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Combining Airborne and Ground Gravity using Collocation.

Abstract. In the Greenland Aerogeophysical Project (GAP) airborne gravity data have been collected covering all of Greenland. Ground data located primarily in the coastal areas have been used for calibration. However improved gravity field modeling results may be obtained by combining the two data sources. Least Squares Collocation (LSC) may be used with advantage for this purpose. The method furthermore allows for (a) the correct modeling of the (filtered) observation functional, (b) the use of an error correlation function, (c) the determination of biases in the airborne data and (d) the estimation of the errors of the quantities calculated for the obtained gravity field models. We describe her the results obtained using LSC in the two areas with good ground data coverage. The basis for using LSC is the estimation and subsequent analytic modeling of the empirical covariance function. However, the modeling was only successful, when a model error correlation function was subtracted from the covariance function of the airborne data. The variances calculated from the ground data were used to control that the variances derived from the airborne covariance had correct values. This made it possible to correctly downward continue the GAP data using LSC. When comparing observed and ground data predicted from the GAP data, biases were found. This is probably due to systematic errors in the airborne data or systematic errors from terrain effects. The bias was removed by assigning a bias-parameter to each track and adding (a part of) the ground data as new observations. The remaining ground data were used to verify that the bias has been removed. The results, however, were in both areas only marginally better than those obtained when using one ground gravity sub-data set to predict the other. On the other hand they were much better than results obtained using LSC or frequency domain collocation for the prediction of ground gravity from the GAP data only.

Introduction.

An airborne gravity survey covering all of Greenland was carried out in 1991 and 1992 by the (US) Naval Research Laboratory in cooperation with Defence Mapping Agency, Naval Oceanographic Office and KMS. The Greenland Aerogeophysics Project (GAP) is described in e.g. Brozena (1992). The data have been analyzed in several publications, e.g. Forsberg & Brozena (1993), Forsberg & Kenyon (1994, 1995), Rubek (1997). Here we study the problem of optimally combining the GAP data with the ground gravity data, with the purpose of determining the best gravity model for Greenland.

The method of Least Squares Collocation (Moritz, 1980) is ideal for this purpose. It not only permits the combination of the two types of data, but also

- the determination of systematic errors (e.g. in the form of biases),
- estimation of prediction errors,
- correct representation of the along-track filtered GAP data
- use of individual and possibly correlated errors .

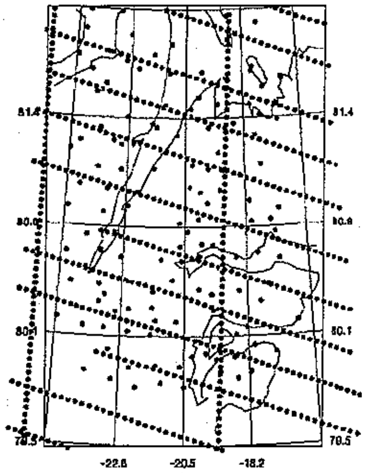

Fig. 1, GAP and ground gravity.

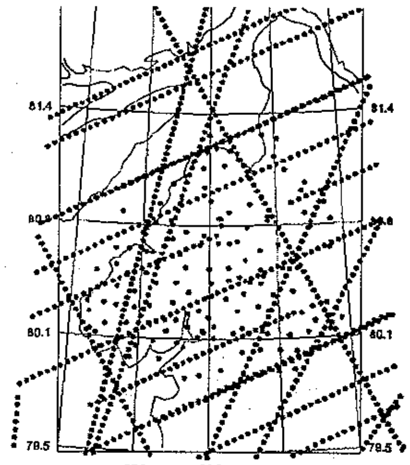

Fig. 2, GAP and ground gravity.

We have selected two areas in Northern Greenland with a good ground gravity coverage and digital elevation model, cf. Table 1 and Fig. 1 & 2. Despite all its nice properties, it is not possible to use LSC to create a gravity model wile using all the GAP and the ground data in one run. A system of equations with as many unknowns as the number of data would have had to be solved, making solutions in smaller blocks necessary.

Table 1. Location of experimental areas, and number of data points.

Min Latitude deg.	Max Latitude, deg.	Min Longitude, deg.	Max Longitude, deg.	Number of airborne data	Number of ground data
79.25	82.1	-26.0	-14.5	691	139
79.0	82.3	-70.0	-47.0	2126	151

Data selection and preprocessing

From the values have been subtracted the contribution from EGM96 (Lemoine et al., 1996) and the contribution of the residual topography using standard procedures. A dense topographic elevation model of 500 m resolution obtained from KMS's digital mapping project was used. The model however does not include accurate information about the depth of the fjords, which are in the test areas. This might be a source of a bias, since the terrestrial gravity data are only measured on land, whereas the airborne data samples the land, fjords and the ice areas randomly. A typical Greenland fjord will have a large negative gravity anomaly due to the bathymetry.

When evaluating the topographic as well as the spherical harmonic contributions, it was taken into account that the airborne data are along-track filtered data. The filter is symmetric and has a resolution around 12 - 15 km and a window length of 60 km. 11 weighted point values were used to represent the filter.

The GAP data are ordered by flight tracks. Values at or very close to track crossing points does not have to be equal, since they are weighted means along track. However, the values should in general be quite close, since a cross-over adjustment has been performed. A calculation of root mean square values of differences between values within 1 km of track cross points gave values of from 6 to 15 mgal for equal-area blocks with side length 6 deg. For the two test areas the values were 11 mgal for the Eastern block and 7 mgal for the western block. This indicated that there might be systematic errors present, larger that the standard error of the data of 4.6 mgal for the single track noise obtained from the cross-over adjustment. That this is the case will become clear from the following.

Covariance estimation.

The use of the LSC method requires the estimation and analytic modeling of a covariance function. In an ideal situation ground data alone would have been used for covariance estimation, due to its low, uncorrelated noise. But for Greenland the data spacing is so large that only the rough shape of the covariance function can be estimated. Consequently the GAP data had to be used, but in such a way that only products of values located on the same track were used to calculate the covariances.

The resulting empirical covariances were unusually smooth for small distances, reflecting the inherent along-track filtering of the airborne data. Despite the correlated noise present we tried to determine an analytic representation of the two covariance functions, using a version of the program COVFIT (Knudsen, 1987). This was not successful, since the derived ground gravity variance became many times too large. The estimated radius of the Bjerhammar sphere in the used T/R-covariance model, (Tscherning & Rapp, 1974) was too close to the Earth mean radius.

When a 1-D covariance function (Sanso' and Schuh, 1987) with 20 mgal^2 zero-lag value, was subtracted, values consistent with the ground gravity variance were obtained. However, the ideal procedure would have been the use of a covariance function derived from the ground data. The difference between the upward-continued (and filtered) covariance function and the GAP covariance function would then have given an estimate of the error-covariance function of the GAP data. This procedure should be tried in an area with a good gravity coverage.

Numerical Experiments.

A number of numerical experiments have been carried out by (Rubek, 1997), but without taking the occurrence of correlated errors, or possible track-related biases into account. Both LSC and frequency domain collocation (FFT) were used, giving similar results. Here we only report on results obtained using LSC and taking the correlated errors into account. The results are summarized in Table 3.

First ground data were predicted from the GAP data. The mean (and also the standard deviation) of the differences between observed and predicted values are quite large, but it should of course be pointed out that estimating point values from along-track averaged airborne data will always be a noisy process. Furthermore a comparison of the prediction error with the absolute value of the differences observed - predicted showed that the percentage of differences larger than the error estimate was too large, see (Tscherning & Knudsen, 1986). This could indicate possible (track related) biases in the GAP data.

The LSC method includes the possibility for the estimation of parameters such as track biases, and this is implemented in the program GEOCOL (Tscherning, 1997). Consequently a bias parameter was associated with each track, and its value as well as the error was estimated. The 5 largest biases and their values as estimated from the GAP data only and from GAP data combined with ground data are found in Table 2. Note the large differences between the results with and without ground data, and the decrease in the error estimate when ground data are added.

(At this point it would have been most correct to re-estimate the empirical covariance function from the LSC-filtered GAP-data, however we have aimed here to show the potential of LSC and not to produce the optimal result).

Table 2. Five largest biases and their standard deviations (units mgal) from the two test areas using GAP data only and GAP data combined with ground data.

Eastern area				Western Area			
Bias from GAP	Error	Bias from GAP + ground	Error	Bias from GAP	Error	Bias from GAP + ground	Error
-14.84	6.33	-27.01	4.66	16.12	7.44	17.17	7.43
-12.40	5.02	-16.17	4.88	-14.49	4.54	-19.70	4.46
-10.86	3.91	-12.24	3.81	-13.70	3.69	-19.37	3.61
-10.00	5.64	-22.72	4.04	-11.69	3.29	-16.22	3.19
-8.31	3.28	-10.87	3.05	-10.22	3.28	-16.03	3.05

The gravity prediction results, cf. Table 3 did also improve when ground gravity were added. Only every second of the ground data points were used in the form of observation data. The remaining values were used as control data. The biases have disappeared, but note that the result is not better than when the ground "observation" data set was used to predict the control data set. The error estimates, however, are 1 - 2 mgal higher than for those where the GAP and the ground data were combined.

Table 3. Mean and standard deviation (mgal) of observed minus computed ground gravity values computed from ground data only, from GAP data only, from GAP data only including bias-estimates and from GAP data and ground gravity including bias estimates.

Eastern area				Western area			
Number of data +parameters		Obs.-predicted		Number of data +parameters		Obs.-predicted	
GAP data	Ground data	Mean	Standard deviation	GAP data	Ground data	Mean	Standard deviation
0	70	0.76	6.07	0	76	2.89	7.07
691	0	5.45	9.09	2126	0	8.54	14.96
691+25	0	3.46	7.95	2126+62	0	7.52	13.82
691+25	70	0.31	5.96	2126+62	70	0.31	5.96

Conclusion.

We have demonstrated how LSC may be used to combine airborne and ground gravity data, especially taking into account the along-track filtering inherent in the airborne gravity data. The LSC method, which includes the possibility for bias estimation, should be used if there is a suspicion that data contain biases. A traditional cross-over analysis will not always remove biases efficiently and a bias-and-tilt procedure might actually degrade data in smaller sample areas. Furthermore, estimation of biases in airborne data from surface data might not always be a good idea, due to the possibility of biases arising from terrain corrections, e.g. from insufficient digital elevation models or unmodeled effects of fjord depths and glaciers.

The use of a model for correlated errors is important, in order that downward continued values have a a correct variance. The combined use of the two data types much improves the result as compared to only using airborne data.

The challenge now is to find a procedure allowing large areas to be covered in one run. In this way we can take advantage of ground data in the coastal regions to remove biases on tracks also passing areas without ground data. The solution here may simply be to only use every third or fourth value. This may give data sets of size 5000 - 8000 observations which are not difficult to handle. Naturally the computational effort will be large, considering that the numerical evaluation of a covariance involves the calculation of 11 x 11 point covariances.

References.

Brozena, J.: The Greenland Aerogeophysics Project. Airborne Gravity, topographic and magnetic mapping of an entire continent. In: O.Colombo (Ed.): From Mars to Greenland. Proc. IAG Symp. G3, Vienna, Austria, Aug. 1997, Springer Verlag, 1992.

Forsberg, R. and J.Brozena: The Greenland Airborne Gravity Project - Comparison of Airborne and Terrestrial Gravity Data. In: W.Torge (Ed.), Geodesy and Physics of the Earth, Proceedings IAG Symp. 112, Potsdam, Oct. 1992, pp. 171-175, Springer Verlag, 1993.

Forsberg, R. & S.Kenyon: Evaluation and downward continuation of airborne gravity data - The Greenland example. Proc. KISS94, Banff, Canada, pp. 531-538, 1994.

Forsberg, R. & S.Kenyon: Downward continuation of airborne data. Proc. Symp. 4, IUGG XX! GA, Boulder, 1995.

Knudsen, P.: Estimation and Modelling of the Local Empirical Covariance Function using gravity and satellite altimeter data. Bulletin Geodesique, Vol. 61, pp. 145-160, 1987.

Lemoine, F.G., D.Smith, R.Smith, L.Kunz, E.Pavlis, N.Pavlis, S.Klosko, D.Chinn, M.Torrence, R.Williamson, C.Cox, K.Rachlin, Y.Wang, S.Kenyon, R.Salman, R.Trimmer, R.Rapp and S.Nerem: The development of the NASA GSFC and DMA joint geopotential model. Proc. Symp. on Gravity, Geoid and Marine Geodesy, Tokyo, 1996.

Moritz, H.: Advanced Physical Geodesy. H.Wichmann Verlag, Karlsruhe, 1980.

Rubek, F.: Comparison of three different methods for harmonic downward continuation of airborne gravity measurements. Master Thesis, Dep. of Geophysics, University of Copenhagen, 86 p., 1997.

Sanso, F. and W.-D. Schuh: Finite Covariance Functions. Bulletin Geodesique, Vol. 61, pp. 331-347, 1987.

Tscherning, C.C.: Current Problems in Gravity Field Approximation. Proceedings 1. Hotine-Marussi Symposium, Rome, June 3-6, 1985, pp. 363-384. Politecnico di Milano, 1986.

Tscherning, C.C.: GEOCOL - A FORTRAN-program for Gravity Field Approximation by Collocation. Technical Note, Geophysical Institute, University of Copenhagen, 12. ed., 1997.

Tscherning, C.C. and P.Knudsen: Determination of bias parameters for satellite altimetry by least-squares collocation. Proceedings 1. Hotine - Marussi Symposium, Rome, June 3-6, 1985, pp. 833-852. Politecnico di Milano, 1986.

Tscherning, C.C. and R.H.Rapp: Closed Covariance Expressions for Gravity Anomalies, Geoid Undulations, and Deflections of the Vertical Implied by Anomaly Degree-Variance Models. Reports of the Department of Geodetic Science No. 208, The Ohio State University, Columbus, Ohio, 1974.