From Eötvös to mGal

Study Team 2 - Workpackage 2

Detailed scientific data processing using the space-wise approach.

Final report.

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Summary

The space-wise approach may be implemented using an integration approach or an optimal estimation approach, Least Squares Collocation (LSC). The latter method can at present only be used on limited data sets, such as data with in limited region, or on data which has been assembled to point or mean normal points. When used in limited regions, however, space and ground data may be combined.

The unknowns and stochastic parameters to be determined in the space-wise approach are identified. A stochastic model (global or regional) is needed, and a set of possible parameters to define such a model are identified.

The spherical harmonic coefficients may be estimated using both methods. However, an alternative to the usual integration method has been found, using Slepian functions, which is recommended.

When LSC is used systems of equations with no zero elements must be solved. However, recent developments using sparse preconditioners are very promising and may at the time of the launch of GOCE make it possible to use a brute-force method like LSC on very large datasets.

2.1. Introduction.

In the following we will describe the elements which must be included in a data processing scheme using the space-wise approach. This is done after first having identified the unknowns to be determined, and the stochastic parameters to be selected. Also the relationship between global and local data processing approaches are discussed.

We will exclusively discuss the data processing having either LSC or the integration approach in mind. However, several other methods implements the space-wise approach, see for example the discussion and extensive bibliography in Tscherning et al. (1990). Since then many other methods have been proposed, most important probably the wavelet methods, see e.g. Freden and Schneider (1998) for an extensive review.

2.2. Identification of unknowns and of stochastic parameters.

The following quantities are the unknowns to be determined by the space-wise approach:

Spherical harmonic coefficients (SHC), C_{ij}, up to a maximal degree (e.g. equal to 240) and their standard errors and error-correlations.

Mean gravity anomalies and geoid heights at the surface of the Earth (not at height zero !) of equiangular or equal area blocks with side-length expressed in latitude difference (e.g. 0.25 degrees).

The (mean) gravity disturbance vector components and vertical gravity gradient, T_{zz}, in pre-defined normal points or in normal areas, at a fixed altitude, e.g. equal to the mean satellite altitude. Other components like T_{xx} and T_{yy} could also be considered.

Instrument bias and drift terms in specific time intervals.

In principle these unknowns are the same as those which must be determined using the time-wise approach.

Stochastic models must be defined when using estimation methods such as Least Squares Collocation (LSC). They will be needed in regional and global stochastic models, if as suitable method for the representation of global non-isotropic functions has not been implemented successfully, (see Tscherning, 1999).

Both the global and the regional (isotropic) models may be defined using the following parameters

Depth to the Bjerhammar-sphere
Mean gravity variance at ground level
Scale-factor on the error-degree-variances of the spherical harmonic model used as reference.

The mathematical model used to represent the stochastic behaviour will, if these parameters are selected, be the so-called T/R- model, see (Tscherning & Rapp, 1974). This model is an isotropic stochastic model.

The observation equations are either related to functionals associated with a point or with a mean value. SST observations are also supposed to be of this kind, which requires that we can consider the GPS satellites as having a known orbit. The functionals are then as described in several publications: linear combinations of first and second derivatives of the anomalous gravity potential, T.

Since the original data are associated with the gravity potential, V, the precise calculation of the functionals applied on T, requires that the position of the satellite and the attitude of the SGG instrument is given. The position has to be known with a vertical position better than 1 m. (T_{zz} changes 1.44 E -3 EU pr. m at satellite altitude).

The attitude should be known better than 10", so that an error in T_{zz} does not project into errors in other gravity gradient components.

2.3. Recovery of unknowns using integration or collocation.

Suppose block mean values of gravity anomalies or disturbances or of T_{zz} covering a sphere which includes the Earth, have been calculated by a local prediction method like least-squares collocation (LSC). In this case numerical integration procedure can be used to recover the spherical harmonic coefficients. Since the mean-values will have errors (also calculated as a by-product of the local prediction process if LSC is used), the numerical integration must take this into account. This leads again to the use of LSC, contingently in a finite-dimensional subspace, exactly as used when calculating standard global models like the OSU91 or the EGM96 model.

The use of LSC for coefficient prediction has been implemented in the GRAVSOFT program GEOCOL as a part of WP3, see Tscherning (2000). The study of computation of block mean values covering a sphere has been carried out in the past years using polynomial interpolation algorithms, see Albertella and Migliaccio (1994).

2.4. Use of alternative representations.

In the last two years the representation of harmonic functions defined only by data on a sphere with polar gaps has been studied, see Albertella et al. (1999), using a set of functions orthonormal both on the sphere and on the data set. The method developed and tested uses generalisation of planar Slepian function to the spherical case. The idea can be applied for instance to derive the anomalous potential T both at satellite altitude and at ground level, from a set of satellite radial gradiometric observations, T_{zz}.

The method has been compared to a simple harmonic analysis, showing a superior capability of retrieving the unknown potential, particularly close to the polar gaps, see Albertella and Sneeuw (1999).

2.5. Simultaneous use of SGG and SST data.

In the general LSC procedure, SSG and SST observations are treated in the same way. SST observations, however, must first be converted to range rate rates, by forming differences twice and dividing by the time interval twice (see Tscherning et al. 1990, section 2) if existing software is to be used without modification. This results in correlated observations, the correlation of which must be taken into account when using LSC.

The two data types will have a high physical correlation because they will be related to the same point or segment of the satellite orbit. This may cause singularities. If this is the case only one data-type should be used. Simulations should be made to see which type of data is the most advantageous to be used.

2.6. Normal-point values or normal mean values as the basic observation ?

When forming "normal" values such as gradiometer data referring to a specific point on the satellite orbit or mean values for a certain segment, it is still an open question which quantity is the most advantageous to use. Point values are easy to represent in a numerical procedure, while track-segment means require a larger numerical effort.

2.7. Brute force or reduction of the number of data by using "normal" values.

According to earlier simulation studies, the most advantageous combination is the brute force one which use all observations simultaneously, including ground data. This will only be possible in regional solutions using LSC. For the integration method, the data must be pre-processed to form means of blocks of e.g. T_{zz} at satellite altitude. LSC can also be used to handle such data. The advantage of this is that error-correlations can be taken into account, and that error-estimates can be calculated.

2.8. Use of symmetries.

If blocks are used, symmetries are created for data blocks having the same latitude, on the condition that the errors are identical. This will probably not be so, since if data have been pre-processed in an optimal way, and even if high frequency information has been removed a-priori, the signal variance will vary. This may force the use of a uniform noise in latitude bands as done when estimating the EGM96 coefficients of higher degree.

The influence of having to assume an uniform noise can be investigated by determining a low degree solution with and without uniform noise. The treatment of a non-uniform signal variance is still waiting a numerical implementation.

2.9. Use of data-redundancies.

The ground tracks of the satellite will cross. This has as a consequence that data close to the cross-over points are strongly (spatially) correlated. One may explicitly take advantage of this as discussed in WP 1, section 2. In LSC there is implicitly taken advantage of this. Since the integration method is expected to have mean block values formed by LSC, this means that the blocks will have a smaller error as compared to a situation where the calculation of the means is made with simpler methods. The redundancies will also be used in data screening, a subject treated in WP 4.

2.10. Global and regional (local) data processing approaches.

In LSC the local and global approaches are seen from the theoretical standpoint identical. The difference arises because the use of LSC in the global case will require that a very large system of equations have to be solved if the basic observations are to be used. If mean values of e.g. T_{zz} are used as normal points, also the global case can be handled by LSC. (Work on improved methods for solving the equations will be reported in WP 3.)

When determining "normal" values a local approach is clearly of advantage. One will be able to use a stochastic model which represents the local signal variation. However, the local/regional solutions should not be merged to a global solution (by e.g. defining a solution for each block of a certain size covering the Earth). This opens up for a step-wise approach, where the regional solutions have as their only purpose the use in the calculation of suitable "normal" values and their error-estimates.

2.11. Loss of information in regional solutions.

Using the developments in the use of LSC for coefficient prediction as described in Tscherning (2000) one may investigate to which extend long wavelength information is lost in a regional approach. One example is described in the paper, which shows that information is lost, but also that some coefficients are determined better than other.

2.12. Use of preconditioning when solving very large systems of equations.

In the space-wise approaches it is generally so that all quantities “reduced” by subtracting the contribution from the best possible set of spherical harmonic coefficients. This has as consequence that the physical variance and correlation lengths becomes significantly shorter than if a standard normal potential is used. This opens up for the use of the conjugent gradient method with preconditioners, which become zero at a short distance.

This was investigated first theoretically (Moreaux, 1999). Subsequently numerical tests with larger data sets (13000 observations) have been made, and further studies are in progress. The data were distributed regionally, and the tests show that large computational savings can be expected for global datasets.

2.13. Conclusion.

We have outlined the elements of the data processing scheme to be used in the space-wise approach. The use of the approach requires several compromises to be made depending on the exact way chosen: use of regional solutions, use of normal point or mean values, use of homogeneous/isotropic statistical models and assumptions of uniform noise. The consequences of these compromises have only been studied in a few cases, and should be studied in the future.

The whole space-wise approach relies on the assumption that in some way the low frequency coefficients are well determined in advance, so that the determination of the full set of coefficients (up to some maximal degree) is affordable as illustrated in the investigation concerning the use of preconditioners.

Moreover the subject of regional and global processing of the data still requires a significant attention. The full exploitation of all components also still needs development.

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