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THE USE OF LEAST SQUARES COLLOCATION METHOD IN GLOBAL GRAVITY FIELD MODELING

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Abstract

Least Squares Collocation (LSC) is a very flexible method in gravity field modelling, which permits the use of data of different kinds and with different noise-characteristics. It also permits the computation of drift parameters and of error estimates of both computed values and parameters. The method has however the drawback that one needs to solve a system of normal equations involving as many unknowns as the number of data and parameters. The global use of the method have therefore been restricted to the use of block mean values as input data. The combined use of point and mean values could be possible if finite instead of full covariance functions could be used. Preliminary prediction results using LSC with finite covariance functions showed satisfactory agreement with corresponding results obtained with full covariance functions in the cases where the finite covariances fit satisfactory the corresponding full covariances. Some new experiments showed that it is possible to find some general procedure to approximate full by finite covariance functions without significant loss in the quality of the prediction. The computational savings which might be gained in finite covariance functions shows that LSC in the future could be used globally. Iterative solutions based on solutions where finite covariance functions are used may be the way to go.

1. Introduction

The Least Squares Collocation (LSC) is a very flexible method in gravity field modelling, which permits the use of data of different kinds and with different noise-characteristics. It also permits the computation of drift parameters and of error estimates of both computed values and parameters. These possibilities have been used extensively in earlier studies (see e.g. Arabelos and Tscherning, 1996a).

The method has however the drawback that one needs to solve a system of normal equations involving as many unknowns as the number of data and parameters. The matrices of normal equations are full matrices because the covariance functions even for very long distances are different from zero due to the harmonicity of the covariance function. This gives a heavy computation burden for simulations as well as for actual computations tasks to be solved in practice. The reason permitting the use of finite instead of full covariance functions is that after a certain distance the auto- and cross-correlation between the quantities related to the gravity field becomes very weak. Taking this into account the corresponding covariance values could be approximated by zeros. Other methods for gravity field modelling use base functions, like finite elements, which results in the use of sparse matrices. The same would be the case if we in LSC gave up the requirements that our base functions are harmonic.

The computational savings which might be gained if finite covariance functions could be used would be large, (see e.g. Schuh, 1994). If we used a grid with N*N values, the full matrices would contain N*N*(N*N-1)/2 elements, i.e. of the order of N raised to the power of 4. If finite covariances are used, -and the observations are ordered in a reasonable manner, the matrices would contain non-zero elements of the order of N raised to the power 3. The computational savings would be even larger.

This fact made Sansò and Schuh (1987) propose to use the so-called finite covariance functions, i.e. covariance functions with values equal to zero for distances larger than a given radius. We have here studied their use for gravity field modelling and it seems that they may be useful in solving some tasks. The consequence of using finite covariance functions is obviously that we disregard long-distance correlations. Consequently for differentiation we could expect good results (gravity gradients from gravity) while for integration (geoid and gravity from gravity gradients) we could expect biased results. This is also what we have found and what is new here is the magnitude of the errors we commit using these functions.

For the first- and second- order derivatives of the anomalous potential T with respect to z of a local (x, y, z) coordinate frame (East, North, Up) we use the notations: $\partial T / \partial z = T_z$, $\partial^2 T / \partial z^2 = T_{zz}$ respectively.

2. Finite covariance functions modelling

The combination of ground and satellite data may be done using several different methods. The optimal one (in a very specific sense, see Moritz, 1980) is LSC. The cost of this "optimality" is as noted above that systems of equations with a dimension as large as the number of observations have to be solved. The equations to be solved have associate matrices, which are full, since the base functions (generally) used are harmonic. This is a consequence of that the anomalous gravity potential T is a harmonic function, and that the only function which is both zero in an open set and harmonic is the function which is identically zero.

One may however use more general functions, like finite elements. But then the observation that the Laplace equation applied to T is harmonic in every point must be added to the observations. One may also use functions which are good approximations to harmonic functions, or in the case of LSC, good approximations to the analytic covariance functions. This was suggested by Sansò and Schuh (1987), who suggested various kinds of covariance functions which were zero for distances larger than a certain distance.

Various methods for obtaining a finite covariance function have been suggested. One method is to multiply a "real" covariance function with one being zero at a given distance. However the functions suggested by Sansò and Schuh all give very strong changes of the original covariance functions.

But if one inspects the finite covariance functions suggested for use, they all look (for their first part) like any other covariance function. We therefore decided to try to use one of the most reasonable suggested. It has the following very simple expression (Sansò and Schuh, 1987, eq. 30):

$$\mathrm{cov}(d) = \frac{1}{3}R^6\pi - \frac{1}{2}R^4 d^2\pi + \frac{1}{3}\left(R^4 d + \frac{4}{3}R^2 d^3 - \frac{1}{12}d^5\right)\sqrt{R^2 - \frac{d^2}{4}} + \left(R^4 d^2 - \frac{2}{3}R^6\right)\mathrm{a}\sin\left(\frac{d}{2R}\right), \quad (1)$$

$$\mathrm{cov}(d) = 0 \text{ for } d \geq 2R, \quad (2)$$

with R a constant and d the variable distance.

The values to be selected in order to fix the covariance function is the quantity R and a scale-factor so the functions agree with the original one for d = 0. R may be selected so that we get the best approximation to the original function but we chose simply to fix R so that we had the same correlation distance d_l for the finite and the original,

$$\mathrm{cov}(0)\,0.5 = \mathrm{cov}(d_l) \tag{3}$$

Consequently, a number of simple subroutines have been written, finding the correlation distance d_l from the original function and subsequently the value of R for the finite covariance function. These subroutines have been included in the program GEOCOL, version 11. Other routines were changed, so that they for a specific covariance function would use the finite one instead of the original.

3. Data

In order to study the performance of LSC with finite covariance functions in different cases, various regional and global experiments were done using different types of data. The following data sets were used:

1. Marine gravity data

From the GEODAS -Marine Geological and Geophysical Data from NGDC CD-ROM, published by NOAA, a number of 29,760 point gravity values have been extracted covering the area bounded by ($70^\circ \leq \phi \leq 83^\circ$, $-10^\circ \leq \lambda \leq 20^\circ$). These gravity values are available from different Institutions and different surveys. For the main part of these data there is the information that the Gravity System is Potsdam and the reference system is International formula 1930. We assume that also in the cases that this information is missing, the gravity system and the reference gravity field are the same (Potsdam/ Int.Form. 1930). However this assumption could introduce compatibility problem when merging all data sources. No information concerning the accuracy of the data is available. The statistics of this data set after the transformation to IGSN71/GRS80 and subtraction of the OSU91 is shown in Table 1.

From this data set a new data set was created by selecting point data closest to nodes of a 10' × 10' grid. 3,710 point values were selected in this way.

In Central Mediterranean we have used the gravity data described in numerous earlier studies (see, e.g., Arabelos and Vermeer, 1996). The test area in this case is bounded by ($33^\circ \leq \phi \leq 37^\circ$, $16^\circ \leq \lambda \leq 20^\circ$).

2. Airborne data:

A set of 5,292 point free-air gravity anomalies covering part of Greenland ($52.25° \le \phi \le 73.68°$, $-58.32° \le \lambda \le -20.25°$) is available from (Forsberg and Brozena, 1992). The measuring height was about 4 km . These values are mean values along the air-plain tracks with a sampling rate corresponding to about 18 arcmin. From these values the contribution of an upgraded version of OSU91 Geopotential model (OSU91aC) to degree 180 was subtracted. In Table 2 the statistics of the original and the reduced data is shown.

3. Satellite altimeter data

A set of common adjusted ERS1/ERM, ERS1/GM and TOPEX data have been used to create a $5' \times 5'$ sea surface height grid in Central Mediterranean. The area bounded by ($33° \le \phi \le 37°$, $16° \le \lambda \le 20°$) is covered by this grid. The editing and adjustment procedure of these data is described in (Arabelos and Vermeer, 1996).

4. Simulated data

For the needs of the global experiment which will be described later, two simulated data sets were produced. Since the goal of this experiment was to compare the results of the collocation prediction using full and finite covariance functions, no special care was taken for the production of the simulated data sets. The procedure in producing this data sets was the following: "Observed" gravity anomalies were computed at 98.45 km altitude, in the centres of 5^0 equal blocks (4,098 values), using the EGM96 geopotential model up to degree 360. The point values at 98.45 km altitude were considered as 5^0 mean values at zero level. Then "reduced" gravity anomalies were computed by subtracting from the "observations" the contribution from the OSU91A1F geopotential model to degree 72. The same procedure was followed for the production of simulated 5^0 mean geoid heights. The statistics of the "observed" and "reduced" gravity anomalies is shown in Table 3 and for the "observed" and "reduced" geoid heights in table 3a.

4. Regional experiments with finite covariance functions

With the software described in section 2 and the data described in section 3, several experiments were carried out. First we computed T_{zz} at 200 km altitude from point surface gravity anomalies in the Arctic zone using full (traditional) and finite covariance functions. In Fig. 1. the full covariance function of free air gravity anomalies at sea level reduced to OSU91A, the corresponding finite as well as their differences are shown. The statistics of the prediction results using (a) full and (b) finite covariance functions as well as for the differences (a) - (b) are shown in Table 4. The differences (a) - (b) are small but not negligible.The results in terms of the mean value and standard deviation of these differences were encouraging enough to continue the finite covariance investigations.

Subsequently, one further upward continuation experiment using (a) full and (b) finite covariance functions was executed using the airborne data described in section 3. T_z field at 400 km altitude was predicted from airborne gravity anomalies in Greenland. The corresponding covariance functions (full, finite and their differences) of gravity anomalies at zero level, reduced to OSU91C are shown in Fig. 2. The results of this experiment from (a), (b) and (a)-(b) are shown in Table 5.

The results of this upward continuation experiment in terms of the statistics of Table 5 are not so encouraging as in the previous experiment. Moreover, there is an -almost constant- difference between the collocation error estimates: In (a) the error estimate is 0.02 mGal larger than in (b).

We also conducted a downward continuation experiment. Gravity anomalies and geoid heights were predicted in Greenland at ground level from simulated T_{zz} at 200 km altitude. This simulated data set was predicted from the airborne gravity data described in section 3, using LSC with full covariance functions (see Arabelos and Tscherning, 1996b). The 496 T_{zz} values were distributed on a 20' × 20' grid and considered to have an accuracy equal to 0.01 EU (Arabelos and Tscherning, 1996). The area covered by the T_{zz} grid (part of Greenland) is bounded by $(64^\circ \leq \phi \leq 69^\circ,\ -50^\circ \leq \lambda \leq -40^\circ)$. In Fig. 3 the model (full) and the finite covariance functions of the T_{ZZ} values at 200 km altitude are shown. The differences between model and finite covariance functions are depicted in the same figure (3).

Free air gravity anomalies and geoid heights were predicted on a 10' × 10' grid covering the area $(65^\circ \leq \phi \leq 68^\circ,\ -48^\circ \leq \lambda \leq -42^\circ)$ (703 values). Using finite covariances 12,675 covariances were set to zero. The collocation error estimation in the case of gravity prediction using full covariance functions was equal to about 21 mGal. The corresponding quantity in the case of finite covariance functions was 2 mGal higher (equal to about 23 mGal). There are large discrepancies between the corresponding quantities predicted using (a) full and (b) finite covariance functions as it is shown in Tables 5 and 6.

The differences between the predicted gravity anomalies using full and finite covariance functions are shown in Fig. 4a, while the corresponding differences for the geoid are shown in Fig. 4b.

In the previous experiment the correlation length was equal to 0.02675 rad. Changing slightly the correlation length we can get a better agreement between the finite and the full covariance functions and consequently slightly better results. As an example, changing the correlation length from 0.02675 to 0.0281 the standard deviation of the differences (a) - (b) in the case of gravity drops from 9.97 to 9.08 mGal and for the differences (a) - (b) in the case of geoid heights from 0.68 to 0.54 m.

Finally we performed experiments in the Central Mediterranean using sea gravity and satellite altimeter data. The experiments concern the prediction of gravity from altimetry and vice-versa, using full and finite covariance functions. The results of these predictions are compared with observed data in order to have a realistic estimation of the effect of cutting the covariance function in such a way as it was described in section

2. We started with the prediction of geoid heights from the gravity data for the Central Mediterranean, using full as well as finite covariance functions. The prediction was made on a 5' $\times$5' grid (1,369 points) covering the inner $3^o \times 3^o$ area bounded by $(33.5^o \leq \phi \leq 36.5^o,\ 16.5^o \leq \lambda \leq 19.5^o)$. The accuracy adopted for the gravity data was 5 mGal in both cases (full and finite covariance functions). In Fig. 5 the model (full) and the finite covariance functions of gravity anomalies are shown. It should be noted that the collocation error estimation was varying between 0.12 and 0.13 m for the prediction with full covariances and between 0.11 and 0.14 m for the prediction with finite covariances.

The statistics of the comparison between predicted geoid heights and satellite sea surface heights is shown in Table 7. From Table 7 it is shown that the results of the prediction in terms of the mean value and standard deviation of the differences between predicted geoid heights and satellite sea surface heights using full and finite covariance functions are very close although we have to do with an integration procedure.

The opposite way, i.e. the prediction of gravity from altimetry using the same covariance functions as in the previous experiment was not an encouraging experience. We started adopting a realistic error estimation for the satellite altimeter grid equal to 0.1 m. The comparison of the predicted using full covariance functions gravity anomalies with ground truth at 1,369 points gave the very good agreement reported in earlier studies (e.g., Arabelos and Vermeer, 1996). The standard deviation of the differences was equal to 4.87 mGal. The collocation error estimation in this case was equal to about 7 mGal. Using finite covariance functions with the same accuracy assumption for the satellite altimeter data the results of the prediction were strongly biased. Supposing that the problem was due to high frequency features of the altimeter data we tried to apply a damping procedure (similar to that used by FFT techniques) by filtering the data. In the case of LSC this could be done by increasing the error estimation of the data. After some experiments with different accuracy assumptions for the altimeter data we produced the results given in Table 8.

The case giving the best results corresponds to an accuracy equal to 0.7 m which is about 7 times higher than the actual accuracy of the altimeter data. In Table 9 the results of the prediction using full (with accuracy assumption for the altimeter data equal to 0.1 m) and finite (with accuracy assumption for the altimeter data equal to 0.7 m) covariance functions are summarized. From Table 9 it is shown that even after the filtering of the high frequencies, the use of the finite covariance function gives no satisfactory results. On the other hand the number of covariances set to zero was 1,845,780. The computational saving gained in this case would be extremely large. Obviously the reason is the considerable discrepancies between full and finite covariance functions (see Fig. 6).

The results - good or bad - of these preliminary experiments are according to expectations. Cutting off the tails of the covariance functions, we disregard long-wavelength correlations. Consequently the finite covariance functions will work well when we differentiate, but not so well when we integrate (e.g. in the case of computation of gravity and geoid at ground level from gravity gradients at satellite altitude).

Considering all the previously reported results it is obvious that further investigation needed for a better modelling of the finite covariance functions. However, in the case of a good agreement between finite and full covariance functions, finite covariance functions work properly also when we integrate.

5. A global experiment

In our first global experiment with collocation we started with 5^0 equal areas data sets since at the time being the number of the data of a such data set (4,098 values) is close to the upper limit of unknowns that the used version of GEOCOL program is able to handle.

The covariance function of the global gravity data set described in section 3.4 is shown in Fig. 7. Using the full and the finite covariance functions, geoid heights were predicted from the simulated 5^0 equal areas gravity anomalies and vice versa, i.e. gravity anomalies were predicted from simulated 5^0 equal areas geoid heights. The 5^0 gravity anomalies and geoid heights were treated as point values located at 98.45 km altitude (Tscherning and Rapp, 1974). The prediction in both cases has been made on a $5^0 \times 5^0$ grid. This means that the prediction was performed in both: points which coincide with input data points, and points lying away from input data points.
The statistics of the prediction results using full and finite covariance functions are summarized in Table 11 for the predicted geoid heights from gravity anomalies and in Table 11a for the predicted gravity anomalies from geoid heights. The sd of the predicted quantities (geoid heights from the gravity data and gravity anomalies from geoid heights) is larger than the sd of the corresponding "observations" (described in section 3.4) since point values were predicted from mean values. As it has been mentioned previously, the goal of this experiment was just to study the problems in using finite covariances. The differences between the predicted quantities using full and finite covariance functions are plotted in Figs.8a and 8b for geoid heights and gravity anomalies respectively

The sd of the differences equals to 20% and to 18% of the sd of the predicted signal in the case of geoid heights and gravity anomalies respectively. On the other hand, the computational gain in this case was enormous: more than 8,3 million covariances were put to zero when finite covariances were used. However it is due to the shape of the global covariance function (Fig.7): The 5^0 values are very smooth, so that for $\psi > 5^0$ the covariances have very small values and consequently they can be approximated by zeros without considerable problems in the prediction results.

6. Conclusion

The first experiments with finite covariance functions were encouraging in the sense that here is a research track to be followed. We only used the most crude finite covariance functions, where there exist several alternatives which it would be interesting to study in the future. Also finite covariance functions which have a consistent height dependence should be tried (Schuh, 1996). The preliminary results confirms the intuitive expectations one might have to the consequence of using finite covariance functions, namely that long wavelength information is lost, or could be damaged.

The global experiment using 5^0 mean values was very encouraging, since the prediction results using finite covariances were very close to the corresponding from full covariances and the computational saving was enormous.

If methods like LSC in the future should be used globally, one has to find ways to reduce the requirements on both storage and computer time. Iterative solutions based on solutions where finite covariance functions are used may be the way to go.

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Table 1. Statistics of the gravity data distributed in an $11^{o} \times 30^{o}$ area in the Arctic zone. Unit is mGal.

	Mean	Std	Min.	Max.
original	34.99	24.94	-63.81	162.88
original -OSU91 (360)	-2.80	22.22	-86.20	133.63

Table 2. Statistics of the original and reduced airborne gravity anomalies in Greenland. Unit is mGal.

	No.	Mean	Std	Min.	Max.
original	5292	25.108	35.338	-103.440	137.880
original - OSU91aC	5292	-2.348	27.851	-108.690	115.220

Table 3a. Statistics of the "observed" and "reduced" 5^0 mean global gravity anomaly data set. Unit is mGal.

	No.	Mean	Std	Min.	Max.
original (EGM96)	4098	0.04	14.44	-68.43	67.90
original - OSU91A1F	4098	0.05	3.97	-31.07	27.88

Table 3b. Statistics of the "observed" and "reduced" 5^0 mean global geoid height data set. Unit is m.

	No.	Mean	Std	Min.	Max.
original (EGM96)	4098	-0.04	29.98	-99.06	75.97
original - OSU91A1F	4098	0.00	0.49	-3.37	4.28

Table 4. Statistics of 496 T_{zz} values predicted from point gravity anomalies in the Arctic zone. (a) full and (b) finite covariance functions were used. Unit is EU.

	Mean value	Std	Min.	Max.
(a)	0.0743	0.0361	-0.0108	0.1459
(b)	0.0575	0.0430	-0.0410	0.1438
(a) - (b)	0.0167	0.0078	0.0012	0.0321

Table 5. Statistics of T_z values (248) predicted from airborne gravity anomalies in Greenland. (a) full and (b) finite covariance functions were used. Unit is mGal.

	Mean value	Std	Min.	Max.
(a)	-0.0178	0.1149	-0.2400	0.1700
(b)	0.1384	0.3338	-0.5800	0.6600
(a) - (b)	-0.1562	0.2214	-0.5000	0.3400

Table 6a. Statistics of the results of gravity prediction at ground level from simulated T_{zz} at 200 km altitude in Greenland. (a) full and (b) finite covariance functions were used. Unit is mGal.

	Mean value	Std	Min.	Max.
(a)	17.05	15.92	-34.13	58.96
(b)	18.31	13.55	-26.44	53.38
(a) - (b)	-1.26	9.97	-27.18	15.79

Table 6b. Statistics of the results of geoid prediction at ground level from simulated T_{zz} at 200 km altitude in Greenland. (a) full and (b) finite covariance functions were used. Unit is m.

	Min.	Max.	Mean value	Std
(a)	-3.40	4.79	1.40	1.43
(b)	-3.30	5.99	2.01	1.88
(a) - (b)	-1.92	0.46	-0.61	0.68

Table 7. Statistics of the differences between geoid heights predicted from sea gravity data and satellite sea surface heights, using full and finite covariance functions. Unit is (m).

		Full covariance functions		Finite covariance function	
	Observations	Predictions	Difference	Predictions	Difference
Mean value	0.20	0.30	-0.10	0.26	-0.06
Std	0.30	0.28	0.10	0.31	0.11

Table 8. Change of the prediction results by adopting different accuracy values of the altimeter data. Unit is mGal.

Error adopted for the altimeter data	Collocation error estimation	Std of the predicted gravity	Std of the differences (observed-predicted)
0.1	500	73.15	65.74
0.4	40	19.36	14.17
0.6	9	10.76	8.65
0.7	10	8.59	8.23
1.0	12	5.32	8.88

Table 9. Statistics of the differences between gravity anomalies predicted from satellite SSH, using full and finite covariance functions. 0.7 m error adopted for SSH. Unit is mGal.

		Full covariance functions		Finite covariance function	
	Observations	Predictions	Difference	Predictions	Difference
Mean value	2.40	1.95	0.44	2.52	-0.11
Std	11.79	10.35	4.87	8.59	8.23

Table 10a. Statistics of the differences between geoid heights predicted from gravity data using full and finite covariance functions. Unit is (m).

	Mean	Sd	Min.	Max.
(a)	0.00	0.76	-5.48	6.07
(b)	0.00	0.76	-6.72	6.42
(a) - (b)	0.00	0.15	-1.15	1.08

Table 10b. Statistics of the differences between gravity data predicted from geoid heights using full and finite covariance functions. Unit is (mGal).

	Mean	Sd	Min.	Max.
(a)	0.03	4.48	-46.89	31.48
(b)	0.03	4.48	-42.70	33.18
(a) - (b)	0.00	0.81	-5.44	5.10

Figure captions

Fig. 1. Covariance functions of gravity anomalies at zero level in the Arctic zone.

Fig. 2. Covariance functions of gravity anomalies in Greenland.

Fig. 3. Model and finite covariance functions of T_{zz} at 200 km altitude, as well as their differences.

Fig. 4a. Differences (a) - (b) between surface free air gravity anomalies in Greenland predicied from T_{zz} at 200 km altitude using (a) full and (b) finite covariance functions. Unit is mGal.

Fig. 4b. Differences (a) - (b) between geoid heights in Greenland predicied from T_{zz} at 200 km altitude using (a) full and (b) finite covariance functions. Unit is m.

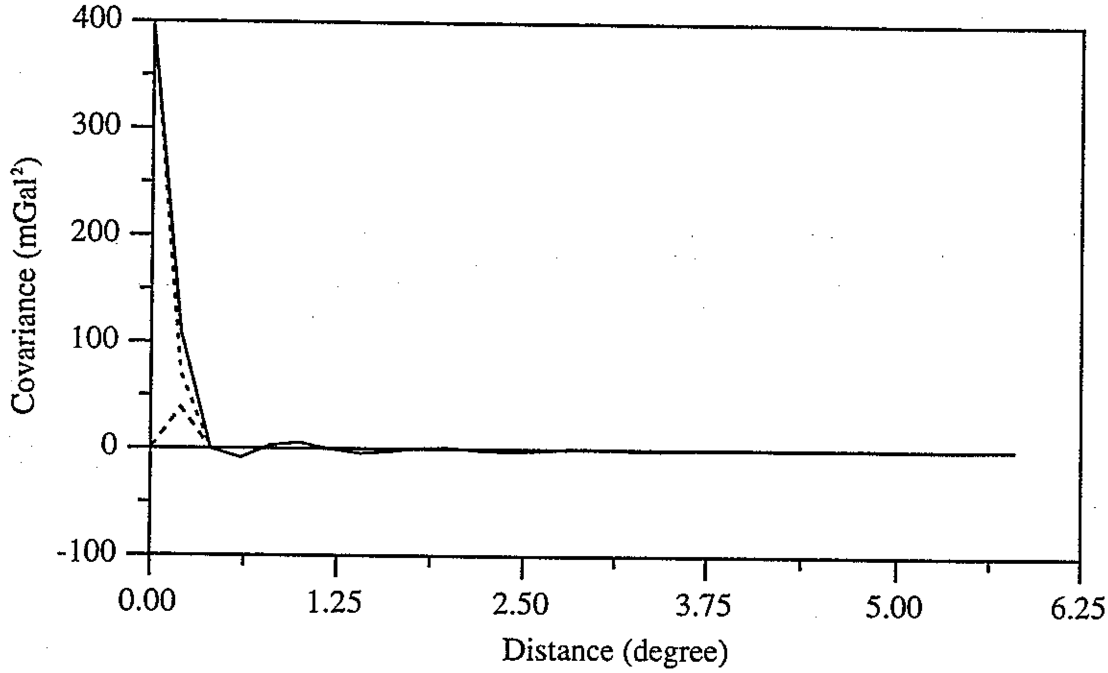
Fig. 5. Covariance functions of gravity anomalies in the Central Mediterranean.

Fig. 6. Covariance functions of satellite altimeter data in Central Mediterranean.

Fig. 7. Covariance functions of the global mean 5^0 gravity anomalies reduced to OSU91A1F to degree 72.

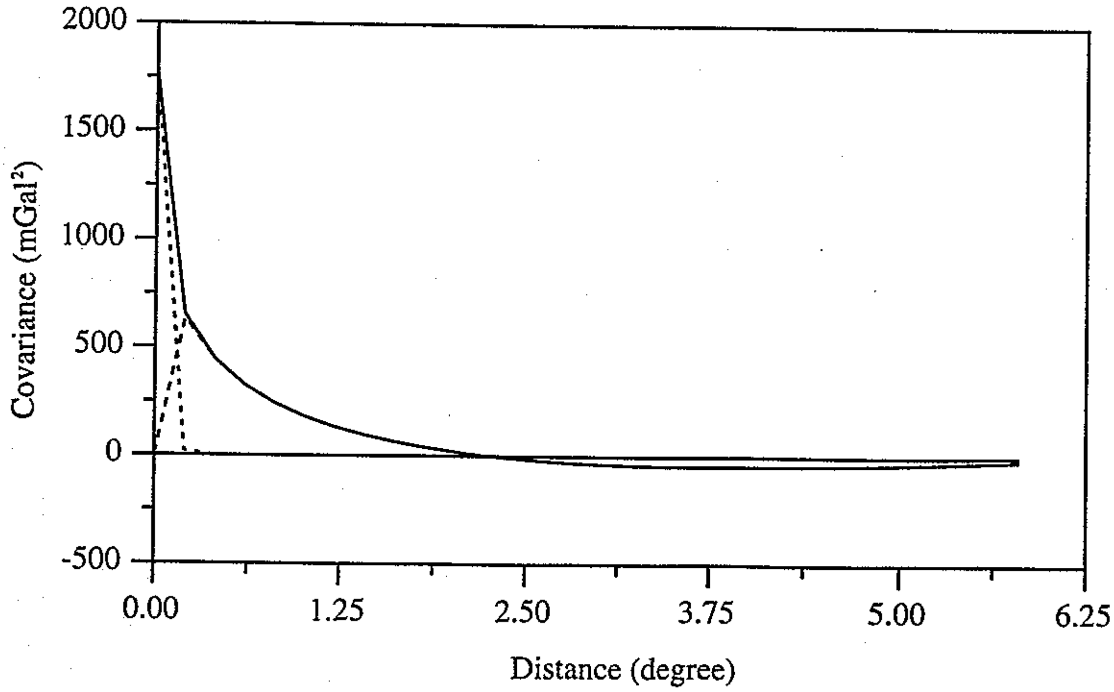
Fig. 8a. Differences between geoid heights computed from gravity anomalies, using full and finite covariance functions. Contour interval = 0.5m.

Fig. 8b. Differences between gravity anomalies computed from geoid heights, using full and finite covariance functions. Contour interval = 1 mGal.



Solid: model, Dashed: finite, Long dashes: difference

Fig. 1. Covariance functions of gravity anomalies at zero level in the Arctic zone.



Solid line: model, Dashed line: finite, Long dashes: difference

Fig. 2. Covariance functions of gravity anomalies in Greenland.

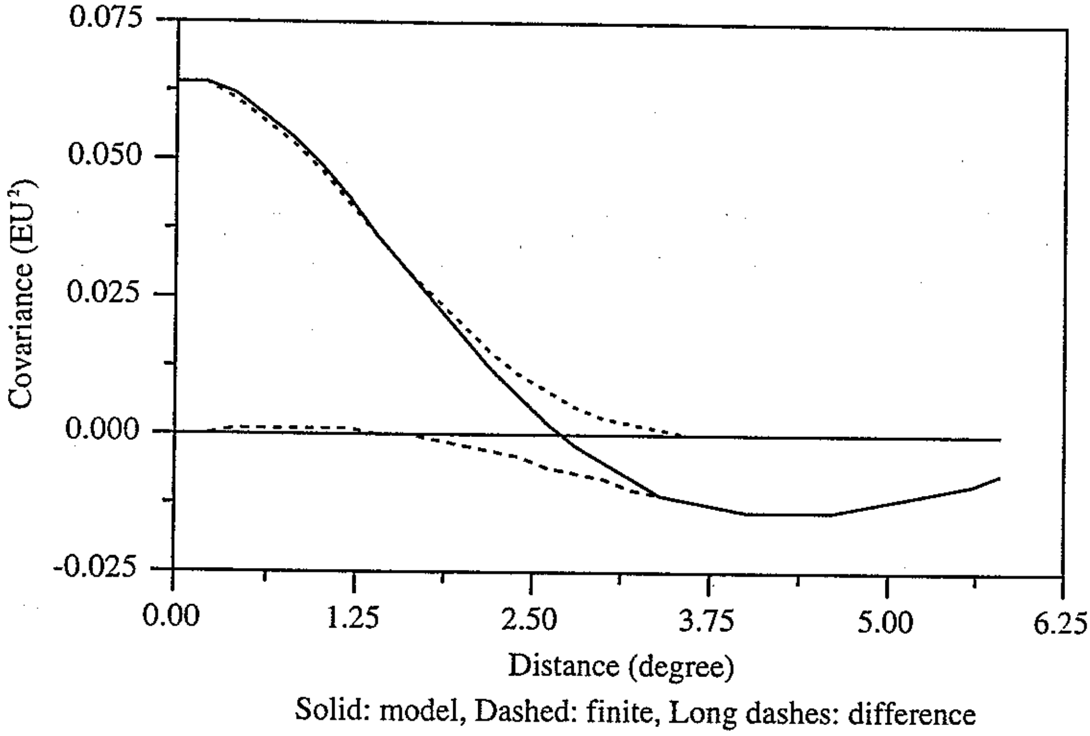


Fig. 3. Model and finite covariance functions of T_{zz} at 200 km altitude, as well as their differences.

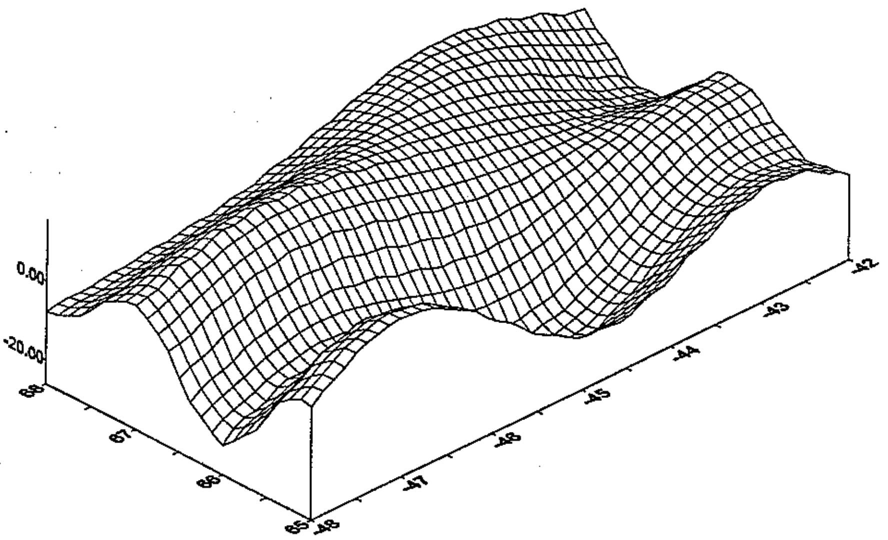


Fig. 4a. Differences (a) - (b) between surface free air gravity anomalies in Greenland predicted from Tzz at 200 km altitude using (a) full and (b) finite covariance functions. Unit is (mGal).

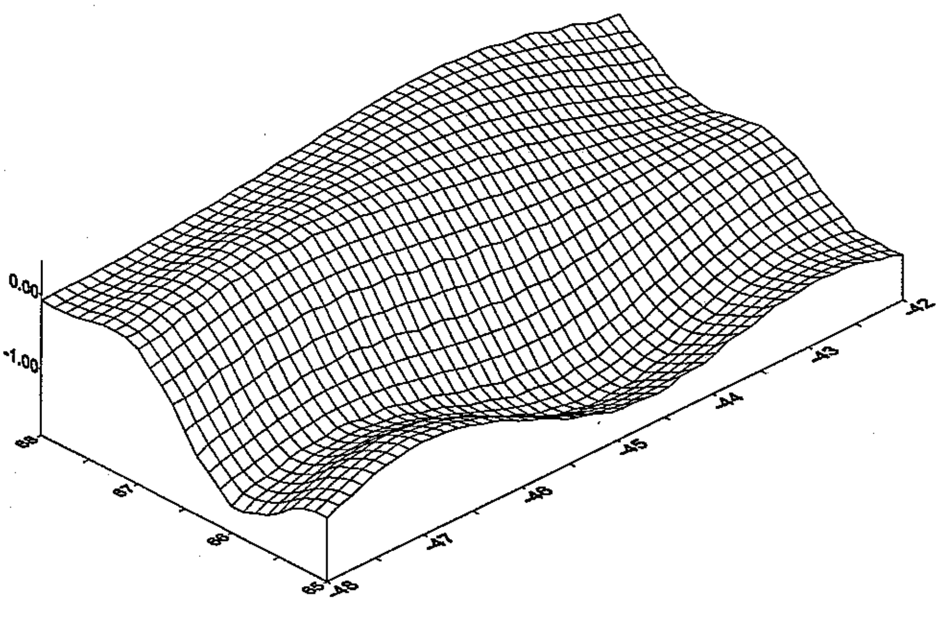
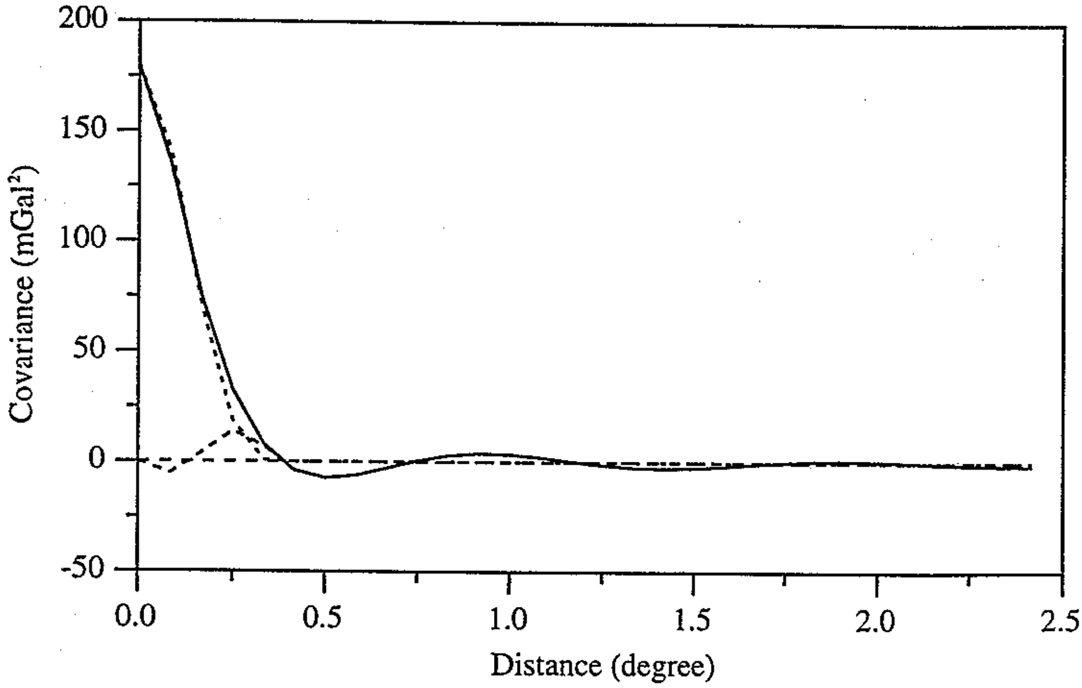
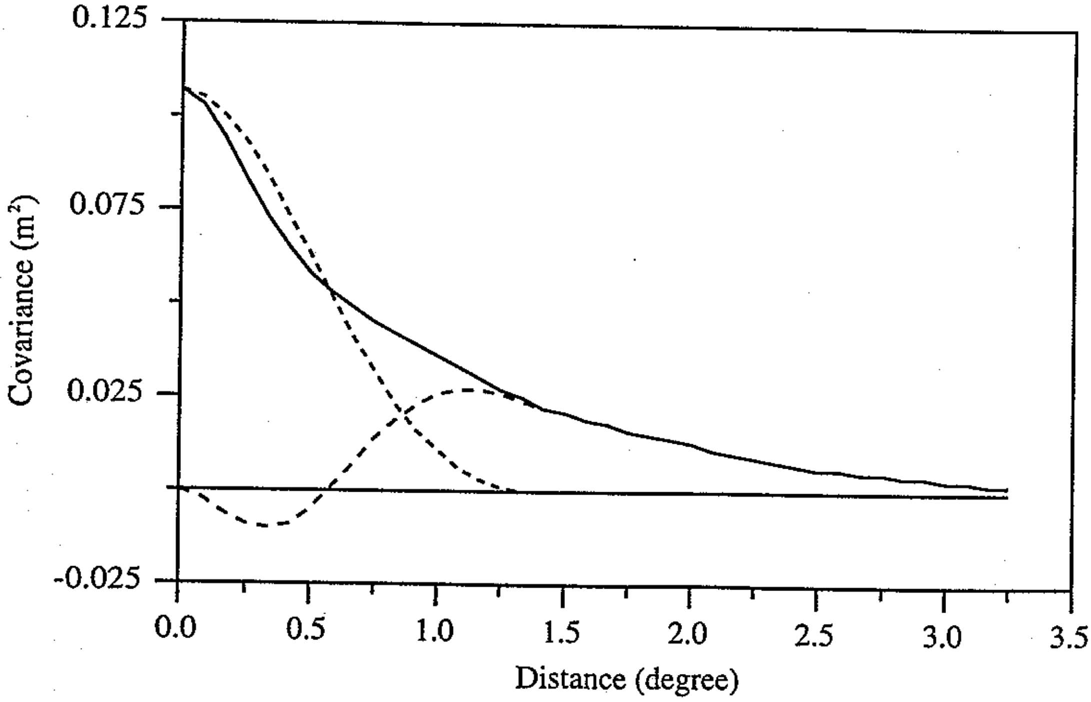


Fig. 4b. Differences (a) - (b) between geoid heights in Greenland predicted from Tzz at 200 km using (a) full and (b) finite covariance functions. Unit is (m).



Solid: model, Dashed: finite, Long dashes: difference

Fig. 5. Covariance functions of gravity anomalies in the Central Mediterranean.



Solid line = model, Dashed = finite, Long dashes = difference

Fig. 6. Covariance functions of satellite altimeter data in Central Mediterranean.

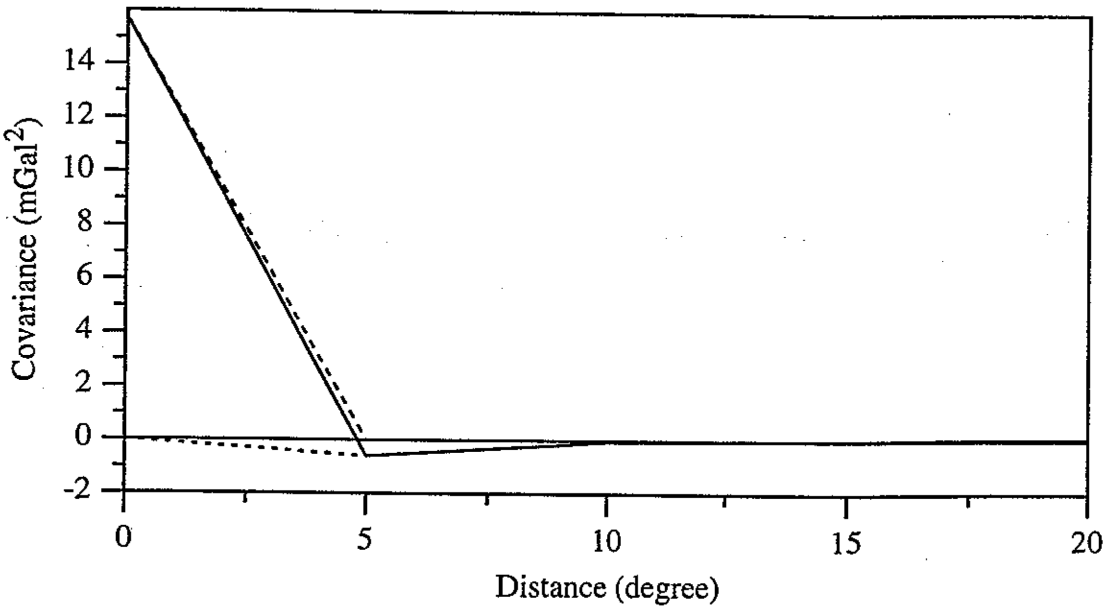


Fig. 7. Covariance functions of the global mean 5^0 gravity anomalies reduced to OSU91A1F to degree 72.

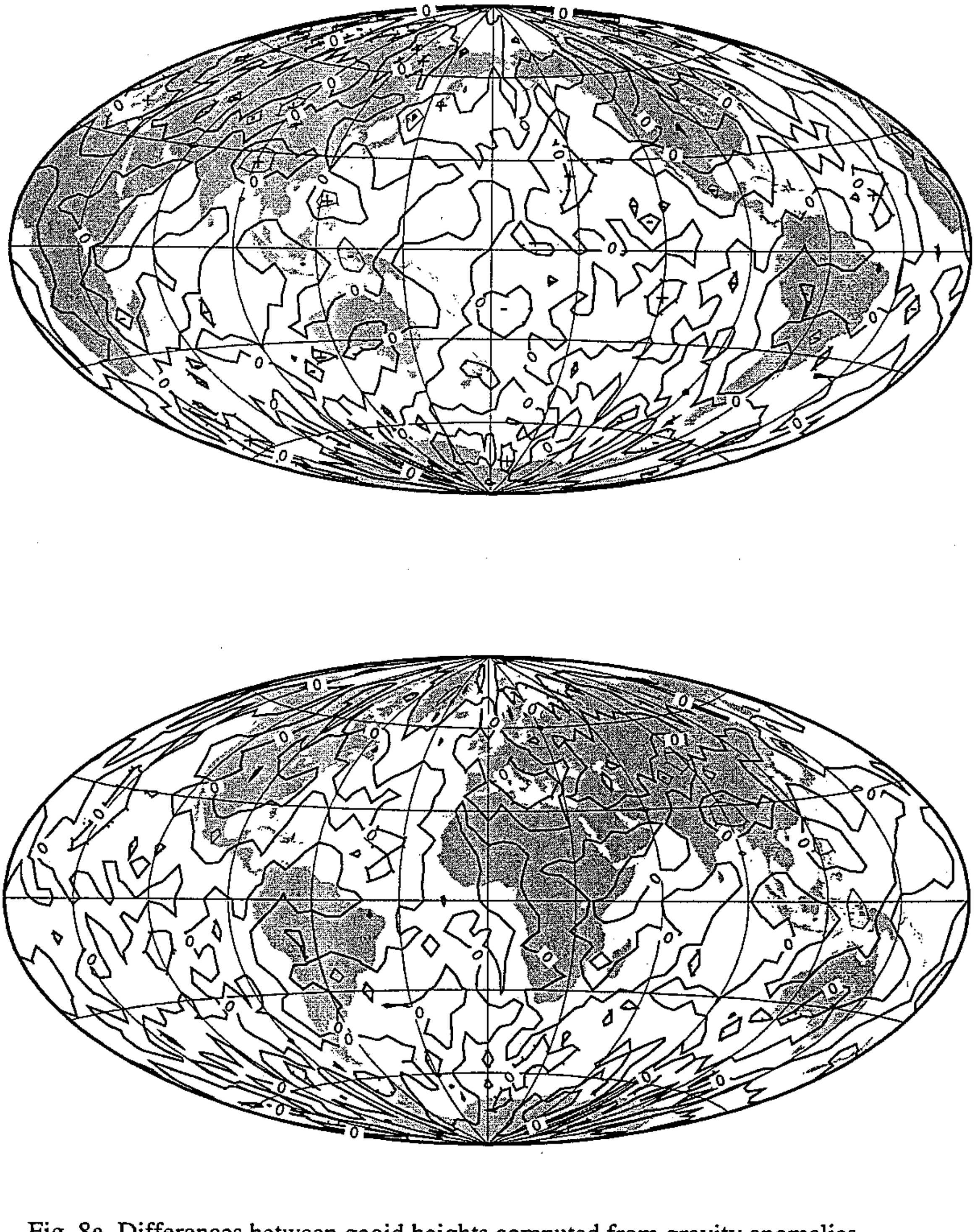


Fig. 8a. Differences between geoid heights computed from gravity anomalies, using full and finite covariance functions. Contour interval=0.5 m.

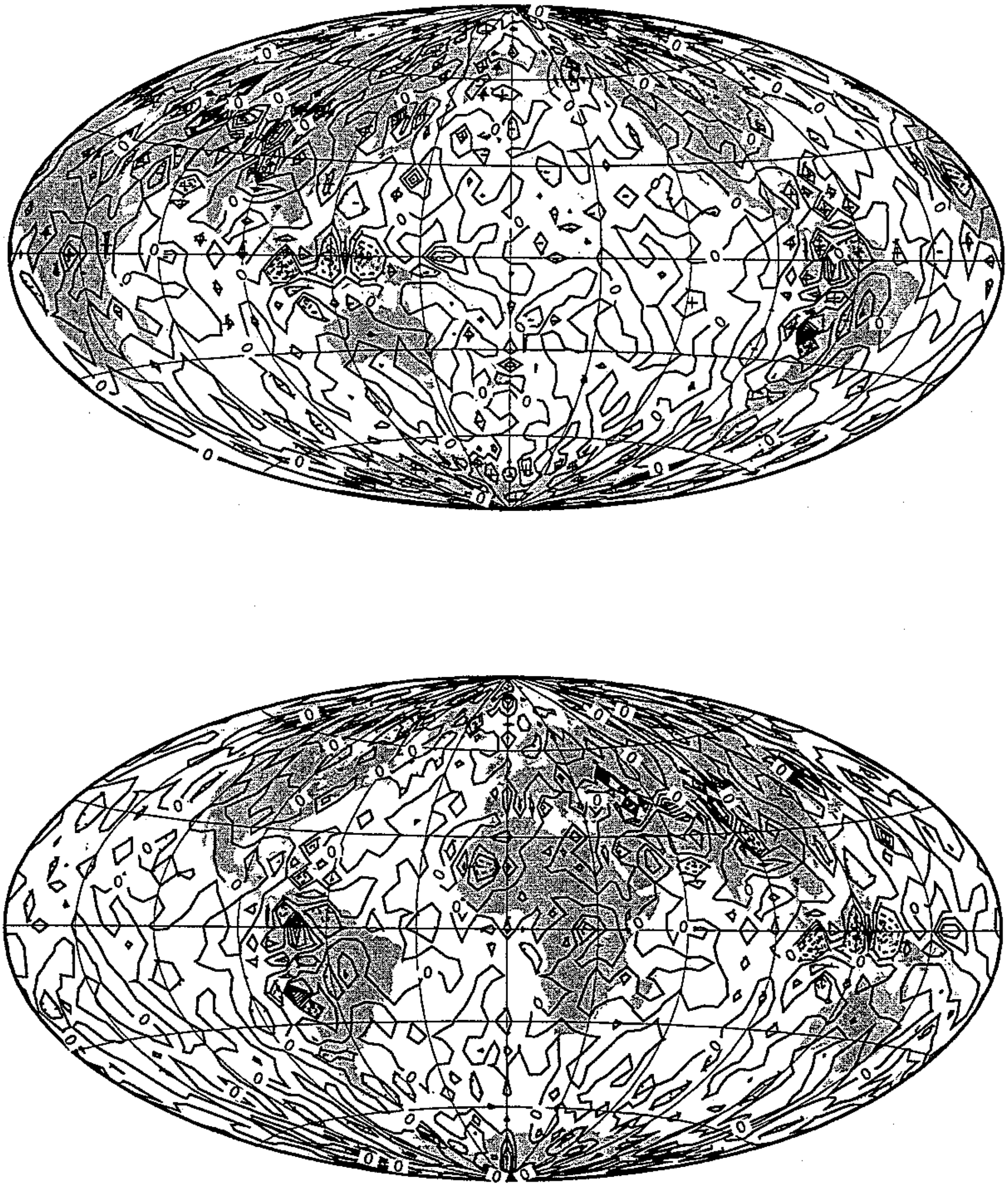


Fig. 8b. Differences between gravity anomalies computed from geoid heights, using full and finite covariance functions. Contour interval=1 mGal.