The 1-cm geoid after GOCE.

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Abstract: The new satellite gravity missions (CHAMP, GRACE and GOCE) will all bring substantial improvements to our knowledge of the gravity field and thereby of the (quasi-) geoid. One of the aims of the Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) is to determine the geoid to within 1 cm at wavelengths down to 100 km.

When determining local or regional geoids, a large part of the error is due to the error in the spherical harmonic expansion used as a reference field in remove-restore calculations. We have estimated this error for the EGM96 and for the future GOCE based model using the so-called error-degree variances associated with these models. These variances have for GOCE been determined in studies analysing the mission performance.

The error-degree variances have been used in a Monte-Carlo like generation of pertubations of the EGM96 and the GOCE gravity field model in various scenarios (moderate topography, medium varying topography and Alpine topography) by scaling the error-degree variances. The error of a derived quantity like a geoid height differences have then been estimated by calculating the root-mean square variation of the results.

Based on typically 50 generated models (for EGM96 and GOCE) we have estimated mean errors of geoid height differences between the continents as well as for typical levelling lines on the continents. For geoid height differences between the continents the EGM96 error is found to be between 0.45 m and 0.60 m, while the corresponding error when using GOCE decreased to between 0.05 and 0.10 m. For levelling lines of distances between 25 km and 500 km a similar improvement was found, so that maximal errors decreased from 0.88 m to 0.09 m in Alpine topography and from 0.44 m to 0.03 m in Scandinavian type topography. Obviously the use of local gravity anomalies, gravity disturbances, GPS/levelling geoid height differences as well as topographic information may further improve the results. Consequently 1-cm geoid height differences should easily be obtained in areas where these types of data are available.

1. Introduction.

The new satellite gravity missions (CHAMP, GRACE and GOCE) will all bring substantial improvements to our knowledge of the gravity field and thereby of the (quasi-) geoid. One of the aims of the Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) is to determine the geoid to within 1 cm at wavelengths down to 100 km, see ESA (1999).

In the preparation of arguments for selecting GOCE in between totally 4 proposed ESA Explorer missions a number of studies were made. One of the studies (Tscherning et. al, 1999) aims at estimating how much we would be able to reduce the error in gravity field quantities such as height anomalies and gravity anomalies if GOCE and not EGM96 (Lemoine et al., 1996) was available. Meanwhile the GOCE mission has also been approved, so we should now prepare for the utilisation of the results of all the space missions. GOCE is, due to its orbit and measurement type, especially well suited for supporting detailed geoid computations. All the space missions together will furthermore improve very much the global gravity field, so that we will be able to improve considerably the connection between the continental levelling datums.

Various methods may be used for geoid determination, but the presently used methods all use a remove-restore technique where a spherical harmonic model is used to represent information with wavelengths longer than the area considered. The error in the spherical harmonic model is thereby one of the major errors in geoid determination.

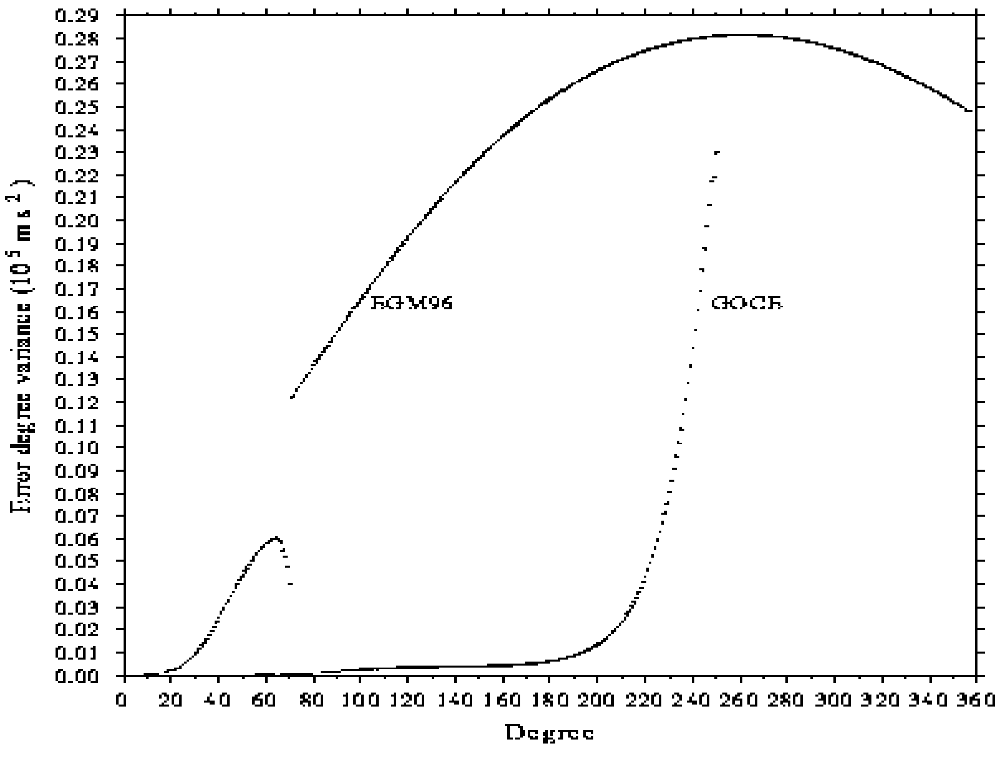
In the following we will present the results of simulations made in order to estimate how much the error in local geoids and levelling datums (tide gauges) connections may be reduced due to the improvements expected from GOCE as compared to the at present generally used EGM96.

2. The expected quality of the GOCE mission gravity field.

The GOCE satellite will carry a gradiometer and a GPS/GLONAS receiver, which will be used for precise orbit determination and high-low satellite to satellite tracking. The latter type of measurements will assure the quality of the low-degree harmonics, which may be further improved if data from other missions are used. The determination of higher degree harmonics is assured due to the low altitude (250 km) and the high precision of the gradiometer when used in space, (0.005 Eötvös Units).

Simulations of the errors of the coefficients have been made, which have been expressed as error-degree variances, see Figure 1.

Fig. 1. Error-degree variances of EGM96 and of GOCE (P.Visser, personal communication).



The root mean square error of a quantity derived from EGM96 or from GOCE may be studied by propagating the error using the full covariance matrix. In this case error-correlations may be taken into account. However the full error-covariance matrix is only available for EGM96 and only up to the degree 72. We therefore used another method to express the errors. Potential fields (typically 50 fields) with coefficients drawn from normal distributions with zero mean and the error-degree variance as its variance were generated. The error in geoid determination and datum connections due to errors in the reference could then be determined by calculating the root mean square variation of the result.

The results do however very much depend on the absolute variation of the gravity field in the area where the calculation is to be made. When the field is very smooth the errors will be smaller than the global mean, and correspondingly if the field has a variation larger than the global mean variation the errors will be larger. We have taken this into account by scaling the error degree-variances with a factor equal to 1 at degree zero and equal to the ratio between the local and the global mean variation at the highest degree.

3. Datum connections between continents.

We have simulated the transfer of the datum from Amsterdam (longitude = 4.9°, latitude = 52.4°, gravity standard deviation 3.4 mgal) to other places of the world, see Table 1.

From			Local std	ΔN	EGM96	GOCE	GOCE
Amsterdam	Long.	Lati.	(mgal)	(m)			Scaled
Greenland	-37.7	65-6	18.9	5.7	0.53	0.07	0.09
New York	-74-0	40.7	19.1	-78.5	0.58	0.06	0.07
Peiraias	23.6	37.9	19.0	-5.3	0.55	0.06	0.07
Osaka	135.4	34.6	24.7	-6.8	0.57	0.05	0.07
Los Angeles	-118.5	34.0	41.3	-78.7	0.57	0.06	0.09
Hong Kong	114.4	22.4	9.1	-45.29	0.56	0.06	0.06
Madras	80.3	13.2	19.3	-134.3	0.47	0.06	0.07
Rio de Jane.	-43.2	-22.9	20.1	-48.5	0.47	0.06	0.07
Cape Town	18.4	-33.9	14.4	-12.8	0.61	0.06	0.07
Melbourne	144.0	-37.8	11.8	-39.2	0.55	0.06	0.07

Table 1. Simulated height difference errors for the height difference between Amsterdam and the selected 10 locations. Column 4 contains the local gravity variation (used to scale the coefficient variances). Column 5 the approximate height difference and in column 6, 7, 8 the results for EGM96 and GOCE with and without scale-factor.

We see in all cases - also when the local gravity variation is large, that GOCE gives an improvement of the order 5 - 6. Thus we believe that GOCE will make a solid foundation for the establishment of a global vertical datum.

4. Geoid height improvement.

We have here studied 2 areas. One with low and medium gravity field variation: South Scandinavia and one with high gravity field variation: the high Alps. In these two areas we have studied geoid height differences over short (25 km), medium (100 km) and long (300/500 km) distances. The points were either on the same parallel or the same meridian. The results are summarized in Table 1 for South Scandinavia and in Table 2 for the Alps.

Same	Distance:	25 km	Distance:	100 km	Distance:	500 km
Parallel	GOCE	EGM96	GOCE	EGM96	GOCE	EGM96
Mean	0.00	-0.01	-0.00	-0.03	0.00	-0.05
Std.var.	0.02	0.19	0.03	0.38	0.03	0.44
Same	Distance:	25 km	Distance:	100 km	Distance:	500 km
meridian	GOCE	EGM96	GOCE	EGM96	GOCE	EGM96
Mean	0.00	0.03	0.01	0.11	0.02	0.12
Std.var.	0.02	0.08	0.09	0.40	0.07	0.18

Table 2. Geoid height difference simulation results for South Scandinavia (in meters).

Same	Distance:	25 km	Distance:	100 km	Distance:	225 km
Parallel	GOCE	EGM96	GOCE	EGM96	GOCE	EGM96
Mean	-0.02	0.02	-0.04	-0.04	-0.07	-0.07
Std.var.	0.06	0.47	0.09	0.88	0.07	0.67
Same	Distance:	25 km	Distance:	100 km	Distance:	225 km
meridian	GOCE	EGM96	GOCE	EGM96	GOCE	EGM96
Mean	-0.03	-0.03	0.03	0.17	0.02	0.11
Std.var.	0.02	0.11	0.07	0.37	0.06	0.46

Table 3. Geoid height difference simulation results for the high Alps (in meters).

In the South Scandinavia GOCE will give results which are about 10 times better than those which could be obtained from EGM96. For the high Alps, the improvement is a factor between 5 and 10.

5. Conclusion.

The main errors in local and regional geoid determinations are caused by errors in the spherical harmonic field used as a reference in the generally used remove-restore techniques. The new satellite missions will reduce these errors 5 to 10 times and thereby establish a solid foundation for the calculation of local and regional geoids with cm errors and will enable the establishment of a global height datum satisfactory for many purposes. Obviously the realisation of the cm-geoid will only be possible if local gravity anomalies, gravity disturbances, digital topographic models and GPS/levelling height differences are used also.

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