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Local geoid determination combining gravity disturbances and GPS/levelling: a case study in the Lake Nasser area, Aswan, Egypt

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Abstract. The use of GPS for height control in an area with existing levelling data requires the determination of a local geoid and the bias between the local levelling datum and the one implicitly defined when computing the local geoid. If only scarse gravity data are available, the heights of new data may be collected rapidly by determining the ellipsoidal height by GPS and not using orthometric heights. Hence the geoid determination has to be based on gravity disturbances contingently combined with gravity anomalies. Furthermore, existing GPS/levelling data may also be used in the geoid determination if a suitable general gravity field modelling method (such as least-squares collocation, LSC) is applied. A comparison has been made in the Aswan Dam area between geoids determined using fast Fourier transform (FFT) with gravity disturbances exclusively and LSC using only the gravity disturbances and the disturbances combined with GPS/levelling data. The EGM96 spherical harmonic model was in all cases used in a remove–restore mode. A total of 198 gravity disturbances spaced approximately 3 km apart were used, as well as 35 GPS/levelling points in the vicinity and on the Aswan Dam. No data on the Nasser Lake were available. This gave difficulties when using FFT, which requires the use of gridded data. When using exclusively the gravity disturbances, the agreement between the GPS/levelling data were 0.71 ± 0.17 m for FFT and 0.63 ± 0.15 for LSC. When combining gravity disturbances and GPS/levelling, the LSC error estimate was ±0.10 m. In the latter case two bias parameters had to be introduced to account for a possible levelling datum difference between the levelling on the dam and that on the adjacent roads.

Keywords: Gravity Disturbances – Geoid – GPS – Levelling

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1 Introduction

The use of GPS for the determination of ellipsoidal heights in areas with levelling control requires the determination of the relationship between the old orthometric heights and the ellipsoidal heights. The problem may be solved in a simple manner by determining ellipsoidal heights in all levelling points. However, if new height control is to be established it may not be possible to extrapolate the GPS/levelling differences. In this case gravimetric information may be used to bridge the gap through the determination of a local geoid. However, the local geoid will have its own zero level, and the levelling datum may be local or national. A global levelling datum is something for the future.

Hence, besides determining the geoid, we must determine the bias with respect to the levelling data. Again this may be done in a simple manner by calculating the mean difference between the GPS/levelling differences and the geoid heights. However, these values may have an uneven spatial distribution, and we need to balance the values against each other taking into consideration the spatial correlation.

Many methods are available for local geoid determination (see e.g. International Geoid Service 1997) but we will here consider only two, both of which both are implemented in the GRAVSOFT software package (see Tscherning et al. 1994): the method of least-squares collocation (LSC) and the Fourier transform-based method, generally called FFT.

The data used for geoid determination are normally gravity anomalies, since the heights associated with old gravity data are exclusively orthometric heights. Levelling is, however, very tedious and may be substituted by ellipsoidal heights determined by GPS. Hence, when using GPS for height determination of gravimetric observation sites we obtain gravity disturbances. We are then in this case faced with the task of combining gravity disturbances, GPS/levelling data and contingently older gravity anomaly data.

In the following we describe how we have handled this problem using data from the Aswan Dam area in Egypt (see Fig. 1). This area has the problem that the gravity data are not regularly distributed: there are no data on the Lake Nasser (see Fig. 2). The data will be described in somewhat more detail in Sect. 2.

LSC is a very flexible method, which may combine many data types in a consistent manner. Its use is described in Sect. 3, and in Sect. 4 we describe the FFT solution. In Sect. 5 we discuss the results of comparisons of the methods.

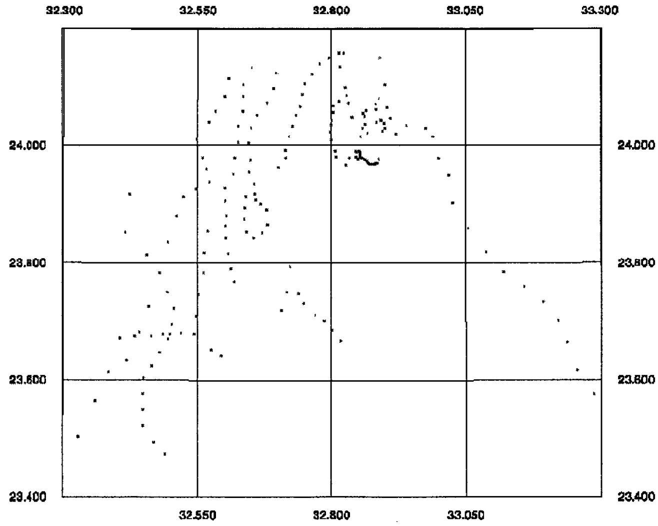

Fig. 2. Gravity points

2 Gravity and GPS/levelling data

In principle geoid determination is a task which requires a global data distribution. However, when working in a local area we may represent the 'surrounding' gravity field using a high degree and order spherical harmonic model such as EGM96 (Lemoine et al. 1996). Its contribution must then be subtracted from the local data and subsequently restored. However, it causes the problem that any geoid determined on the basis of local data will have a bias.

In Egypt there are limited freely available gravity data. We had Bouguer gravity maps available covering the Lake Nasser area, but unfortunately we only had a very crude digital terrain model (ETOPO5) available, which made it rather impossible to convert the Bouger anomalies into free-air anomalies, so that they were compatible with the EGM96 model. It was therefore decided to collect new gravity values in a $1° \times 1°$ area surrounding the dam, and determine the associated heights using GPS. The distribution of a subset of the gravity data, corresponding to a subset with

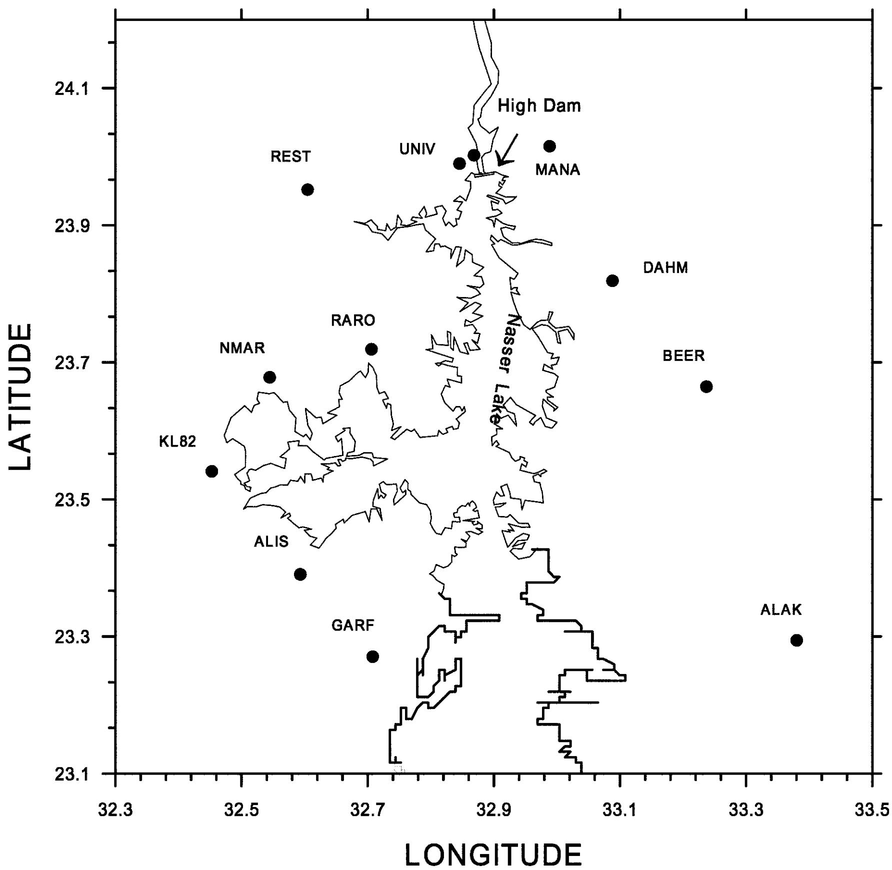

Fig. 1. Lake Nasser area

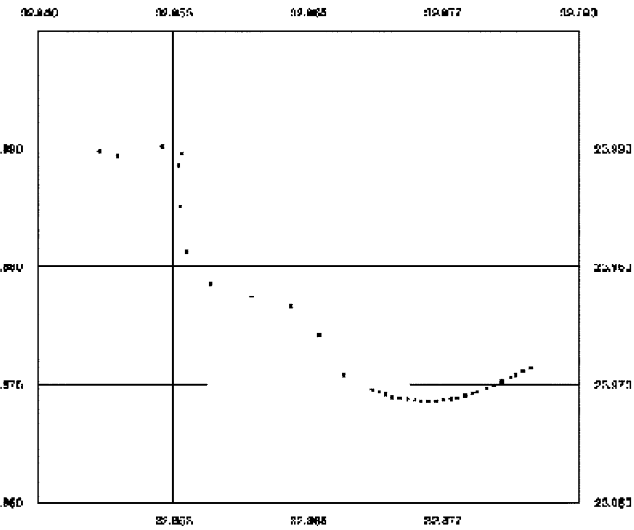

Fig. 3. GPS/levelling points

0.05-degree spacing, is shown in Fig. 2. Note that large gaps are present, primarily at the Lake Nasser.

Limited levelling information is available, primarly control levelling on the Aswan Dam (see Fig. 3). For these points ellipsoidal heights were determined using GPS.

The use of GPS for ellipsoidal height determination is in our case problematic. No national reference station data are available. The closest International GPS Service (IGS) station is 757 km away in Israel. Fortunately data from this station could be downloaded, together with precise orbits. The program G-P Survey was used for the data processing, despite the fact that it should not be used for such long baselines. One base station was positioned using sessions of 8–12 hours, which has a formal standard deviation of 0.03 m. Local levelling points were then positioned using 30-minute sessions differentially with respect to the base station. The height differences between stations having distances of between 5 and 60 km were found to be of the order of 0.02 m. The GPS data were processed by one of the authors in Denmark, i.e. it was not possible to re-observe any of the stations. (This situation is not unusual for surveying projects in polar areas and in developing countries.)

The contribution of EGM96 (complete to degree 360) was then subtracted from the data. Statistics of the differences are shown in Tables 1 and 2. Note the remarkable agreement between EGM96 and the local GPS/levelling data and the large disagreement in the gravity disturbance data.

Figure 4 shows the gravity disturbances after having subtracted the EGM96 contribution.

3 LSC solution

The LSC solution is obtained in the following form (using the differences with respect to EGM96):

Table 1. Statistics of 198 gravity disturbances. Unit: mGal

	Mean	Standard deviation
Original	−0.9	5.7
EGM96	3.4	6.2
Difference	−4.4	9.1

Table 2. Statistics of 35 GPS/levelling heights. Unit: m

	Mean	Standard deviation
Original	10.74	0.15
EGM96	10.08	0.02
Difference	0.66	0.16

$$T(P) = \sum_{i=1}^{N} b_i \cdot \mathrm{cov}(T(P), L_i) \qquad (1)$$

$$\{b_i\} = \{\mathrm{cov}(L_i, L_j) + \sigma_{ij}\}^{-1} \cdot \{x_j\} = \bar{C}^{-1} \cdot x \qquad (2)$$

where T is the local approximation to the anomalous potential, x_i are the observations and σ_{ij} are the error covariances. The covariance is represented by the following expression in which the constants R (the radius of the Bjerhammar sphere), c and c' are determined from the local gravity disturbance data:

$$\mathrm{cov}(P,Q) = \mathrm{cov}(r,r',\psi) = \sum_{k=2}^{K} c \cdot \sigma^2_{\mathrm{EGM}} \left(\frac{R^2}{rr'}\right)^{k+1} P_k(\cos\psi) + \sum_{k=K+1}^{\infty} \frac{c'}{(k-1)(k-2)(k+4)} \left(\frac{R^2}{rr'}\right)^{k+1} P_k(\cos\psi) \qquad (3)$$

P and Q are two points with spherical distance ψ; r and r' are the distances of the two points from the origin; and $P_k(\cos\psi)$ are the Legendre polynomials.

Initially an empirical covariance function was determined from the gravity disturbances. The mean value was not subtracted from the data. Then the estimated values were fitted to the model in a non-linear iterative adjustment with the three parameters using the program COVFIT (Knudsen 1987). This gave the values $c' = 1231893$ $(\mathrm{m}^2\,\mathrm{s}^{-2})^2$, $c = 0.312$ and $R = 6361$ km. The summation limit K was set equal to 190, indicating that the coefficients of degree and order larger than 190 used when subtracting the EGM96 model may not give reliable information in the area. Figure 5 shows the empirical and the fitted covariance functions.

When data depend on parameters, such as the difference between the datum of the geoid (rather arbitrary) and the local levelling datum(s), these parameters may be determined using LSC. The observations x are now related to T and the parameter vector X through the following equation:

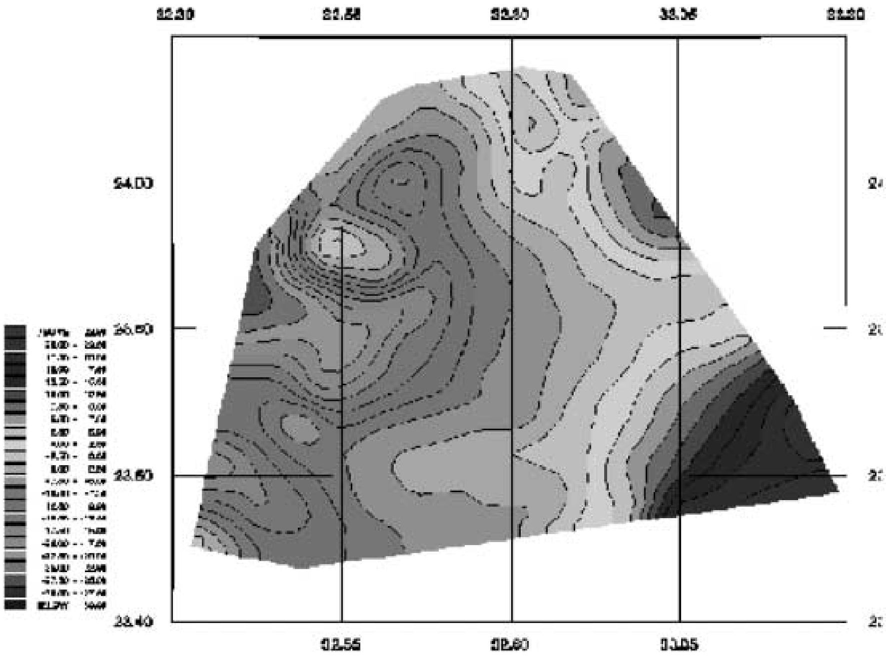

Fig. 4. Gravity disturbances EGM96. Contour interval 2.5 mGal

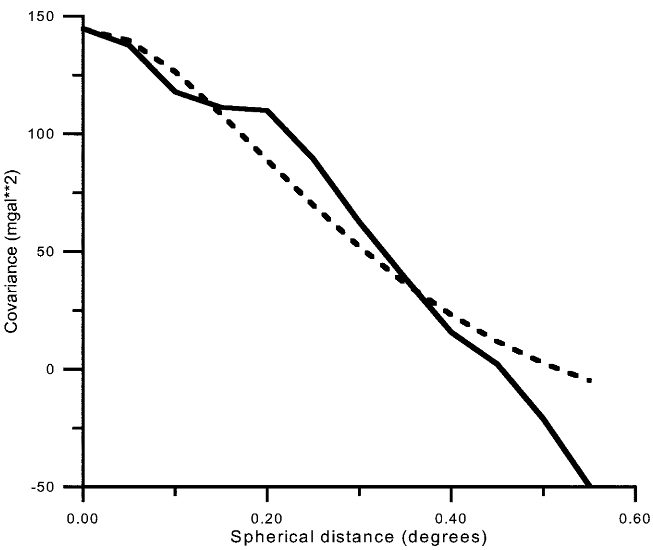

Fig. 5. Empirical (full line) and model (dashed line) auto-covariance function of residual gravity disturbances

$$x_k = L_k(T) + A_k\,X + \varepsilon_k \tag{4}$$

where L_k is the functional associated with the observation, A_k is a vector with elements 0 or 1, X is the parameter vector (here later of dimension 2) and ε_k is the observation error. Then

$$X = (A^T \bar{C}^{-1} A)^{-1} \cdot A^T \bar{C}^{-1} x \tag{5}$$

Derivation of these equations as well as expressions for the estimation errors can be found in Moritz (1980).

The 198 (residual) gravity disturbances were then used to determine an estimate of T, from which estimates of the geoid height in the GPS/levelling points were obtained. The results are given in Table 3. When inspecting the residuals, it became clear that the differences between observed and predicted values fell into two groups: the data on the dam and the data collected on the adjacent roads. These differences are also shown in Table 3.

Table 3. Comparison of observed and predicted GPS/levelling heights predicted from 198 gravity disturbances. Unit: m

		Mean	Standard deviation
All data	Original	10.74	0.15
	Predicted	10.11	0.01
	Difference	0.63	0.15
Dam	Original	10.83	0.09
	Predicted	10.82	0.00
	Difference	0.72	0.09
Road	Original	10.57	0.05
	Predicted	10.12	0.01
	Difference	0.45	0.05

The GPS/levelling data were then added to the data to be used for the geoid determination. However, each set was associated with one unknown parameter. The GPS/levelling data were assigned a standard error of 0.03 m with respect to the unknown bias.

The parameters were then estimated; the results are shown in Table 4.

Table 4. Results of parameter estimation for the GPS/levelling data at the Aswan Dam and the adjacent roads. Unit: m

Parameter	Value	Estimated error
Dam	0.75	0.10
Road	0.48	0.10

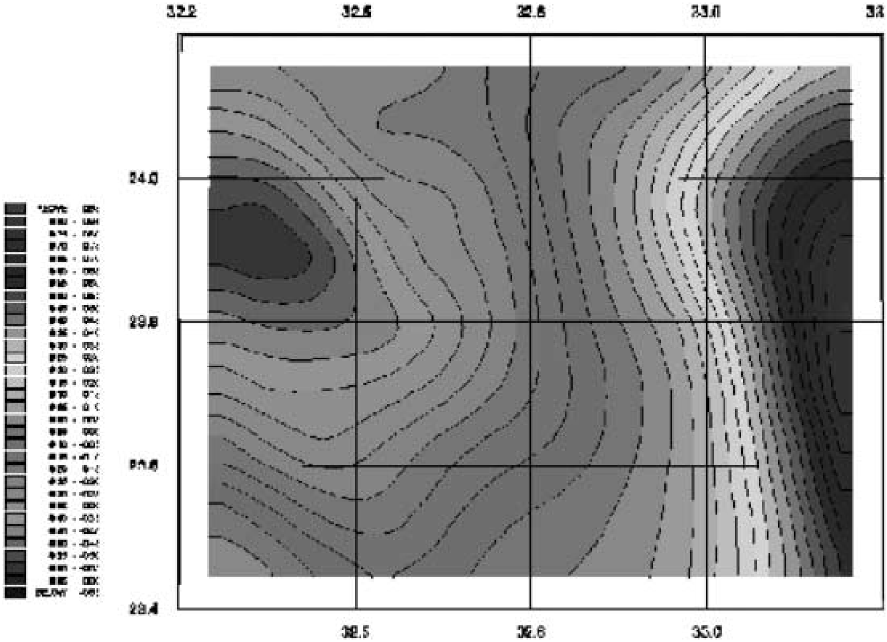

Fig. 6. Residual LSC geoid from gravity and GPS/levelling c.i. 0.05 m

The residual geoid is shown in Fig. 6. The error of the predicted geoid undulations has (cf. Table 5) now decreased slightly from 0.10 to 0.09 m in the centre of the area (see Fig. 7).

4 FFT solution

The 198 gravity disturbance values were gridded using LSC to form a grid bounded by 23.2° and 24.3° latitude and 32.1° and 33.4° longitude with a spacing of 0.05°. Using the FFT program GEOFOUR, a geoid was determined with a zero bias with respect to EGM96. Its difference with respect to the LSC geoid where gravity and GPS/levelling were used is shown in Fig. 8 and Table 6.

The best agreement between the FFT and the LSC solutions occurs in the case when only gravity disturbances have been used.

The difference with respect to the GPS/levelling data is shown in Table 7. We see that the standard deviation of the difference (Table 3, third row minus Table 7, third row) is slightly larger (0.02 m) than the one obtained using LSC.

5 Discussion and conclusion

The FFT and LSC geoids are not in good agreement. This is probably caused by the data distribution.

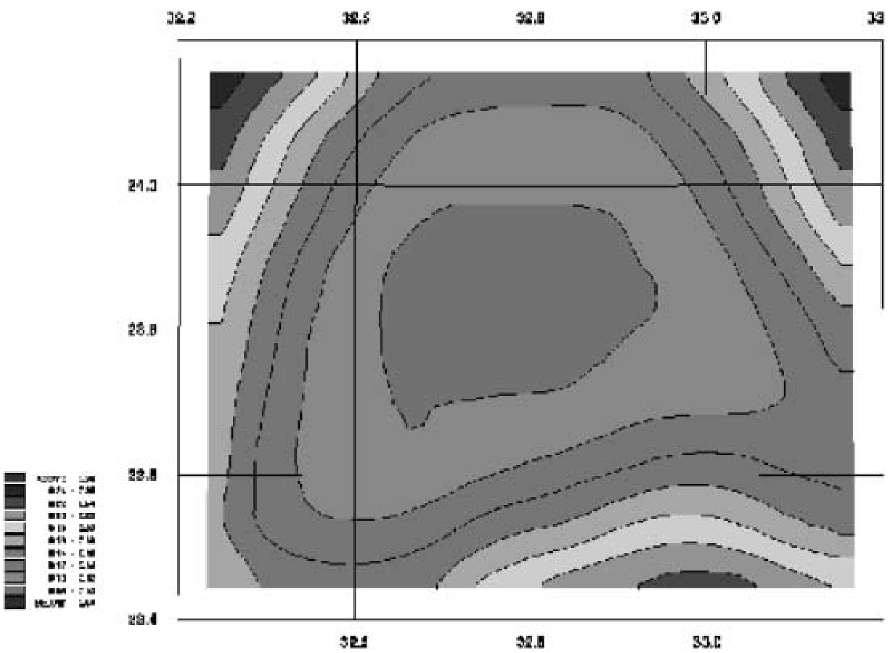

Fig. 7. LSC Geoid errors from EGM96 and gravity c.i. 0.02 m

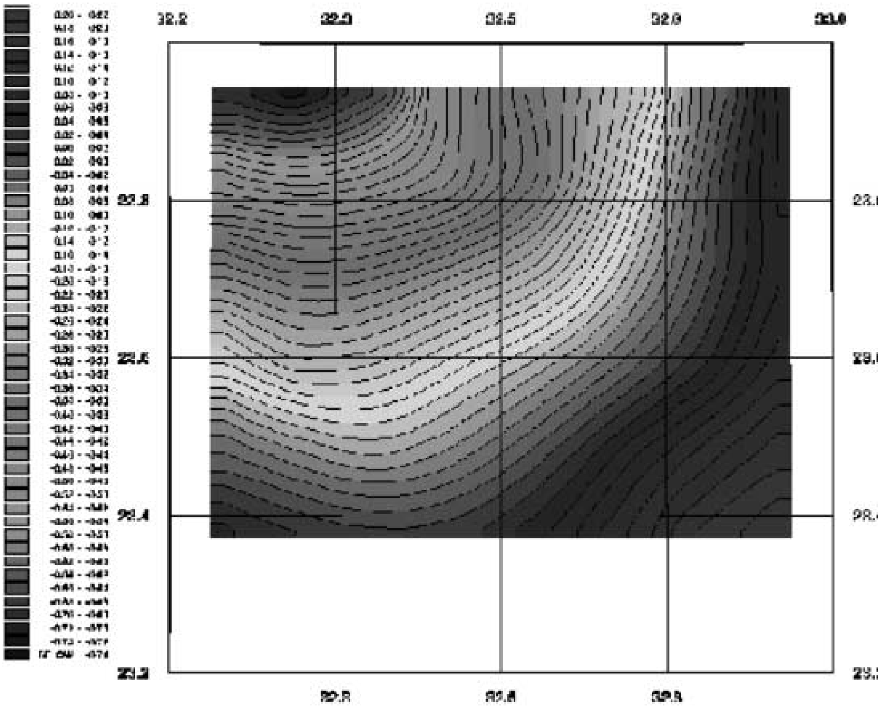

Fig. 8. Differences LSC (gravity and GPS/levelling) and FFT. c.i. 0.02 m

Table 5. Results of prediction of GPS/levelling values. Note that the data used for comparison were also used when determining the geoid. Unit: m

		Mean	Standard deviation
Dam	Original	10.83	0.09
	Predicted	10.82	0.00
	Difference	0.00	0.09
Road	Original	10.57	0.05
	Predicted	10.58	0.08
	Difference	−0.01	0.05

Table 6. Difference between FFT and LSC geoids in a grid. Unit: m

Data used	Method	Mean	Standard deviation
Gravity only	LSC	10.08	0.31
	FFT	10.09	0.52
	Difference	0.00	0.36
Gravity + GPS/levelling	LSC	10.08	0.26
	FFT	10.09	0.52
	Difference	0.00	0.40

Table 7. Difference between GPS/levelling and FFT. Unit: m

	Mean	Standard deviation
Original data (all 39 values)	10.74	0.15
FFT values	10.02	0.03
Difference	0.71	0.17

FFT is very simple to use, but its difficulties are in the estimation of a grid based on contingently very irregularly distributed data. LSC is rather more complicated, since the covariance function has to be modelled. However, the great flexibility of LSC, which permits the inclusion of the GPS/levelling, is a major benefit.

It is obvious that by adding the GPS/levelling data explicitly we have not (in this case) determined an improved geoid. However, we have in addition determined bias values for the differences between the geoids and the levelling datums. Whether there are in reality two datums is being investigated. Here we have only aimed at demonstrating that it is possible to determine the datum biases.

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