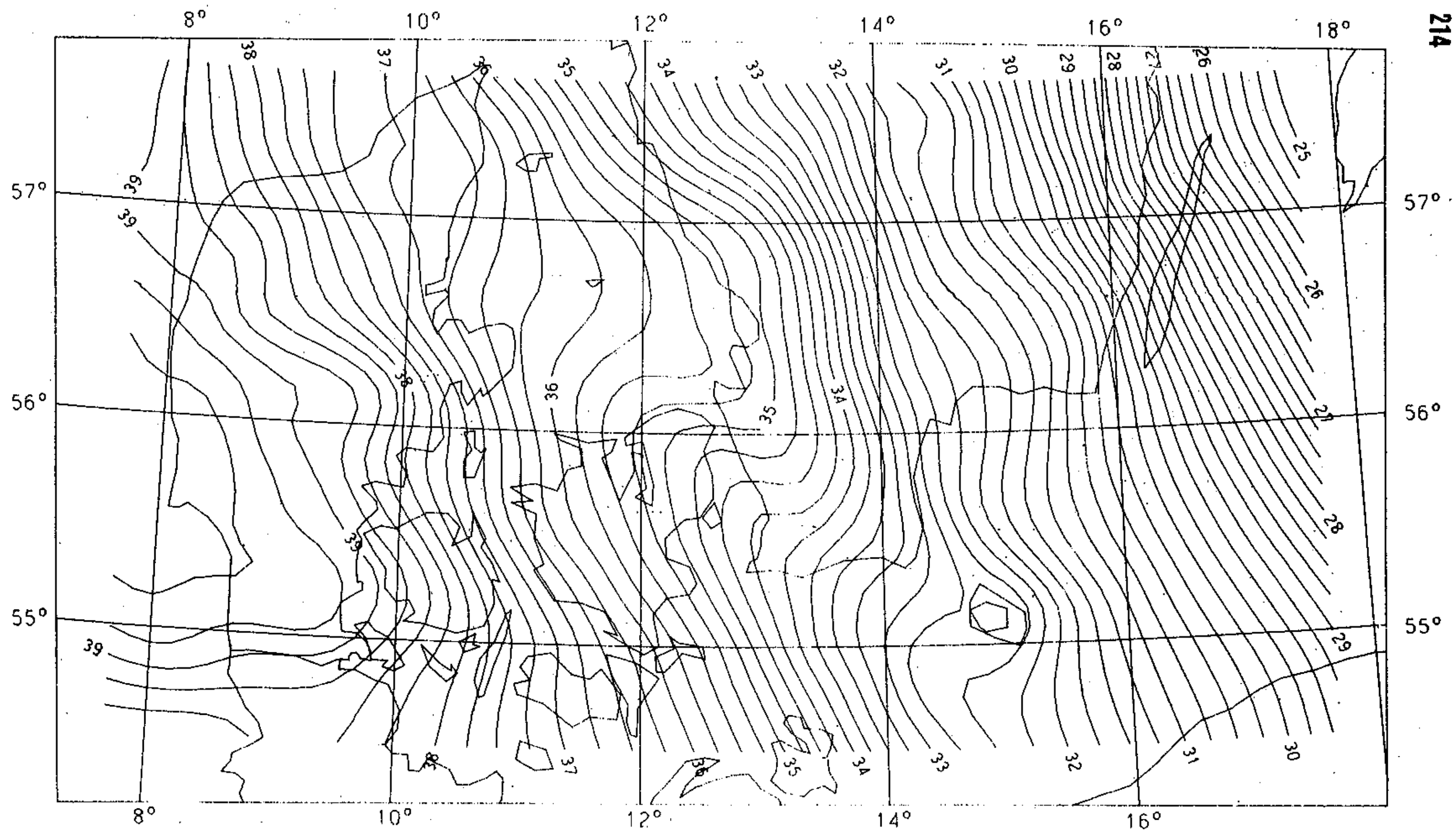

Fig. 2. Astrogravimetric height anomalies evaluated on the ellipsoid. GRS 1980 used in this and all the following figures. Contour interval 0.25 m. x local minimum. Scale: 1.3 000 000.

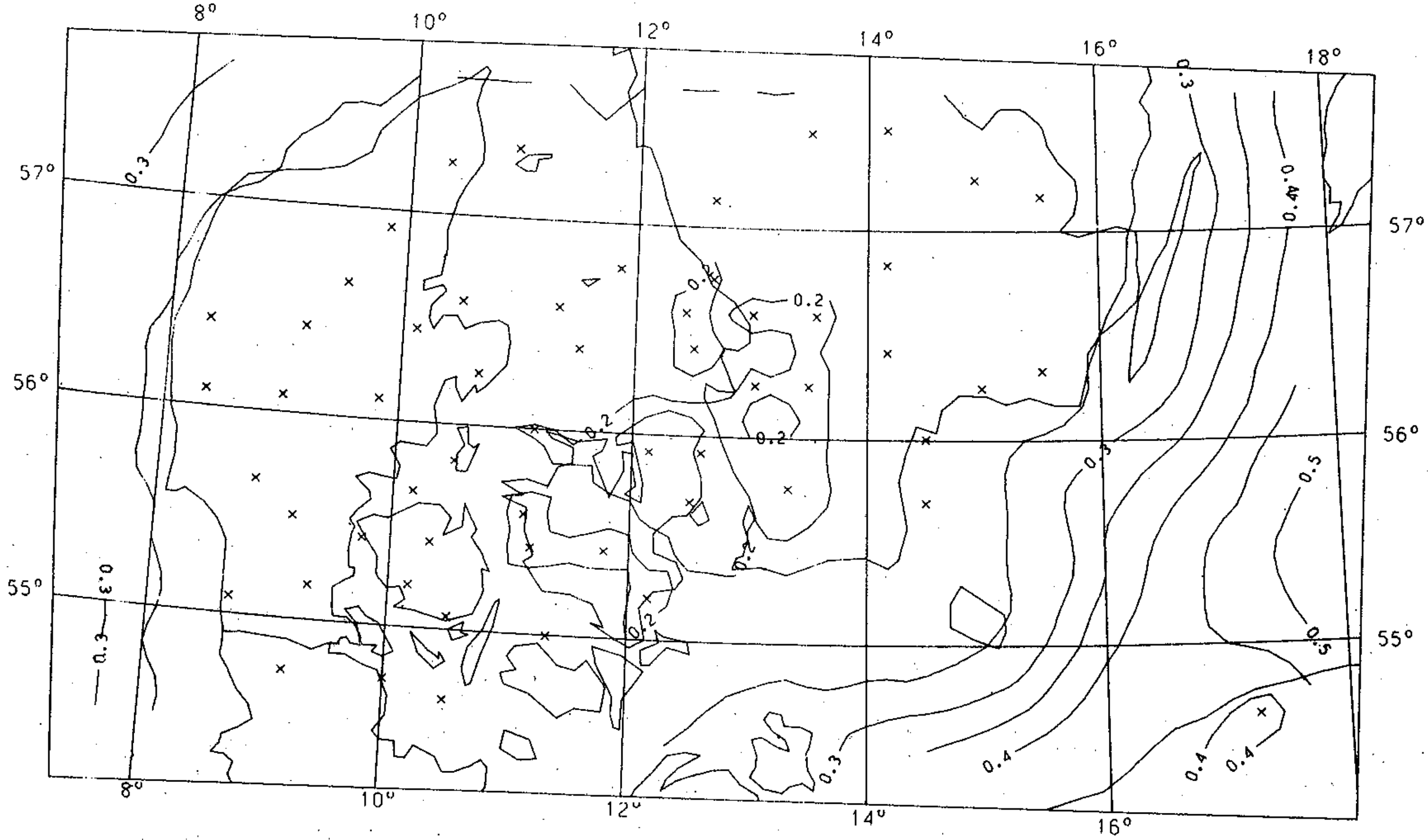

Fig. 3. Error estimates of the height anomalies. Contour interval 0.05 m.
Note, that the EDOC-II station in Buddinge ($\varphi = 55^{o}44'$, $\lambda = 12^{o}30'$) has been used with $\sigma = \pm\ 0.3$ m.

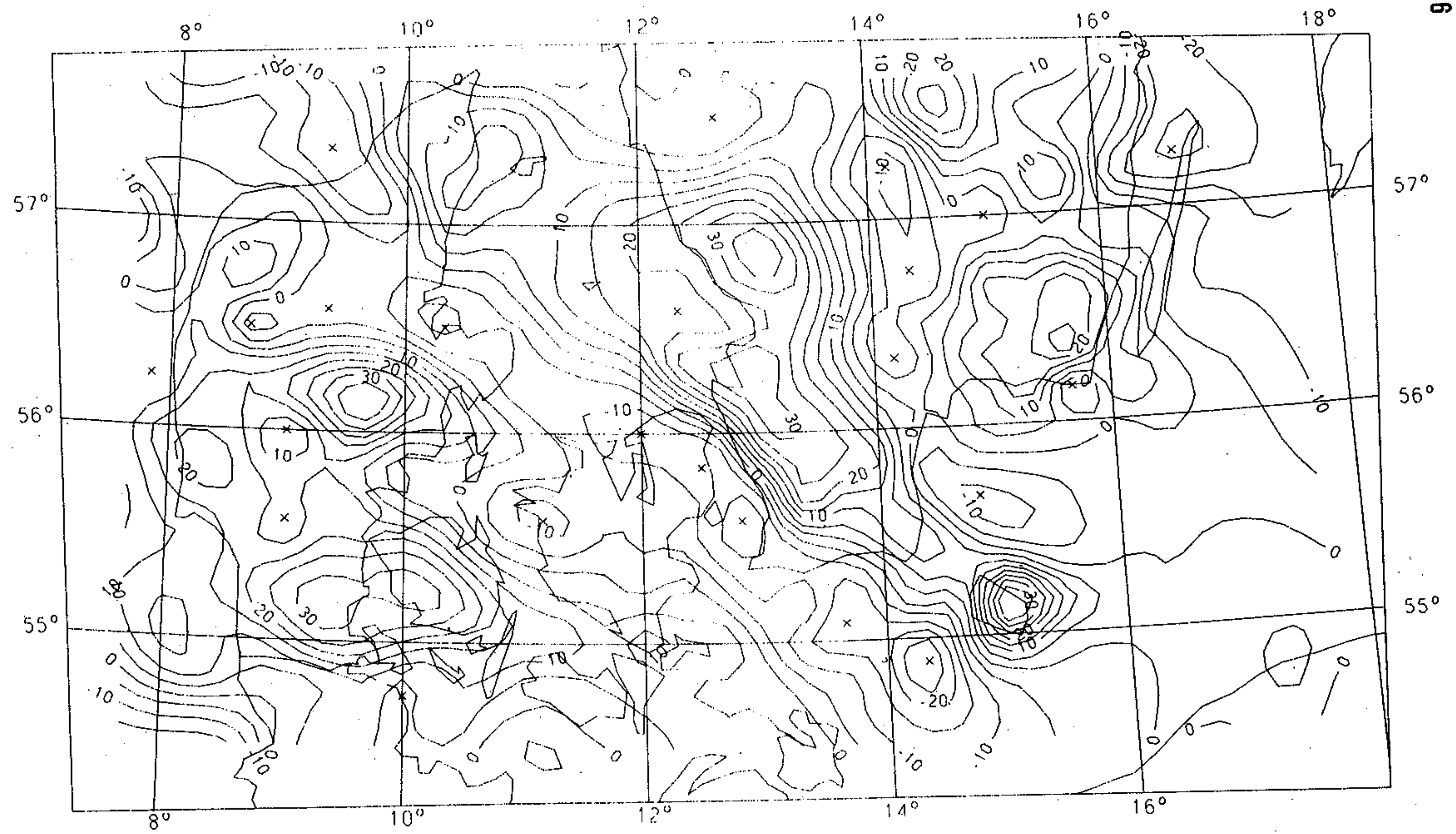

Fig. 4. Free-air gravity anomalies in GRS 1980. Contour interval 5 mgal.

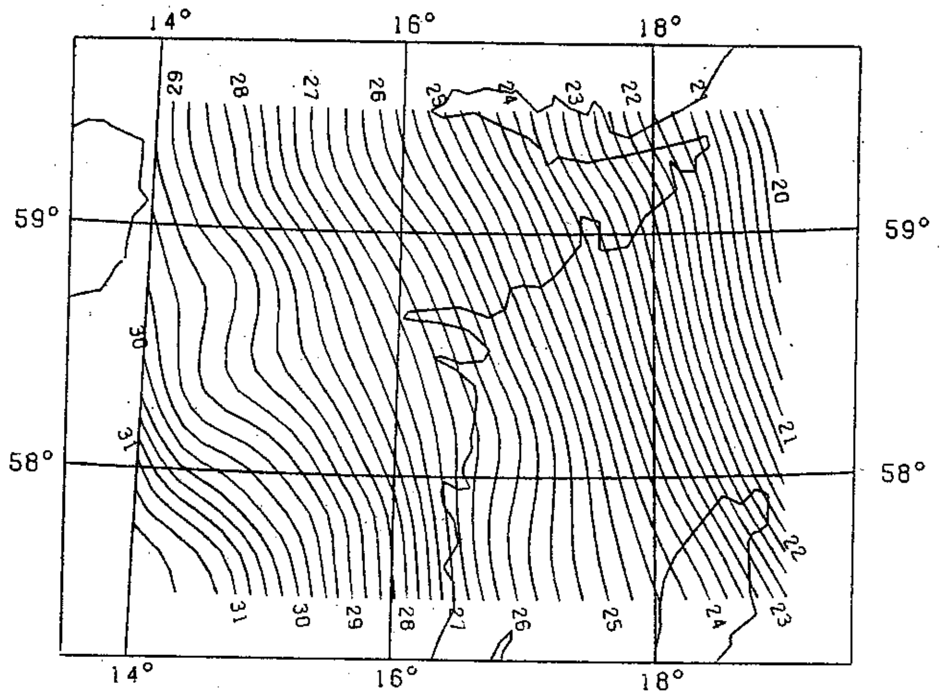

Fig. 5.
Height anomalies evaluated on the ellipsoid. Contour interval 0.25m.

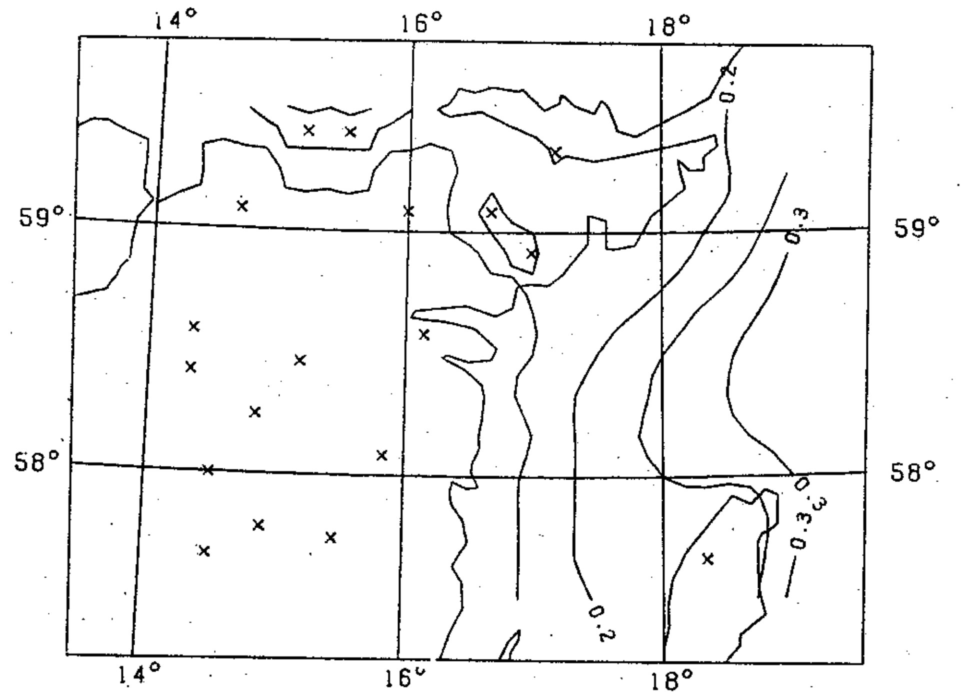

Fig. 6.
Error estimates of the height anomalies. Contour interval 0.05m.

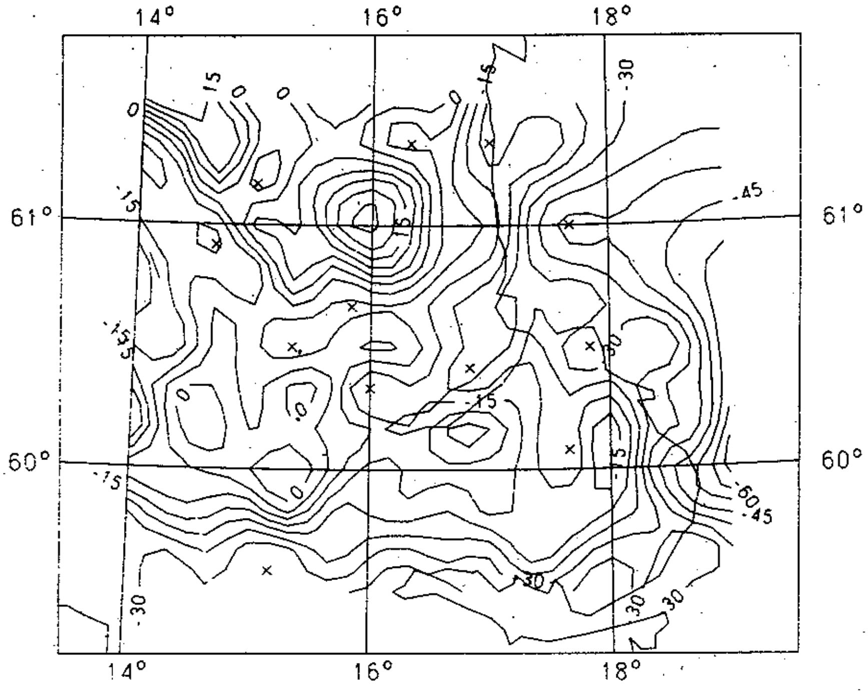

Fig. 7. Free-air gravity anomalies. Contour interval 5 mgal.

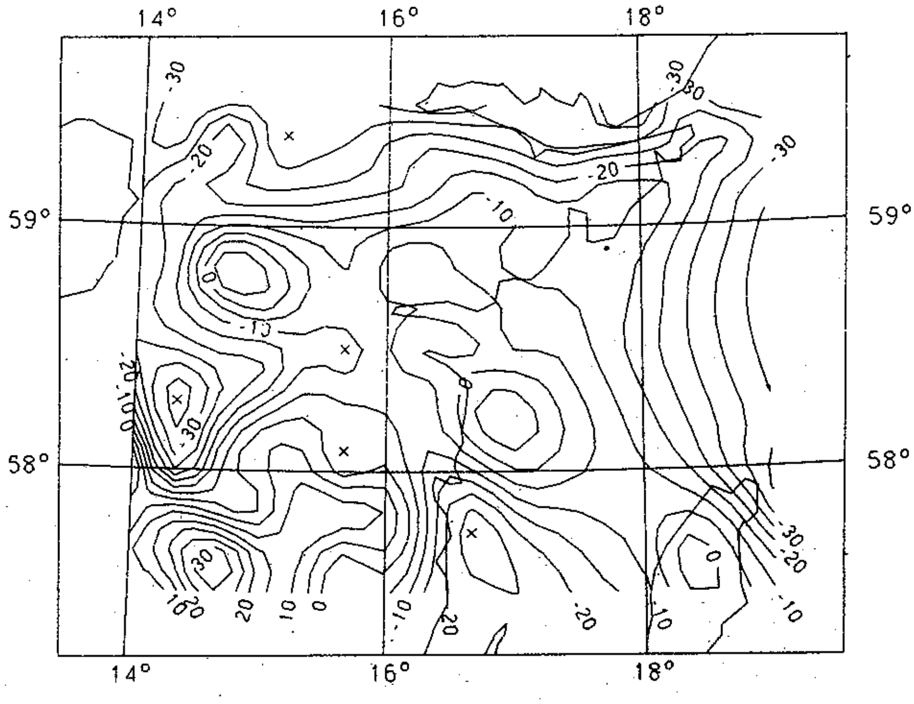

Fig. 8. Free-air gravity anomalies. Contour interval 5 mgal.

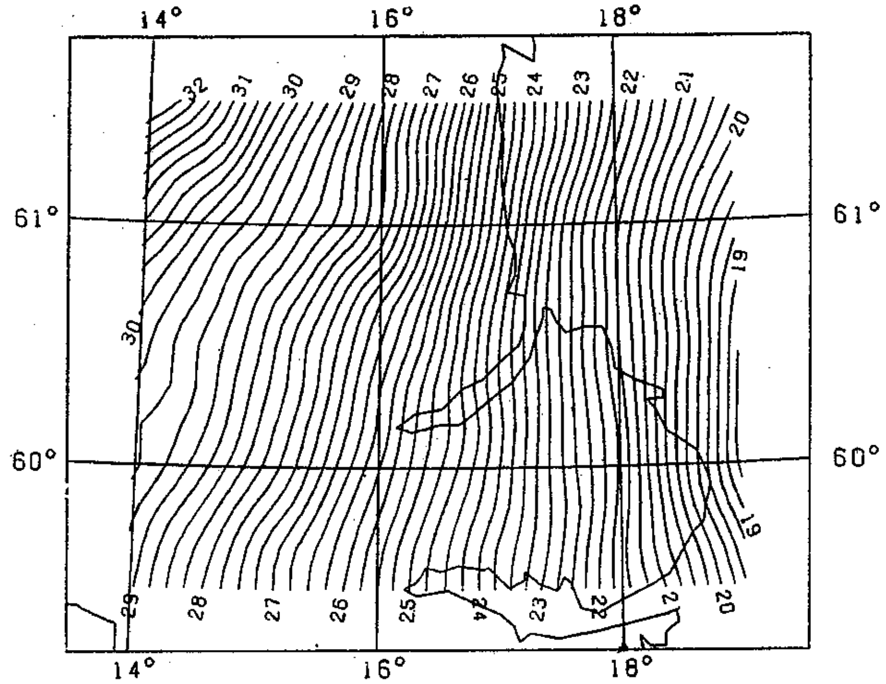

Fig. 9.
Height anomalies evaluated on the ellipsoid. Contour interval 0.25 m.

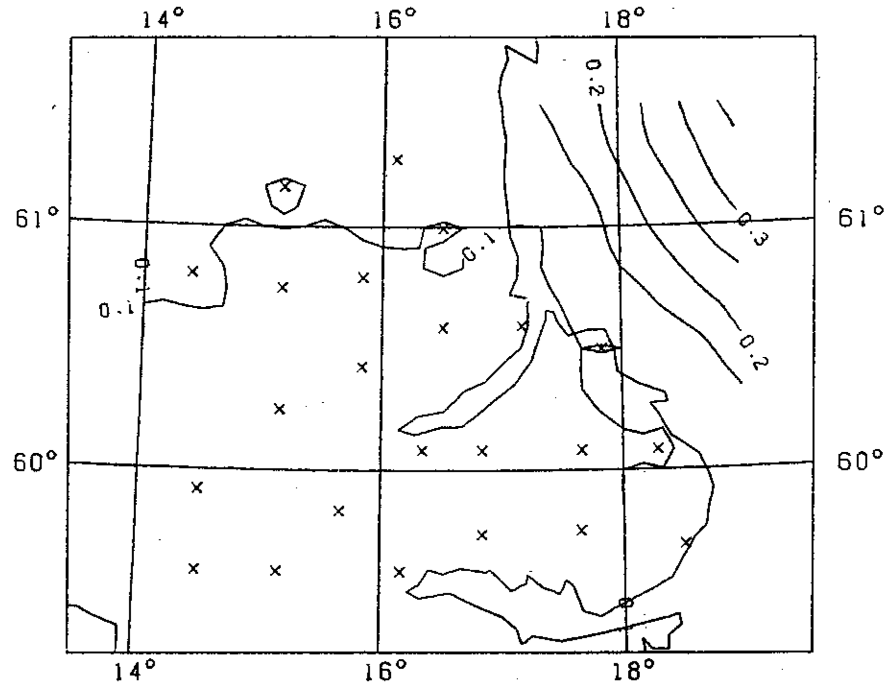

Fig. 10.
Error estimates of the height anomalies. Contour interval 0.05 m.

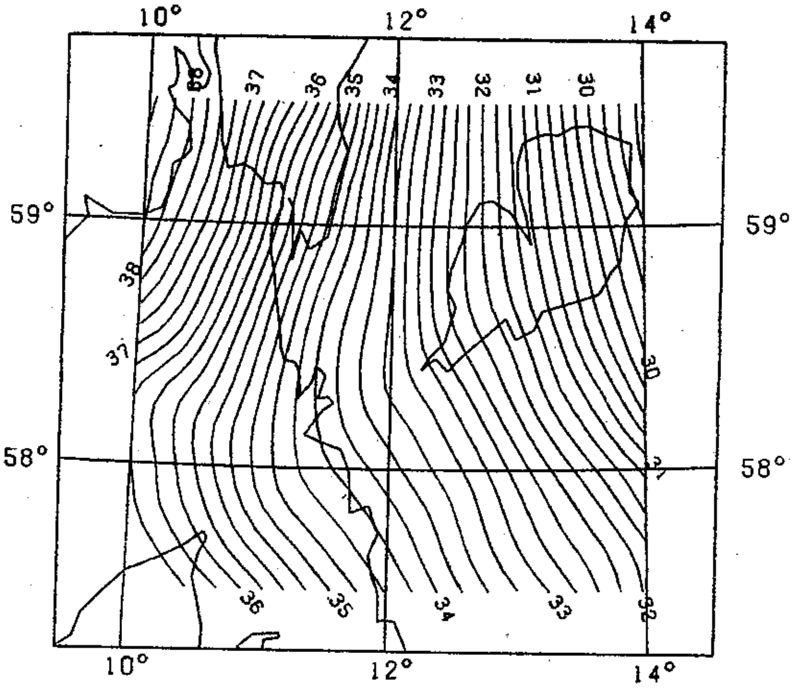

Fig. 11. Height anomalies evaluated on the ellipsoid. Contour interval 0.25 m.

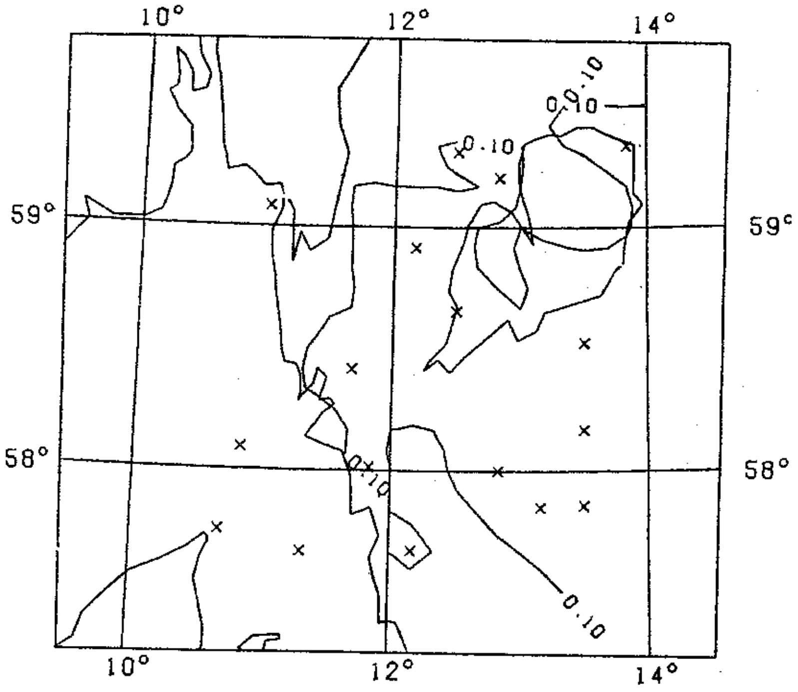

Fig. 12. Error estimates of the height anomalies. Contour interval 0.05 m.

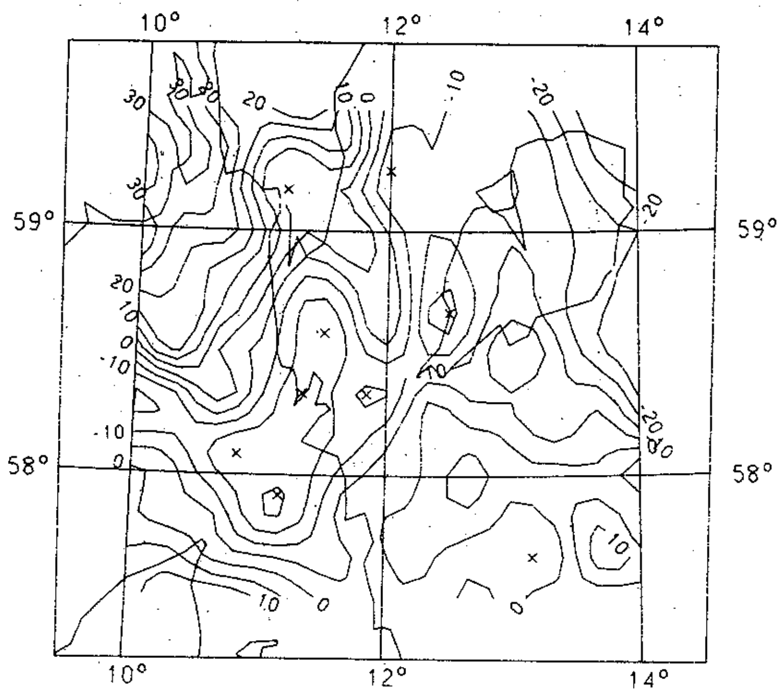

Fig. 13. Free-air gravity anomalies. Contour interval 5 mgal.